# Midterm Exam

Ch En 533 – Transport Phenomena

Due: Oct. 11, 2022

## Instructions

- You can download a pdf of the exam from the Learning Suite assignment "Exam 1."
- You should submit a *single scanned pdf* of your exam answers to the Learning Suite assignment "Exam 2" before the Exam closes.
- Please name the file you submit using the following format: `Exam1_Lastname_Firstname.pdf`. You should double-check that you have submitted the correct file.
- You should start each problem on a separate piece of paper. Write neatly and clearly, and explain all assumptions and steps of your derivations. Box your answers. Unclear writing or answers will result in a loss of points.

## Rules

- There is no time limit for completing the exam.
- You **may** use your notes, your book, your previous homework, and a calculator.
- You **may** use computer math tools such as Wolfram Alpha, Mathematica, Python, etc.
- You **may** use scratch paper or write on your test. However, we will only accept your submitted materials for credit.
- You **may not** communicate with anyone but the instructor or TA about the exam until after the due date has passed. This includes (but is not limited to) students in the class and graduate students outside of the class.
- You **may not** use internet searches, internet fora (e.g. stack exchange), or similar to search or request help, partial solutions, or keys for the exam problems.
- If you have any questions about the above rules, contact Dr. Tree. Violations of the rules of the exam will result in loss of points, exam failure, or course failure.

On my honor, I signify that this exam represents my work, and that I have faithfully followed the above rules to the best of my ability.

**Please Sign Here** ________________________________________

## Contents

This exam contains:

- 4 free response questions

## Problem 1. Flow around a cylinder (30 pts)

Suppose that for an upcoming research project, you need to know the velocity field at high Reynolds number around a cylinder. You haven't read far enough in your transport book yet to learn how to solve this problem, but after some expert Googling, you find a possible solution on an old website from some professor in Australia. The website gives the solution as

$$v_r(r,\theta) = U\cos\theta\left[1-\left(\frac{R}{r}\right)^2\right]$$
$$v_\theta(r,\theta) = -U\sin\theta\left[1+\left(\frac{R}{r}\right)^2\right]$$
$$v_z = 0$$

where $v_r$, $v_\theta$, and $v_z$ are the velocities in the $r$-, $\theta$-, and $z$-directions respectively. $U$ is the velocity far away from the cylinder, and $R$ is the cylinder radius. You want to verify that the solution is correct, so you decide you had better at least verify that the solution conserves mass. Your labmate suggests that this means you should evaluate $\nabla \cdot \boldsymbol{v}$.

(a) Justify why and under what conditions $\nabla \cdot \boldsymbol{v}$ would demonstrate that the velocity field conserves mass. Use a combination of words (full sentences please) and/or short mathematical derivations to help make your point. Do not calculate $\nabla \cdot \boldsymbol{v}$ yet.

(b) Suppose that you left your transport book at home (gasp!), and you can't find a reliable formula for $\nabla \cdot \boldsymbol{v}$ on the internet. However, Wikipedia has the expressions

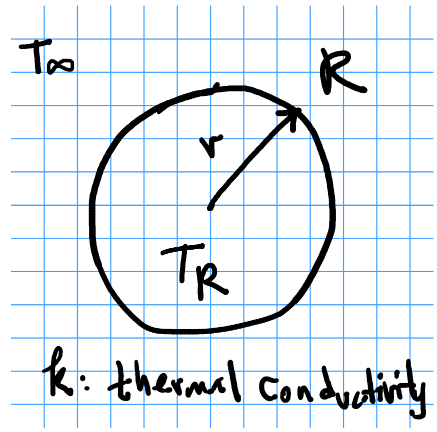
$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$
$$\boldsymbol{e}_r = (\cos\theta)\boldsymbol{e}_x + (\sin\theta)\boldsymbol{e}_y + (0)\boldsymbol{e}_z$$
$$\boldsymbol{e}_\theta = (-\sin\theta)\boldsymbol{e}_x + (\cos\theta)\boldsymbol{e}_y + (0)\boldsymbol{e}_z$$
$$\boldsymbol{e}_z = (0)\boldsymbol{e}_x + (0)\boldsymbol{e}_y + (1)\boldsymbol{e}_z$$
$$\nabla = \boldsymbol{e}_r\frac{\partial}{\partial r} + \boldsymbol{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} + \boldsymbol{e}_z\frac{\partial}{\partial z}$$

for cylindrical coordinates. Use these expressions to derive the formula for $\nabla \cdot \boldsymbol{v}$ in cylindrical coordinates. Please help us give you full credit by writing neatly and documenting the steps in your derivation. Again, do not calculate $\nabla \cdot \boldsymbol{v}$ yet, just find the formula.

(c) Evaluate $\nabla \cdot \boldsymbol{v}$ for the velocity field given above. Does it conserve mass?

## Problem 2. Heat conduction from a sphere (30 pts)

A heated sphere of radius $R$ is suspended in a large, motionless body of fluid with thermal conductivity $k$. It is desired to study the heat conduction in the fluid around the sphere in the absence of convection. Assume that the temperature of the sphere remains constant at $T_R$ and that the temperature of the bath is $T = T_\infty$ at $r = \infty$.




(a) Starting from the general differential balance, write a simplified differential equation and a set of boundary conditions that describe the steady state temperature $T$ of the surrounding fluid as a function of $r$, where $r = 0$ is the center of the sphere. State all assumptions you use while obtaining this differential equation.

(b) Solve for the temperature distribution.

(c) Perform an order-of-magnitude analysis to find the scale of the heat flux for the problem in parts (a)-(b). When convection is introduced to the problem, the heat flux is given instead by $q_{r,\text{conv}} = h(T_R - T_\infty)$, where $h$ is a heat transfer coefficient. Use the flux scale you just found to non-dimensionalize $q_{r,\text{conv}}$. Identify the physical meaning of the dimensionless group that appears. (It is not necessary to know the name of the dimensionless group.)

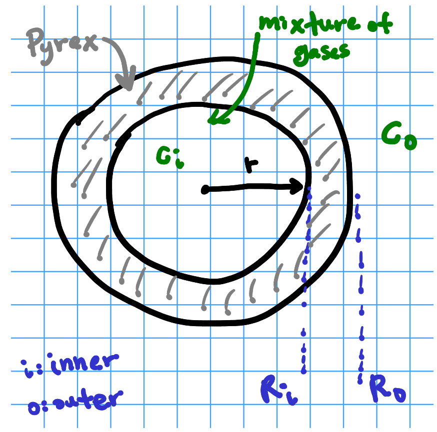
## Problem 3. Divergence Theorem (10 pts)

Prove the divergence theorem for second-order tensors by using the divergence theorem for vectors and letting the vector $\boldsymbol{v}$ be $\boldsymbol{T} \cdot \boldsymbol{b}$, where $\boldsymbol{T}$ is a second-order tensor and $\boldsymbol{b}$ is a constant vector (i.e. $\boldsymbol{b}$ is not a function of space or time).

## Problem 4. Helium Separation using Pyrex (30 pts)

Suppose you have a cylindrical reaction vessel made of Pyrex that contains a mixture of gases including helium. The diffusion coefficient of helium in Pyrex is relatively large. You want to calculate the time it will take for the helium to diffuse out of your vessel, to make sure it won't screw up your experiment.

The vessel geometry is described in the drawing with an inner diameter of $R_i$ and an outer diameter $R_o$. The concentration of helium inside the vessel at the beginning of your experiment is $c_i$. There is very fast air circulation outside the vessel, so you can assume that the helium concentration outside the vessel is a constant $c_o$.




(a) Starting from the general property balance, write a pseudosteady state differential equation (with boundary conditions) for the concentration of helium in the vessel walls. Clearly state and justify all assumptions you use in obtaining your simplified differential balance.

(b) Solve the differential balance to find the concentration profile of helium in the vessel wall.

(c) Use your solution in part (b) to find the "process time" that it will take for the helium to leak out of the vessel. Additionally, compare this time to the diffusion time to determine when your pseudosteady state assumption from part (a) is valid.