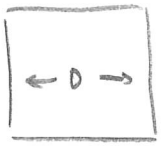
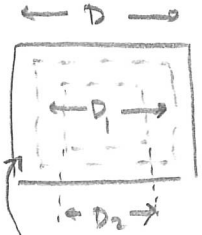
Flow Rate in a constant velocity field



$A = D^2$

$$Q = u \cdot A = u \cdot D^2$$

Flow Rate in a non-constant velocity field.



Assume we have a small slice where the velocity is constant.

$$Q_1 = v_x(y = D_1) \cdot (D^2 - D_1^2) = v_x(D_1)(A - A_1)$$

$$Q_2 = v_x(y = D_2)(D_1^2 - D_2^2) = v_x(D_2)(A_1 - A_2)$$

$$\vdots \quad \text{and so on}$$

$$Q_n = v_x(D_n)(A_{n-1} - A_n) = v_x(D_n)\,\Delta A$$

So, add them all up to get the total:

$$Q_{total} = Q_1 + Q_2 + \cdots + Q_n$$

$$= v_x(D_1)\,\Delta A + v_x(D_2)\,\Delta A + \cdots + v_x(D_n)\,\Delta A$$

$$= \sum_{i=1}^{n} v_x(D_i)\,\Delta A$$

Take the limit as $\Delta A \to 0$

$$\boxed{Q = \int v_x \, dA}$$

why could we use the average?

$$u_{avg} = \frac{1}{A}\int v_x \, dA \quad \leftarrow \text{definition of average}$$

$$u_{avg} \cdot A = \int v_x \, dA = Q$$