Lec 8 - Pipe Flow II Supplement

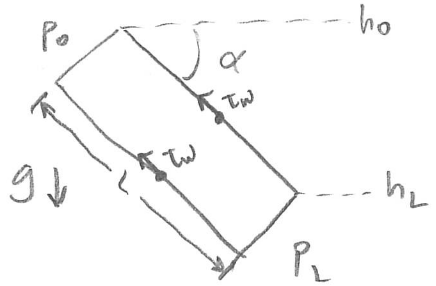
Derivation of Pressure drop vs. τ_w with

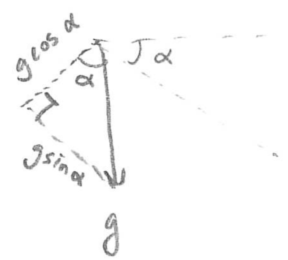
- Height change
- Arbitrary cross section (constant down length of pipe)

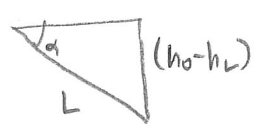
A : cross-sectional area

C : circumfrence / perimeter (already used P)

Side view, on an incline







$\sin\alpha = \frac{h_o - h_L}{L}$

Force Balance:

Force from pressure @ $x=0$: $P_o \cdot A$

Force from pressure @ $x=L$: $-P_L A$

Force on walls from shear : $-\tau_w C L$

Force from gravity : $mg \sin\alpha = mg \frac{h_o - h_L}{L}$

Putting it all together:

$$P_0 A - P_L A - \tau_w C L + mg\,\frac{h_0 - h_L}{L} = 0 \quad \leftarrow m = \rho V = \rho A L \text{ (mass of fluid)}$$

$$(P_0 - P_L) A - \tau_w C L + \rho A \cancel{L} g\,\frac{h_0 - h_L}{\cancel{L}} = 0$$

$$(P_0 - P_L) A + (h_0 - h_L)\rho g A = \tau_w C L$$

$$\tau_w = \frac{A}{C}\,\frac{P_0 - P_L + \rho g (h_0 - h_L)}{L}$$

$$P_L - P_0 = \Delta P$$

$$h_L - h_0 = \Delta h$$

$$\boxed{\tau_w = -\frac{A}{C}\,\frac{\Delta P + \rho g \Delta h}{L}}$$

"Dynamic Pressure"

$$\mathcal{P}_i = P_i + \rho g h_i$$

$$\downarrow$$

$$\Delta \mathcal{P} = \Delta P + \rho g \Delta h$$

$$\boxed{\tau_w = -\frac{\Delta \mathcal{P}}{L}\,\frac{D_H}{4}}$$

$$D_H = 4\frac{A}{C} \quad \text{"Hydraulic Diameter"}$$

$$\uparrow$$

when circle:

$$A = \pi D^2/4 \quad C = \pi D$$

$$D_H = \cancel{4}\cdot\frac{\cancel{\pi} D^{\cancel{2}}}{\cancel{4}}\,\frac{1}{\cancel{\pi}\cancel{D}} = D \checkmark$$