DIFFUSION FLAMES

CH EN 533 Transport Phenomena

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PAPER: Diffusion Flames

S.P.Burke and T.E.W. Schumann(1928)

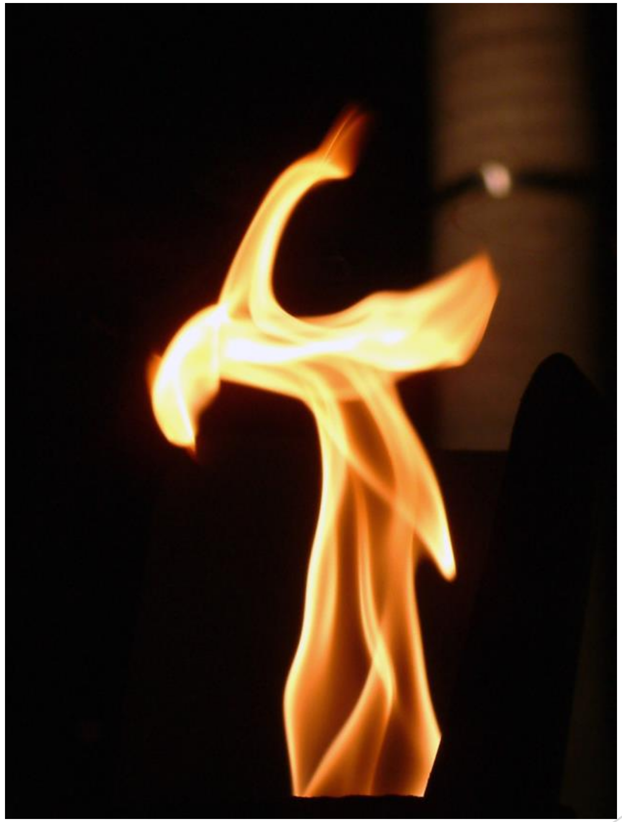
What is a Diffusion flame?

- In combustion, a diffusion flame is a flame in which the oxidizer and fuel are separated before burning.

Applications of Diffusion flames

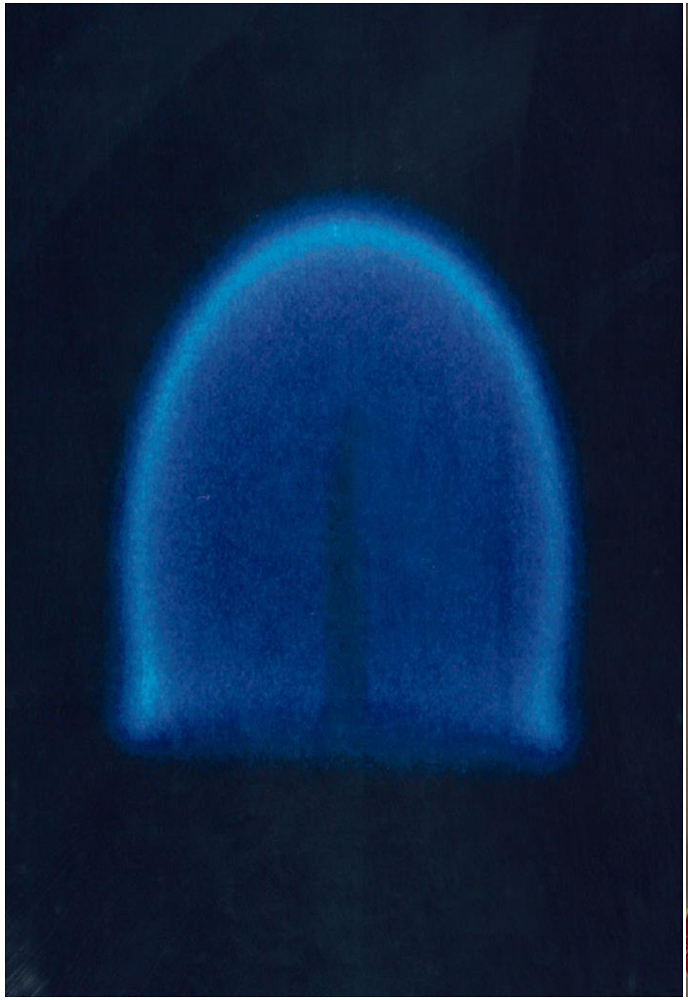
- Industrial burners in furnaces
- Flares

Examples of Diffusion Flames



https://en.wikipedia.org/wiki/Diffusion_flame

Examples of Diffusion Flames Cont…



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EXPERIMENTAL SETUP

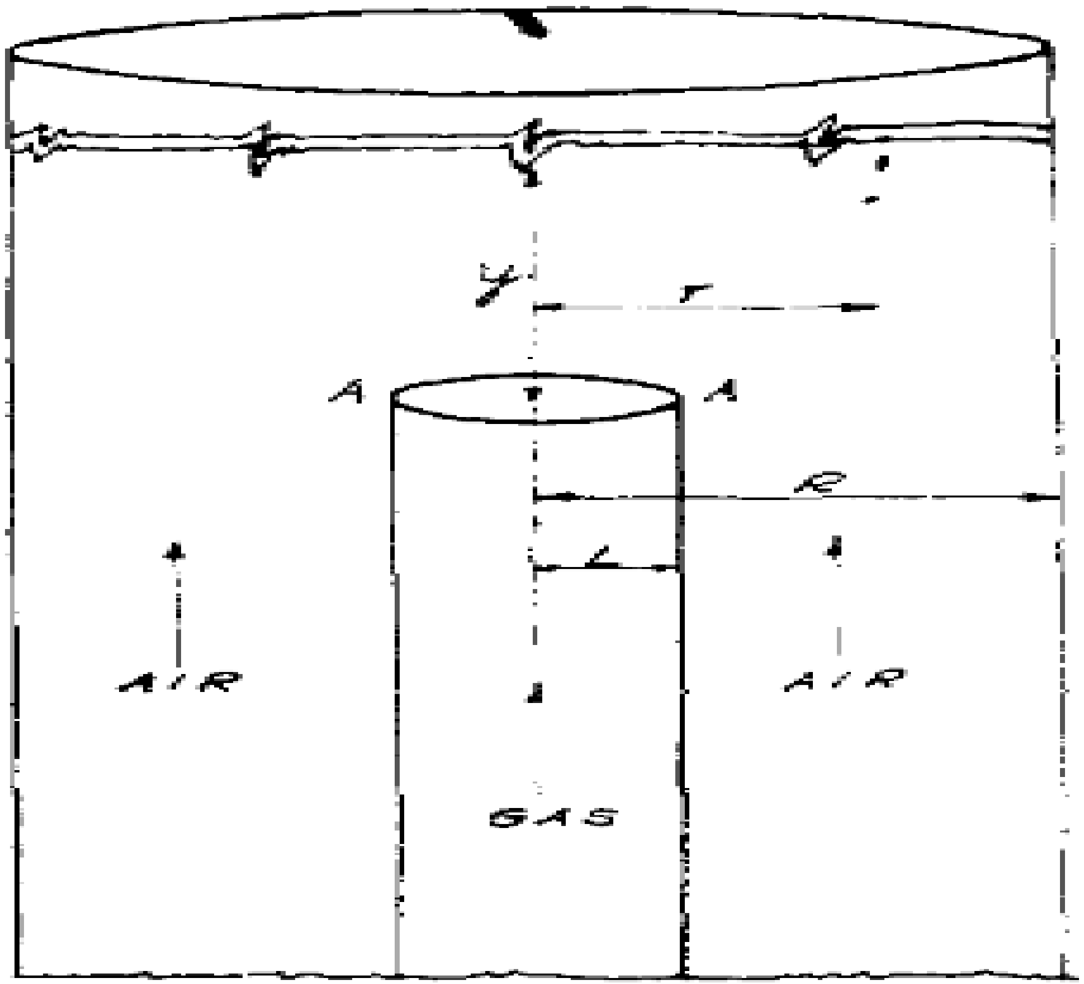


Figure 1—Diagram of Experimental Burner

OBJECTIVE

Flat Flames

Using the same notation as before and assuming that the combustible gas flows through a duct bounded by two parallel walls whose distance apart is $2L$, and that the air flows in an outer duct whose width is $2R$, we have the equation of diffusion

$$\frac{\delta C}{\delta y} = \frac{k}{v}\frac{\delta^2 C}{\delta r^2}$$

The solution of this equation fitting the boundary conditions is

$$C = C_0\frac{L}{R} - \frac{C_2}{i} + \frac{2C_0}{\pi}\sum^{\infty}\frac{1}{n}\sin\frac{n\pi L}{R}\cos\frac{n\pi r}{R}\,e^{-\frac{kn^2\pi^2 y}{vR^2}} \qquad (4)$$

Putting $C = 0$ and $r = x$, we have the equation for the flame front

$$\sum_{1}^{\infty}\frac{1}{n}\sin\frac{n\pi L}{R}\cos\frac{n\pi x}{R}\,e^{-\frac{kn^2\pi^2 y}{vR^2}} = \frac{\pi}{2}\left\{\frac{C_2}{iC_0} - \frac{L}{R}\right\} = E \qquad (5)$$

METHOD

- State assumptions
- State initial and boundary conditions
- State governing equation based on rectangular coordinate system
- Solve for steady state part
- Solve for transient part
- Solve for concentration

ASSUMPTIONS

- v_{gas} and v_{air} up the tube in the region of the flame is constant
- The coefficient of interdiffusion of the two gas streams is constant
- The interdiffusion is wholly flat
- Mixture of the two gas streams occurs by diffusion only

DEFINITIONS

- r = distance in the x-direction
- y = vertical distance above orifice of inner parallel plates
- k = coefficient of interdiffusion
- C_0 = initial concentration of combustible gas
- $C_2 = - C_2/i$ concentration of oxygen ,negative combustible gas
- i is the number of molecules of oxygen which combine with one molecule of combustible gas to effect complete combustion

INITIAL AND BOUNDARY CONDITIONS

- $C = C_0$ from r=0 to r=L at y=0 I.C
- $C = -C_2$ from r=L to r=R at y=0 I.C
- $\frac{dC}{dr} = 0$ when r=0 and r=R B.Cs

- Governing Equation

$$\frac{\partial C}{\partial t} + v_x \cancel{\frac{\partial C}{\partial x}} + v_y \cancel{\frac{\partial C}{\partial y}} + v_z \cancel{\frac{\partial C}{\partial z}} = k\left(\frac{\partial^2 C}{\partial x^2} + \cancel{\frac{\partial^2 C}{\partial y^2}} + \cancel{\frac{\partial^2 C}{\partial z^2}}\right) + \cancel{R_{vi}}$$

- since v is constant, therefore v = yt and let x = r

$$v\frac{\partial C}{\partial y} = k\frac{\partial^2 C}{\partial r^2}$$

$$\theta = \theta_{ss} + \theta_t$$

- Considering θ_{ss} part ; Let r = $\frac{L}{R}$, $\theta = C/ C_0$

Transforming ICs

$C_0 \theta_1 = C_0 \therefore \theta_1 = 1$, $C_0 \theta_2 = C_2 \therefore \theta_2 = - C_2/ iC_0$

$$\frac{\partial^2 \theta_{ss}}{\partial r^2} = 0$$

- Integrating twice :

$$\theta_{ss} = K_1 r + K_2 \, , \qquad K_1 = 1\, , K_2 = -\frac{C_2}{iC_o}$$

$$\boldsymbol{\theta_{ss} = \frac{L}{R} - \frac{C_2}{iC_o}}$$

Considering θ_t part using the FFT Method

- *Table 5-2 Case 4(page 168)*: $\phi_n = \sqrt{\frac{2}{R}} cos \frac{n\pi r}{R}$, n = 1,2,………………

$$\theta(r,y) = \sum_{n=1}^{\infty} C_n(y) \phi_n(r) dr$$

$$\frac{\partial C}{\partial y} = \frac{k}{v} \frac{\partial^2 C}{\partial r^2}$$

$$C_n(y) = \int_0^1 \theta(r,y)\ \phi_n(r) dr$$

$$C_n(y) = \int_0^1 \theta \sqrt{\frac{2}{R}} cos \frac{n\pi r}{R} dr$$

$$\frac{\partial \theta}{\partial y} = \frac{k}{v} \frac{\partial^2 \theta}{\partial r^2}$$

- LHS

$$\int_0^1 \frac{\partial\theta}{\partial y}\sqrt{\frac{2}{R}}\cos\frac{n\pi r}{R}dr$$

$$\frac{\partial}{\partial y}\int_0^1 \theta\sqrt{\frac{2}{R}}\cos\frac{n\pi r}{R}dr$$

$$\longrightarrow \frac{\partial C_n(y)}{\partial y}$$

- RHS

$$\int_0^1 \frac{k}{v}\frac{\partial^2\theta}{\partial r^2}\sqrt{\frac{2}{R}}\cos\frac{n\pi r}{R}dr$$

$$u = \sqrt{\frac{2}{R}}\cos\frac{n\pi r}{R} \qquad du = -\sqrt{\frac{2}{R}}\frac{n\pi}{R}\sin\frac{n\pi r}{R}$$

$$dv = \frac{\partial^2\theta}{\partial r^2} \qquad v = \frac{\partial\theta}{\partial r}$$

- $Applying\ Integration\ By\ Parts : \int udv = uv - \int vdu$

$$\sqrt{\frac{2}{R}} \cos\frac{n\pi r}{R}\frac{\partial\theta}{\partial r} \Big|_0^1 + \int_0^1 \frac{\partial\theta}{\partial r}\sqrt{\frac{2}{R}}\,\frac{n\pi}{R}\,\sin\frac{n\pi r}{R}dr$$

$$u = \sin\frac{n\pi r}{R} \qquad du = \frac{n\pi}{R}\cos\frac{n\pi r}{R}$$

$$dv = \frac{\partial\theta}{\partial r} \qquad v = \theta$$

$$\theta\sqrt{\frac{2}{R}}\frac{n\pi}{R}\sin\frac{n\pi r}{R} \Big|_{r=0}^{r=L} - \int_0^1 \theta\sqrt{\frac{2}{R}}\,\frac{n^2\pi^2}{R^2}\cos\frac{n\pi r}{R}dr$$

$$\frac{k}{v}\sqrt{\frac{2}{R}}\,\frac{n\pi}{R}\sin\frac{n\pi L}{R} - \frac{k}{v}\,\frac{n^2\pi^2}{R^2}C_n(y)$$

- Combining both LHS and RHS(Obtain a PDE)

$$\frac{\partial C_n(y)}{\partial y} = \frac{k}{v}\sqrt{\frac{2}{R}}\frac{n\pi}{R} sin\frac{n\pi L}{R} - \frac{k}{v}\frac{n^2\pi^2}{R^2}C_n(y)$$

- General solution for Homogeneous Differential Equations solution with constant coefficients
- $C_n(y) = Se^{rx}$ According to table B-1(A) page 641

$$Cn(y) = 0\ @y = 0\ ; S = \sqrt{\frac{2}{R}}\frac{R}{\pi n} sin\frac{\pi nL}{R} \quad , \quad e^{rx} = e^{-\frac{\pi^2 n^2 ky}{vR^2}}$$

- $C_n(y) = \sqrt{\frac{2}{R}}\frac{R}{\pi n} sin\frac{\pi nL}{R}\sqrt{\frac{2}{R}} cos\frac{n\pi r}{R} e^{-\frac{\pi^2 n^2 ky}{vR^2}}$

$$C_n(y) = \frac{2}{\pi n} sin \frac{\pi n L}{R} cos \frac{n\pi r}{R} e^{-\frac{\pi^2 n^2 k y}{v R^2}}$$

$$\boldsymbol{\theta_t = \sum_{n=1}^{\infty} \frac{2}{\pi n} sin \frac{\pi n L}{R} cos \frac{n\pi r}{R} e^{-\frac{\pi^2 n^2 k y}{v R^2}}}$$

$$\theta = \theta_{ss} + \theta_t$$

$$\theta = \frac{L}{R} - \frac{C_2}{iC_o} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} sin \frac{\pi n L}{R} cos \frac{n\pi r}{R} e^{-\frac{\pi^2 n^2 k y}{v R^2}}$$

$$\boldsymbol{C = \frac{C_o L}{R} - \frac{C_2}{i} + \frac{C_o 2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} sin \frac{\pi n L}{R} cos \frac{n\pi r}{R} e^{-\frac{\pi^2 n^2 k y}{v R^2}}}$$

- Putting C = O and r = x

$$\sum_{n=1}^{\infty}\frac{1}{n}sin\frac{\pi nL}{R}cos\frac{n\pi x}{R}e^{-\frac{\pi^2 n^2 ky}{vR^2}} = \frac{\pi}{2}\left\{\frac{C_2}{iC_o}-\frac{L}{R}\right\} = E$$

RESULTS AND DISCUSSION

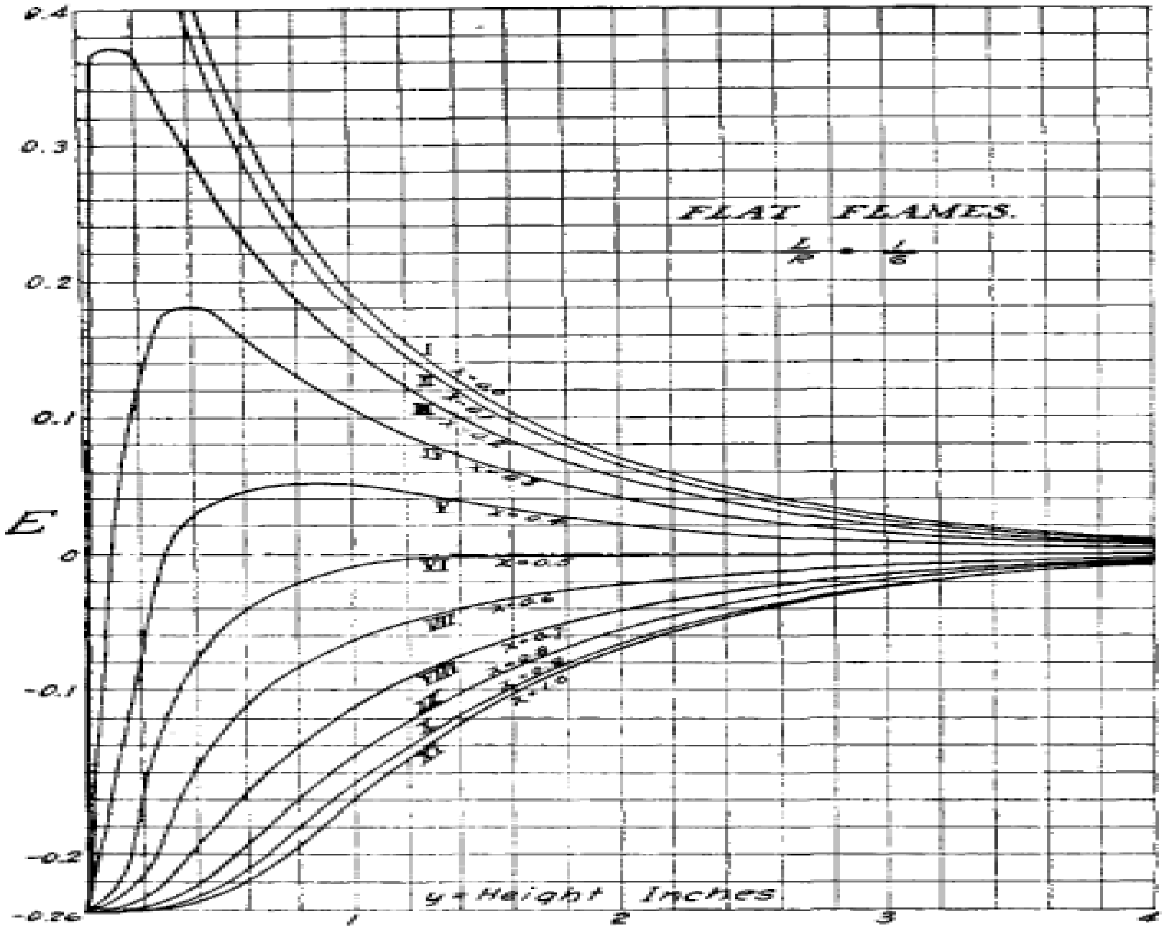
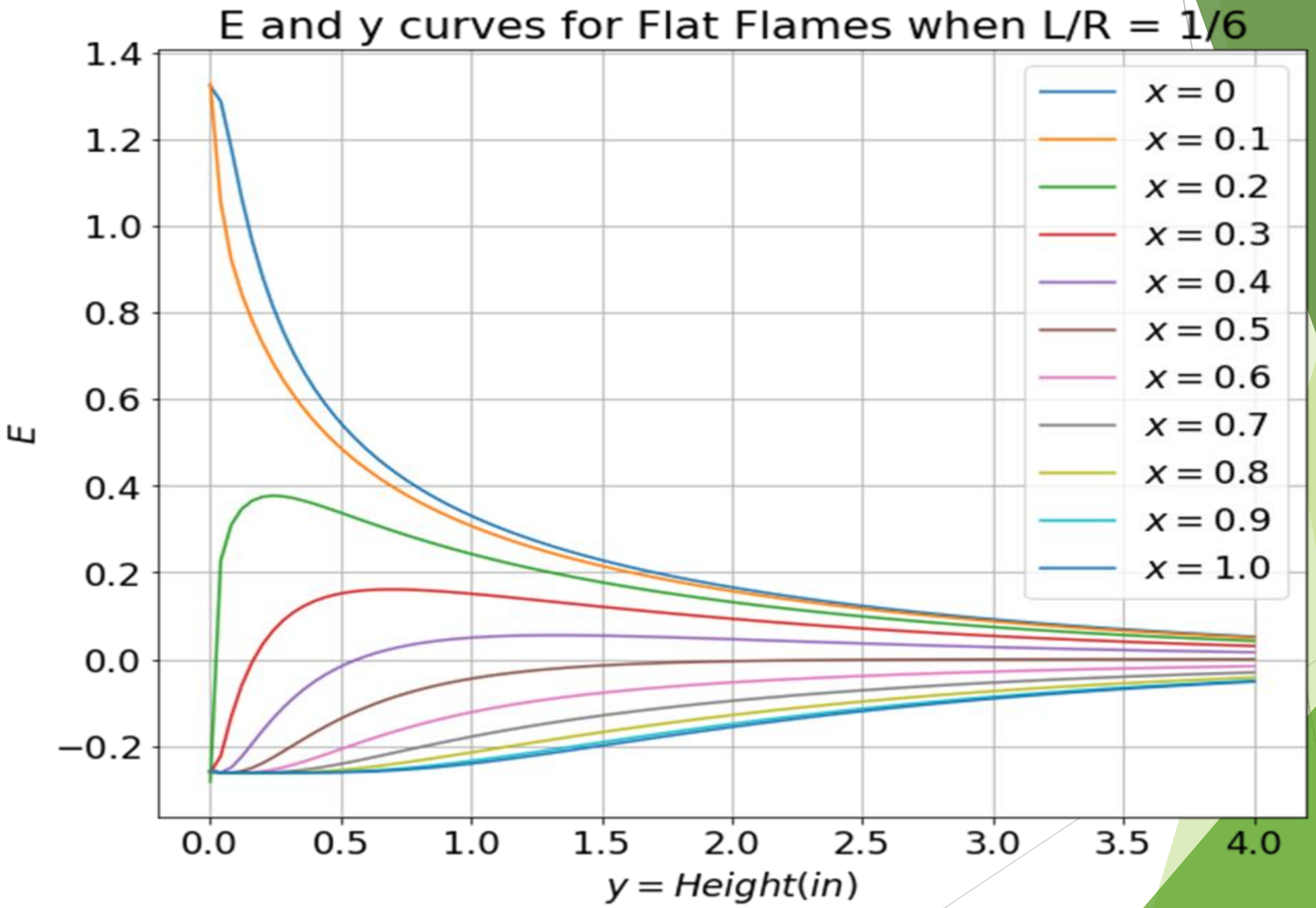


Figure 4—E and y Curves for Flat Flames when L/R = 1/6


E and y curves for Flat Flames when L/R = 1/6
E
y = Height(in)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
x = 0
x = 0.1
x = 0.2
x = 0.3
x = 0.4
x = 0.5
x = 0.6
x = 0.7
x = 0.8
x = 0.9
x = 1.0

CONCLUSIONS

- The theory of diffusion flames proposed herein shows such good agreement with the experimental facts that we feel justified in the hope that its adoption ,inessentials atleast, may lead to a better understanding of and further contributions concerning ,this very common and interesting class of flames.
- The authors feel that a more fundamental investigation of the phenomena described here on the basis of the kinetics of the chemical reactions involved might yield both interesting and profitable results.

THANK YOU