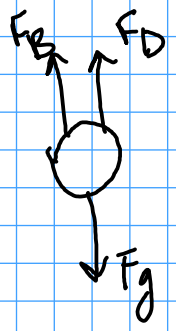
Lecture 11 – Supplemental Example on Terminal Velocity

3 ways to pick C_D

- nasty $C_D = C_D(Re)$ ☹
- Guess one ☺ → check Re
- "smart" Guess



$$\underbrace{\rho g V}_{F_B} + \underbrace{\frac{1}{2}\rho u^2 C_D A_\perp}_{F_D} - \underbrace{\rho_0 g V}_{F_G} = 0$$

$$\frac{1}{2}\rho u^2 C_D A_\perp = \rho_0 g V - \rho g V$$

$$u^2 C_D = \frac{g V (\rho_0 - \rho)}{\frac{1}{2}\rho A_\perp}$$

$u^2 C_D$ ⤳ depends on u i.e. Re

$$= \frac{2gV}{A_\perp}\left(\frac{\rho_0 - \rho}{\rho}\right)$$

$C_D = C_D(Re)$

$Re = \frac{\rho u D}{\mu}$

$V = \pi D^3/6$

$A_\perp = \pi D^2/4$

$$= \frac{2g\,\pi D^3/6}{\pi D^2/4}\left(\frac{\rho_0 - \rho}{\rho}\right)$$

$$u^2 C_D = \frac{4}{3} g D \left(\frac{\rho_0 - \rho}{\rho}\right) \times \frac{\rho^2 D^2}{\mu^2}$$

$\mu/\rho = \nu$

$$\frac{\rho^2 u^2 D^2}{\mu^2} C_D = \frac{4}{3} g D^3 \left(\frac{\rho_0 - \rho}{\rho}\right)\frac{\rho^2}{\mu^2}$$

$\frac{\rho^2 u^2 D^2}{\mu^2} = Re^2$

$$Re^2 C_D = \frac{4}{3}\frac{gD^3}{\nu^2}\left(\frac{\rho_0-\rho}{\rho}\right)$$

Ar : Archimedes #

$Re^2 C_D$ = big $\Rightarrow$ Re is big

$Re^2 C_D$ = small $\Rightarrow$ Re is small

$$Ar = \frac{gD^3}{\nu^2}\left(\frac{\rho_0-\rho}{\rho}\right)$$

$\rho = 1.2\ kg/m^3$ (air)

$\rho_0 = 630\ kg/m^3$

$D = 0.076\ m$

$\nu_{air} = 1.6 \times 10^{-5}\ m^2/s$

$\approx 10^{10}$

$Re^2 \sim 10^{10}$

$Re \sim 10^5$

$$Re^2 C_D = \frac{4}{3}(9.81 \times 10^2)$$

$$= 1.17 \times 10^{10}$$

$Re^2 C_D$

$$Re = \sqrt{\frac{1.17 \times 10^{10}}{0.445}} = 1.62 \times 10^5$$

0.445

$$u = 34\ m/s$$