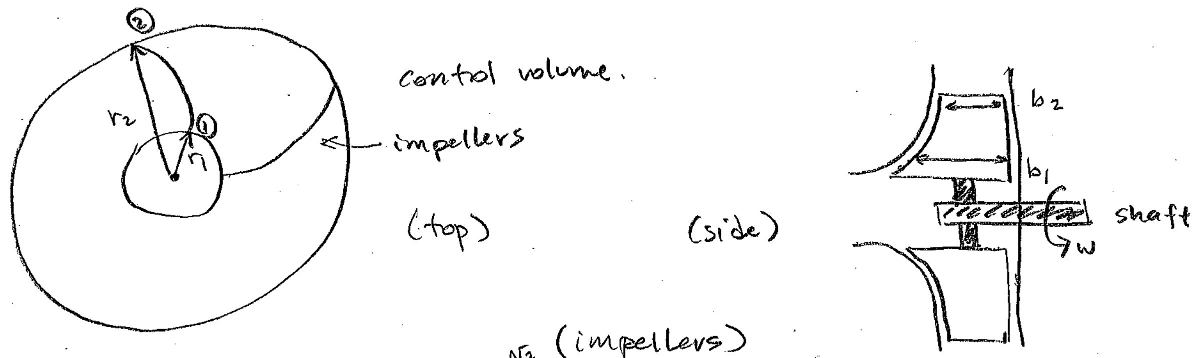
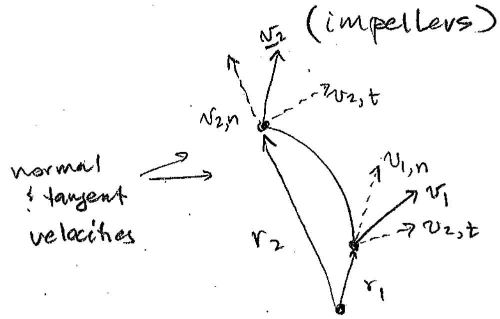
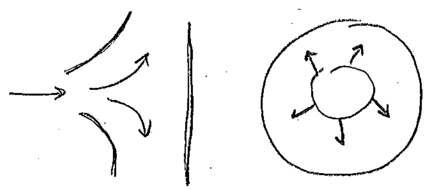
Pump curve for an ideal centrifugal pump.





A. mass balance



$$Q = 2\pi r_1 b_1 v_{1,n} = 2\pi r_2 b_2 v_{2,n}$$

known

knowns, turbine specs.

(radial velocity is carrying fluid out of turbine)

* solve for $v_{1,n}$ & $v_{2,n}$

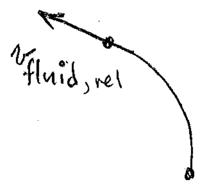
$$v_{i,n} = \frac{Q}{2\pi r_i b_i} \quad \text{(A)}$$

B. Velocities

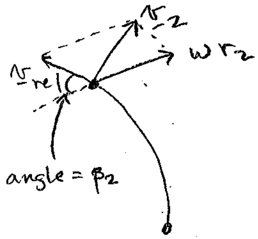
* The normal velocity is fixed by the volumetric flow rate, and by turbine specs.

* we need to know a little bit more to determine what the rest of the velocities are.

* Assume that the relative velocity of the fluid is tangential to the impeller in a rotating frame.



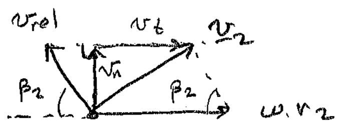
(rotating frame w/ impeller)



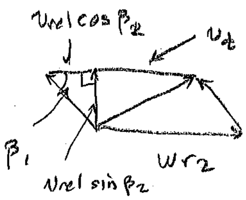
(fixed frame.

v_2 is the velocity of the fluid in the fixed frame (the one we want)

Can we relate ωr_2, v_2 to $v_{2,n}$ & $v_{2,t}$?



* This is just trigonometry:



$$v_{rel} \cos \beta_2 + v_t = \omega r_2$$

$$v_{rel} \sin \beta_2 = v_n$$

$$v_{rel} = \frac{v_n}{\sin \beta_2}$$

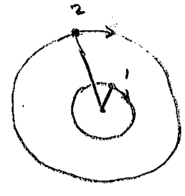
$$\frac{v_n}{\sin \beta_2} (\cos \beta_2) + v_t = \omega r_2$$

$$v_t = \omega r_2 - \frac{v_n}{\tan \beta_2} \qquad \text{(B)}$$

C. Conservation of angular momentum.

$$T = \dot{m} \left(r_2 v_{2,t} - r_1 v_{1,t} \right)$$

torque — angular momentum "out" — angular momentum "in"



"Euler's turbine Equation"

* Assume that $\underline{v_{1,t} = 0}$

* This amounts to an assumption that the impeller β_1 is such that all flow is normal. $(\beta_1 = \pi/2 = 90°)$

$$T = \dot{m}\,(r_2 v_{2,t}) \quad \text{C}$$

D. Find power.

$$E_m = \eta_p \omega T \qquad \text{*assume ideal}, \ \eta_p = 1$$

$$= \omega T \longrightarrow \text{use C}$$

$$= \omega \dot{m}\,(r_2 v_{2,t}) \longrightarrow \text{use B}$$

$$= \omega \dot{m}\left[r_2 \left(\omega r_2 - \frac{v_n}{\tan\beta_2} \right)\right] \longrightarrow \text{use A}$$

$$= \omega \dot{m}\left[r_2(\omega r_2) - \frac{r_2}{\tan\beta_2}\,\frac{Q}{2\pi r_2 b_2} \right]$$

$$\frac{E_m}{\dot{m}} = (\omega r_2)^2 - \frac{\omega Q}{2\pi b_2 \tan\beta_2}$$

$$\boxed{h_p = \frac{(\omega r_2)^2}{g} - \frac{\omega Q}{2\pi b_2 \tan\beta_2\, g}}$$

* linear in Q (decreases as Q gets big).

- $h_{p,max}$ when $Q = 0$, $h_p = \dfrac{(\omega r_2)^2}{g}$.

recall kinetic energy of a simple pendulum

$$\frac{E_k}{m} = \frac{1}{2}\omega^2 r^2$$

- $h_p = 0$ when Q is max,

$$Q_{max} = \omega r_2^2\,(2\pi b_2 \tan\beta_2)$$

* This gets even easier if we define:

$$h_{max} = \frac{\omega^2 r_2^2}{g}$$

and then non-dimensionalize:

$$C_H = \frac{h_p}{h_{max}} \qquad C_Q = \frac{Q}{Q_{max}}$$

$$C_H h_{max} = \underbrace{\frac{(\omega r_2)^2}{g}}_{h_{max}} - \frac{\omega C_Q Q_{max}}{2\pi b_2 \tan\beta_2 g}$$

$$C_H h_{max} = h_{max} - \frac{\omega}{2\pi b_2 \tan\beta_2 g} C_Q Q_{max}$$

$$C_H = 1 - \underbrace{\frac{\omega}{2\pi b_2 \tan\beta_2 g} \frac{Q_{max}}{h_{max}}}_{\frac{\cancel{\omega}}{\cancel{2\pi b_2 \tan\beta_2 g}} \frac{\omega r_2^2 (\cancel{2\pi b_2 \tan\beta_2})}{\omega^2 r^2 / \cancel{g}} = 1} C_Q$$

$$\boxed{C_H = 1 - C_Q} \; !$$

or

$$\frac{h_p}{h_{max}} = 1 - \frac{Q}{Q_{max}}$$

$$\boxed{h_p = h_{max} - \frac{h_{max}}{Q_{max}} Q}$$