Homework 6

Ch En 374 – Fluid Mechanics

Due date: 25 Oct. 2019

Survey Question

Please report how long it took you to complete this assignment (in hours) in the "Notes" section when you turn your assignment in on Learning Suite.

Practice Problems

1. [Lecture 17 – Dynamics II: Navier-Stokes]. For each equation, write the name of the equation, any relevant restrictions to when it applies, and what each term in the equation represents physically.

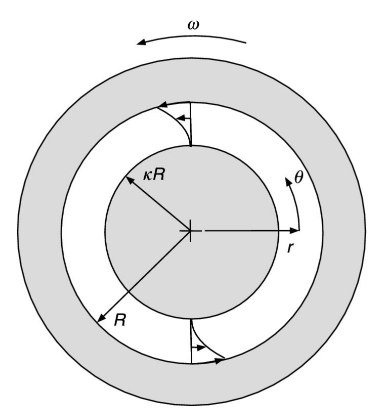
   (a) $\rho \dfrac{D\boldsymbol{v}}{Dt} = \rho \boldsymbol{g} + \nabla \cdot \boldsymbol{\sigma}$

   (b) $\boldsymbol{\tau} = 2\mu \boldsymbol{\Gamma}$

   (c) $\rho \left(\dfrac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} \right) = \rho \boldsymbol{g} - \nabla P + \mu \nabla^2 \boldsymbol{v}$

   (d) Additionally, consider an airplane flying through the sky at a constant velocity, U. Discuss the velocity boundary conditions on the air adjacent to the surface of the airplane from the perspective of an observer moving with the airplane.

2. [Lecture 18 – Unidirectional Flow]. In a Couette viscometer the liquid to be studied fills the annular space between two cylinders, as shown in the figure to the right. The inner and outer radii are κR and R, respectively. The viscosity is determined by measuring the torque required to keep the inner cylinder stationary when the outer one is rotated at a constant angular velocity ω. The cylinders are long enough that the end effects associated with the top and bottom can be neglected and it can be assumed that $\boldsymbol{v}$ and $\mathcal{P}$ are independent of z as well as of θ. Show that a unidirectional velocity field with $v_\theta = v_\theta(r)$ is consistent with continuity and determine $v_\theta(r)$.



3. [Lecture 19 – Non-Newtonian Pipe Flow]. In class, we showed that a power-law fluid flowing in a pipe has a velocity profile

$$v_z(r) = \frac{n}{n+1} \left(\frac{R^{n+1} |\Delta \mathcal{P}|}{2mL} \right)^{1/n} \left[1 - \left(\frac{r}{R} \right)^{(n+1)/n} \right] \tag{1}$$

   where R is the pipe radius, m and n are coefficients in the power-law constiutive model, L is the pipe length, and $|\Delta \mathcal{P}|$ is the magnitude of the pressure drop.

   (a) Calculate the average velocity U.

(b) Calculate the wall shear stress τ_w in terms of U, R, m and n. We have seen this expression before! We used it in HW 3 problem 2 to find n and m from a set of pressure drop data.

(c) Combine the expressions you get from (a) and (b) to find the friction factor of a power-law fluid in terms of the average velocity U, R, m, n and the density ρ.

(d) Using the expression for the friction factor you found in part (a) and the fact that $f = 16/\mathrm{Re}_{\mathrm{PL}}$ for laminar flow of a power-law fluid, find an expression for $\mathrm{Re}_{\mathrm{PL}}$.

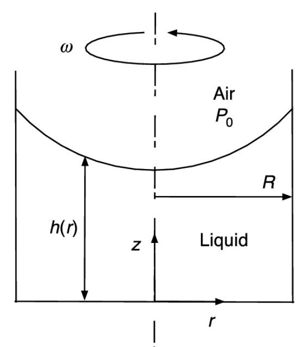
Challenge Problems

4. In this problem we are going to explore the velocity field of a power-law fluid flowing in a pipe.

    (a) Make Eq. 1 from problem 3 dimensionless using R as the length scale and U as the velocity scale.

    (b) Use Python to plot the 1D dimensionless velocity field v_z/U versus r/R for $n = 0.2$, 1.0 and 5.0. Does the profile become more sharply peaked in the middle or more blunt as n increases?

5. Suppose that an open container of radius R is filled with liquid to an initial height h_0. It is then rotated at a constant angular velocity ω as shown in the figure to the right. After an initial transient, the liquid rotates as if it were a rigid body. Assuming that the air pressure is constant at P_0 and that the surface tension and viscous stresses are negligible, determine the liquid pressure $P(r, z)$. *Hint: Because it is a rigid body, we know that the velocity of the fluid is unidirectional with velocity $v_\theta(r) = \omega r$.*



6. Consider pressure-driven flow in an annular channel. The inner and outer radii are the same as the figure from problem 2, but now both cylinders are stationary and there is a mean fluid velocity U in the z direction (into the page). The conduit length is L. Solve for $v_z(r)$ in terms of the pressure drop. It is convenient to lump the viscosity and pressure together as $B = |\Delta\mathcal{P}|/(\mu L)$ and write the final result in terms of a dimensionless radial coordinate $\eta = r/R$.

Bonus Problem (no points). Derive the Navier-Stokes equation in Cylindrical coordinates by making a momentum balance on the cylindrical shell shown on the left. *If you bring me the solution, I will buy you a King-Size Candy Bar of your choice.*