# Course Concepts

Ch En 533: Transport Phenomena
Winter 2025

## I. Course Outcomes

A. Conservation Laws: Derive the equations of change for total mass, species mass, momentum, and energy from conservation principles for Cartesian or orthogonal curvilinear coordinate systems.

B. Dimensional Analysis: Apply the principles of dimensional analysis to the equations of change to generate terms containing dimensionless parameters such as Re, Pr, Sc, Gr, and Pe.

C. Physical Intuition: Understand the origin and physical meaning of terms in the equations of change and when and how to use appropriate simplifying assumptions for a given engineering problem.

D. Vectors and Tensors: Interpret and manipulate transport equations and corresponding solutions that are expressed using vector and tensor notation in Cartesian or orthogonal curvilinear coordinate systems.

E. Solutions to the Equations of Change: Generate analytic solutions to the equations of change using various techniques.

F. Transport Properties: Explain the molecular origins of observed transport properties and predict the transport properties from traditional theories.

G. Transport Applications: Become exposed to one or more advanced topics in transport phenomena including interfacial transport, fluid turbulence, multicomponent energy and mass transfer, electrolyte transport, and buoyancy-driven transport.

H. Innovation: Analyze the viability of proposed solutions to engineering problems in terms of transport processes. Apply knowledge of transport processes to generate new solutions to engineering problems.

## II. Unit 1: Math Review

### Lecture 1: Vector and Tensor Algebra (Deen §A.1-§A.3)

Things you should know

(1.D.1) Concept of a vector space, basis, coordinates

(1.D.2) Concept of scalars, vectors, and tensors and relation to basis (e.g. unit dyad for tensor)

(1.D.3) Gibbs notation and index notation

(1.D.4) Vector and tensor properties, special tensors

Calculations you should be able to do

(1.D.a) Vector and tensor operations: sums, products, transpose, etc.

(1.D.b) Proofs of vector and tensor algebraic identities

### Lecture 2: Coordinate Systems (Deen §A.6-§A.7)

Things you should know

(2.D.1) Definitions and properties of coordinate systems (e.g. homogeneous, curvilinear, orthonormal)

(2.D.2) Examples of coordinate systems (e.g. Cartesian, oblique, spherical)

Calculations you should be able to do

(2.D.a) Conversion between coordinate systems

(2.D.b) Derivation of basis vectors and scale factors in curvilinear coordinates

### Lecture 3: Vector and Tensor Calculus (Deen §A.4-§A.8)

Things you should know

(3.D.1) Meaning and proper use of expressions for vector/tensor derivatives and integrals in Cartesian and curvilinear coordinates

Calculations you should be able to do

(3.D.a) Evaluate vector/tensor derivatives and integrals in Cartesian and curvilinear coordinates

(3.D.b) Derive expressions for vector/tensor calculus in Cartesian and curvilinear coordinates

(3.D.c) Proofs of vector and tensor differential identities

### Lecture 4: Differential Equations (Deen §B.1-§B.5)

Things you should know

(4.E.1) Classify ordinary differential equations by linearity, order, homogeneity, and complexity of coefficients

(4.E.2) Identify Bessel's equations (regular, modified, spherical), equidimensional equations, Legendre's equation, and Gaussian integrals and be familiar with the properties of the corresponding special functions

(4.E.3) The differences between initial value problems and boundary value problems

Calculations you should be able to do

(4.E.a) Solve any first order linear ODE (separable or not)

(4.E.b) Solve homogeneous and non-homogeneous $n^{\text{th}}$ order ODEs with constant coefficients (with real, imaginary, or repeated roots)

(4.E.c) Use initial conditions and boundary conditions to find the particular solution to an ODE

# III. Unit 2: Transport Equations

## Lecture 5: Constitutive Equations (Deen §1.1-§1.3)

Things you should know

(5.A.1) Transport equations are made of balances and constitutive laws, and constitutive laws are made of "forces" and "fluxes."

(5.A.2) The mathematical expressions for diffusive fluxes: Newton's law of viscosity, Fourier's law of conduction, and Fick's law of diffusion

(5.A.3) Relationships between the mass and molar flux for species diffusion

(5.C.1) What the transport phenomena are: momentum, mass, energy

Calculations you should be able to do

(5.A.a) Elementary manipulations of the momentum, energy, and diffusive flux expressions (including mass and molar forms)

(5.A.b) Tensor/vector operations to obtain fluxes/stresses/forces

## Lecture 6: Transport Coefficients (Deen §1.4-§1.6)

Things you should know

(6.C.1) Meaning of viscosity, thermal conductivity, binary mutual diffusion coefficient, and the Prandtl (Pr), Schmidt (Sc) and Lewis (Le) numbers

(6.C.2) Intuition about transport coefficients: units, order-of-magnitude, pressure/temperature dependence

(6.F.1) Methods for estimating transport coefficients: experiments, simulations, corresponding states, kinetic theory (random walk model)

Calculations you should be able to do

(6.F.a) Estimate transport coefficients and related dimensionless ratios using intuition, tables, charts, and simple theories

## Lecture 7: Conservation Equations (Deen §2.1-§2.3)

Things you should know

(7.C.1) General integral and differential conservation equation: various forms and physical meaning of terms

(7.C.2) The continuity equation: various forms, common simplifications, and physical meaning of terms

(7.C.3) The definition and meaning of the material derivative

(7.C.4) Difference between the total flux, convective flux, and diffusive flux

Calculations you should be able to do

(7.A.a) Derive an integral or differential conservation equation for a scalar quantity (e.g. total mass)

(7.E.a) Manipulate general conservation equations and boundary conditions to solve simple transport problems

### Lecture 8: Conservation of Energy (Deen §2.4-§2.5)

Things you should know

(8.C.1) Forms of the internal energy balance equation, the meaning of its terms, and the underlying assumptions

(8.C.2) Physical meaning and mathematical expressions of common boundary conditions for heat transfer

Calculations you should be able to do

(8.A.a) Derive the equation of change for temperature using the general property balance using various assumptions

(8.E.a) Manipulate the equation of change for temperature to solve simple transport problems

### Lecture 9: Conservation of Species Mass (Deen §2.6-§2.8)

Things you should know

(9.C.1) Forms of the species mass balance equation (in mass or molar units), the meaning of its terms, and the underlying assumptions

(9.C.2) Physical meaning and mathematical expressions of common boundary conditions for mass transfer

Calculations you should be able to do

(9.A.a) Derive the equation of change for species mass using the general property balance using various assumptions

(9.E.a) Manipulate the equation of change for species mass to solve simple transport problems

## IV. Unit 3: Solutions of 1D Transport Equations

### Lecture 10: Solving 1D Transport Equations (Deen §3.1-§3.2)

Things you should know

(10.E.1) Motivation for generating analytical solutions to transport problems

(10.E.2) Examples: (i) temperature in a tube wall, (ii) electrically heated wire and (iii) irreversible homogeneous reaction in a liquid

Calculations you should be able to do

(10.E.a) Identify appropriate transport equations (energy/species mass) and boundary conditions in Cartesian or curvilinear coordinates for common physical situations (e.g. binary, incompressible, stationary, heterogeneous/homogeneous reactions, etc.)

(10.E.b) Solve 1D heat or binary mass transfer problems in Cartesian or curvilinear coordinates

### Lecture 11: Scaling (Deen §3.3)

Things you should know

(11.B.1) The purpose of scaling analyses (rational approximations, reduce number of parameters, physical insight)

(11.B.2) Common dimensionless numbers in heat/mass transfer (e.g. Da, Bi, etc.). What they are and what they mean physically.

Calculations you should be able to do

(11.B.a) Order of magnitude analysis and dimensional analysis of a heat/mass transport equation

(11.B.b) Interpret solutions of 1D heat/mass transport problems using scaling analysis techniques

### Lecture 12: Simplifying 1D Transport Equations (Deen §3.4-§3.5)

Things you should know

(12.C.1) Rationales for simplifying transport equations (symmetry, length scales, resistance scales, time scales)

(12.C.2) What the diffusive time scale is

(12.E.1) Example: Pseudosteady evaporation of a liquid column

Calculations you should be able to do

(12.C.a) Simplify a heat/mass transport equation using scaling analysis techniques

(12.E.a) Solve pseudosteady 1D heat or binary mass transfer problems in Cartesian or curvilinear coordinates

## V. Unit 4: Solutions of Multidimensional Transport Equations

### Lecture 13: The Similarity Method (Deen §4.1-§4.2)

Things you should know

(13.C.1) The concept of a penetration depth

(13.E.1) Dimensional analysis for infinite/semi-infinite domains

(13.E.2) Example: Diffusion into a semi-infinite domain

Calculations you should be able to do

(13.E.a) Use the similarity method to solve a diffusion equation in an infinite/semi-infinite domain

## Lecture 14: Perturbation Analysis (Deen §4.3-§4.5)

Things you should know

(14.E.1) The concept of an asymptotic series and asymptotic convergence

(14.E.2) Example: Heated wire with temperature-dependent conductivity

Calculations you should be able to do

(14.E.a) Using regular perturbation analysis to solve a nonlinear transport problem

## Lecture 15: Linear PDEs (Deen §5.1-§5.10)

Things you should know

(15.E.1) Properties of a linear PDE: uniqueness of solutions, superposition

(15.E.2) The vector–function analogy, what an inner product is, basis functions and orthogonality

(15.E.3) The forward and inverse FFT, i.e. Generalized Fourier Series and coefficients

Calculations you should be able to do

(15.E.a) Identify a PDE as linear/nonlinear, indentify differential operator and boundary conditions

(15.E.b) Calculate the basis functions for a linear operator with homogeneous boundary conditions (solve an eigenvalue problem)

(15.E.c) Calculate the Fourier coefficients for a function using an inner product

## Lecture 16: The FFT Method (Deen §5.1-§5.10)

Things you should know

(16.E.1) N/A

Calculations you should be able to do

(16.E.a) Solve a linear PDE using the FFT method in Cartesian, cylindrical, or spherical coordinates

# VI. Unit 5: Fluid Mechanics

## Lecture 17: The Navier Stokes Equation (Deen §6.1-§6.9)

Things you should know

(17.A.1) The Cauchy momentum equation and the Navier–Stokes equation, their relationship, and what the terms mean in each

(17.A.2) Newton's law of viscosity and its relation to the modes of motion in a fluid (translation, rotation, deformation, dilation)

(17.A.3) Be familiar with concepts such as streamfunctions and vorticity

(17.B.1) Dimensional analysis and simplifications of Navier-Stokes at low/high Re

Calculations you should be able to do

(17.A.a) Derive and perform dimensional analysis on Cauchy's momentum equation and the Navier–Stokes equation

(17.E.a) Compute quantities related to flow fields (deformation tensory, vorticity, & streamfunction)

## Lecture 18: Unidirectional Flow (Deen §7.1-§7.6)

Things you should know

(18.E.1) The difficulties solving Navier–Stokes problems and how unidirectional flow addresses them

(18.E.2) What problems can and cannot be solved via the unidirectional flow assumption

Calculations you should be able to do

(18.E.a) Solve unidirectional Navier-Stokes problems including Poiseuille flow and Couette flow

## Lecture 19: Creeping Flow (Deen §8.1-§8.5)

Things you should know

(19.C.1) Qualitative physical (no inertia, pseudosteady, reversible) and mathematical (linear) features of Stokes' flow.

(19.E.1) What the streamfunction is and what it means

Calculations you should be able to do

(19.E.a) Solve a bidirectional Stokes' flow problem using streamfunctions

### Lecture 20: Inviscid Flow (Deen §9.1-§9.3)

Things you should know

(20.C.1) Qualitative physical (inertia, obey's Bernoulli's equation) and mathematical (non-linear) features of inviscid and irrotational flow.

(20.E.1) Relationship between inviscid and irrotational flow and the definition of the velocity potential

Calculations you should be able to do

(20.E.a) Solve a bidirectional irrotational flow problem using streamfunctions or the velocity potential

### Lecture 21: Boundary Layer Theory (Deen §9.4-§9.5)

Things you should know

(21.C.1) What D'Alembert's paradox is and what it has to do with a singular perturbation problem.

(21.C.2) Dimensional analysis of the boundary layer equations

Calculations you should be able to do

(21.E.a) Derivation of Blasius' equation and subsequent calculations using values from the numerical solution.

### Lecture 22: Forced Convection (Deen §10.1-§10.7)

Things you should know

(22.X.1) Coming Soon

Calculations you should be able to do

(22.X.a) Coming Soon

## VII. Unit 6: Transport Project

The transport project aims to fulfill Course Outcomes G (Transport Applications) and H (Innovation).