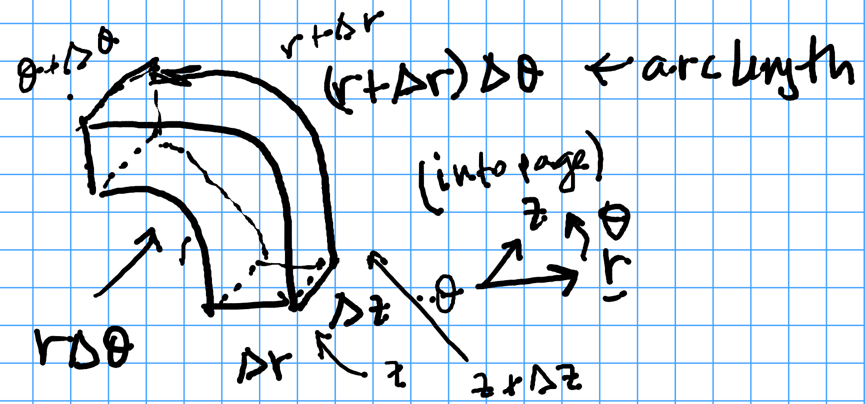
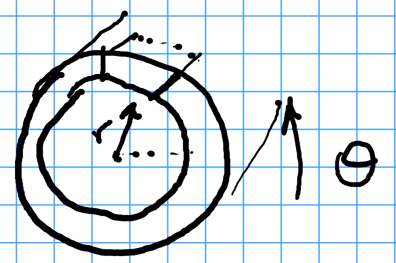
# Lecture 15 - Example Shell Balance in Cylindrical Coordinates




$V_{cyl} = \pi R^2 H$

$= \frac{2\pi}{2} R^2 H$

$= \frac{\theta}{2} R^2 H$

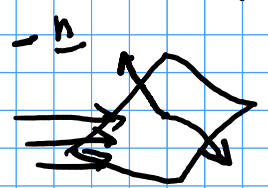
IOGA $\Rightarrow$ in - out + $\cancel{\text{gen}}^{\,0}$ = accum (balance)

mass balance

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm}{dt}$$

$\dot{m} = \rho Q = \underbrace{\rho u}_{\text{flux}} A$

$$\dot{m}_{in} - \dot{m}_{out} = \rho \underline{v} \cdot (-\underline{n} A)$$




## (a) Accumulation term

$$\frac{dm}{dt} = \frac{d}{dt}(\rho \Delta V) = \text{big} - \text{little}$$

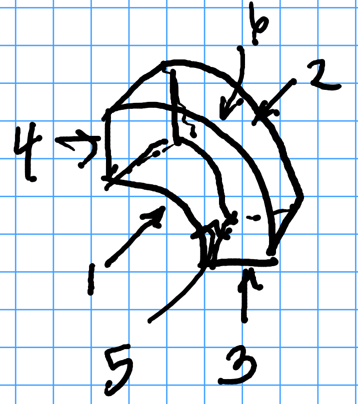
$$= \frac{d}{dt}\left(\rho\left[\frac{\Delta\theta}{2}(r+\Delta r)^2 \Delta z\right] - \rho\left[\frac{\Delta\theta}{2}(r)^2 \Delta z\right]\right)$$

$$= \frac{d}{dt}\left(\rho\left[\frac{\Delta\theta}{2}(\cancel{r^2} + 2r\Delta r + \underbrace{\cancel{\Delta r^2}}_{\Delta r \to 0})\Delta z - \frac{\Delta\theta}{2}\cancel{r^2 \Delta z}\right]\right)$$

$$= \frac{d}{dt}\left(\rho \frac{\Delta\theta}{\cancel{2}} \cdot \cancel{2} r \Delta r \Delta z\right)$$

$$= \frac{d}{dt}\left(\rho \Delta\theta \, r \Delta r \Delta z\right)$$

(b)




1,2 : $\pm \underline{e}_r$

3,4 : $\pm \underline{e}_\theta$

5,6 : $\pm \underline{e}_z$

} you can do this!

$$\rho \underline{v} \cdot (-\underline{n} A)$$

(1) $\rho \underline{v} \cdot \left(-(-\underline{e}_r)\, r\Delta\theta\Delta z\right) = \rho \overbrace{\underline{v} \cdot \underline{e}_r}^{v_r} r\Delta\theta\Delta z$

$= \rho v_r r \Delta\theta \Delta z$

(2) $\rho \underline{v} \cdot \left(-\underline{e}_r (r+\Delta r)\Delta\theta \cdot \Delta z\right)$

$= -\rho v_r (r+\Delta r)\Delta\theta \cdot \Delta z$

$$\frac{d}{dt}\left(\frac{\rho\,\Delta\theta\, r\,\Delta r\,\Delta z}{r\Delta r\Delta\theta\Delta z}\right) = \frac{\rho v_r\, r\,\Delta\theta\Delta z - \rho v_r (r+\Delta r)\Delta\theta\Delta z}{r\Delta r\Delta\theta\Delta z} + \text{o.t.} \quad , \theta, z$$

$$\frac{d}{dt}(\rho) = \frac{\rho v_r r - \rho v_r (r+\Delta r)}{r\Delta r} + \text{o.t.}$$

$$f(r) = \rho v_r r \qquad f(r) = \rho v_r r$$

$$f(r+\Delta r) = \rho v_r (r+\Delta r)$$

$$\lim_{\Delta r \to 0} \frac{f(r+\Delta r) - f(r)}{\Delta r} = \frac{\partial f}{\partial r}$$

$$\frac{\partial \rho}{\partial t} = \frac{1}{r}\left(-\frac{\partial}{\partial r}(\rho v_r r)\right)$$