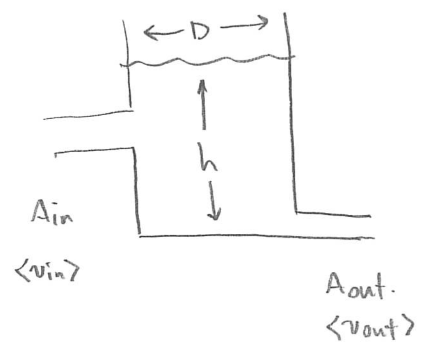
IV. Examples

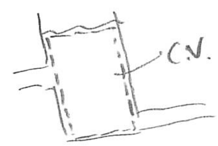
A. Determine the height of fluid in a tank that is draining



- what is $h(t)$?
- $h(t=0) = h_o$

(1) Pick control volume.

- Try to find one where the control surface is perpendicular to the velocity.



(2) write Balance & simplify

$$\frac{d}{dt}\int_{V(t)} \rho \, dV = -\int_{S(t)} \rho \, \underline{n} \cdot (\underline{v} - \underline{u}) \, dS$$

- constant density
- discrete inlets & outlets
- C.V. is not fixed, and is not steady

$$\frac{dm_{cv}}{dt} = \rho A_{in} \langle v_{in} \rangle - \rho A_{out} \langle v_{out} \rangle$$

$$m_{cv} = \rho \frac{\pi D^2}{4} h \quad \Rightarrow \quad \frac{dm_{cv}}{dt} = \rho \frac{d}{dt}\left(\frac{\pi D^2}{4} h\right) = \rho \frac{\pi D^2}{4} \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{\pi D^2}\left(A_{in}\langle v_{in}\rangle - A_{out}\langle v_{out}\rangle\right), \quad h(0) = h_o$$

(3) Solve the math problem

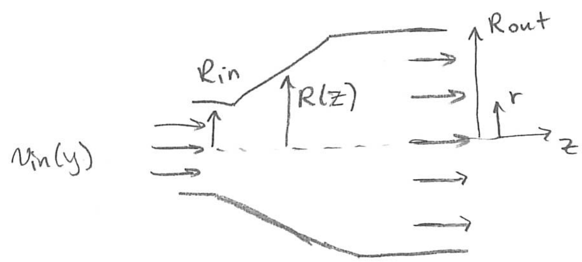
*if v_{in} & v_{out} are not functions of time, this is an easy ODE

$$h(t) = \frac{4t}{\pi D^2}\left(\underbrace{A_{in}\langle v_{in}\rangle}_{Q_{in}} - \underbrace{A_{out}\langle v_{out}\rangle}_{Q_{out}} \right) + C$$

$$h(0) = h_0 = C$$

$$\boxed{h(t) = h_0 + \frac{4(Q_{in} - Q_{out})}{\pi D^2}\, t}$$

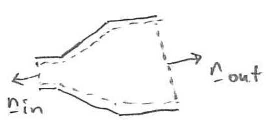
B. Diversing Channel



• given $v_{in}(r)$ can you determine $\langle v_{out}\rangle$?

$$v_{in}(y) = v_0\left(1 - \frac{r}{R_{in}}\right)^{1/7}$$ (turbulent flow)

(1) Draw Control volume:



(2) Write Balance & Assumptions

- constant density
- steady
- fixed control volume
- discrete inlet & outlet

$$\frac{d}{dt}\int_{V(t)} \rho \, dV = -\int_{S(t)} \rho(\underline{v} - \underline{u}) \cdot \underline{n} \, dS$$

$$\downarrow$$

$$0 = \rho \int_{in} v_{in}(r)\, dS - \rho \int_{out} v_{out}\, dS$$

$$\int_{in} v_{in}\, dS = \int_{out} v_{out}\, dS$$

(3) solve the math problem

$$2\pi \int_0^{R_{in}} v_0 \left(1-\frac{r}{R_{in}}\right) r\,dr = \pi R_{out}^2 \langle v_{out} \rangle$$

$$\langle v_{out} \rangle = \frac{2 v_0}{R_{out}^2} \int_0^{R_{in}} \left(1-\frac{r}{R_{in}}\right)^{1/7} r\,dr$$

$\eta = 1 - r/R_{in}$ $\quad d\eta = -\frac{1}{R_{in}} dr$

$r = R_{in}(1-\eta)$ $\quad \eta(0) = 1,\ \eta(R_{in}) = 0$

$$= \frac{2 v_0}{R_{out}^2} \int_0^1 \eta^{1/7} (1-\eta) R_{in}^2\, d\eta$$

$$= 2 \frac{v_0 R_{in}^2}{R_{out}^2} \underbrace{\int_0^1 \eta^{1/7} (1-\eta)\, d\eta}_{49/120}$$

$$\boxed{\langle v_{out} \rangle = \frac{49}{60} \left(\frac{R_{in}}{R_{out}}\right)^2 v_0}$$

$\langle v_{in} \rangle = \frac{49}{60} v_0$