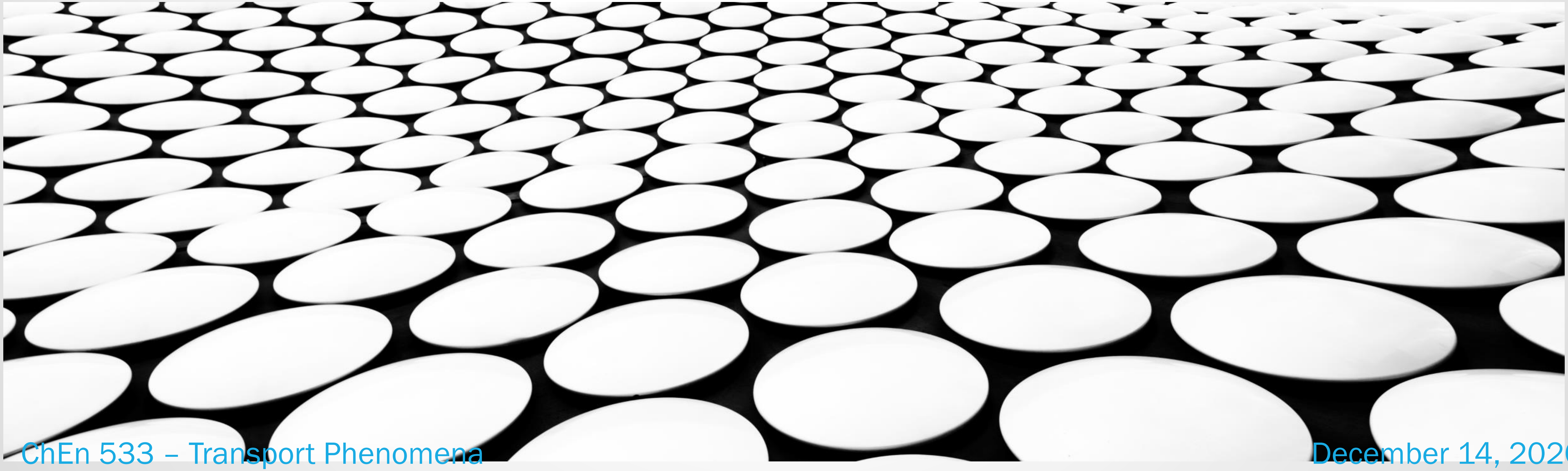
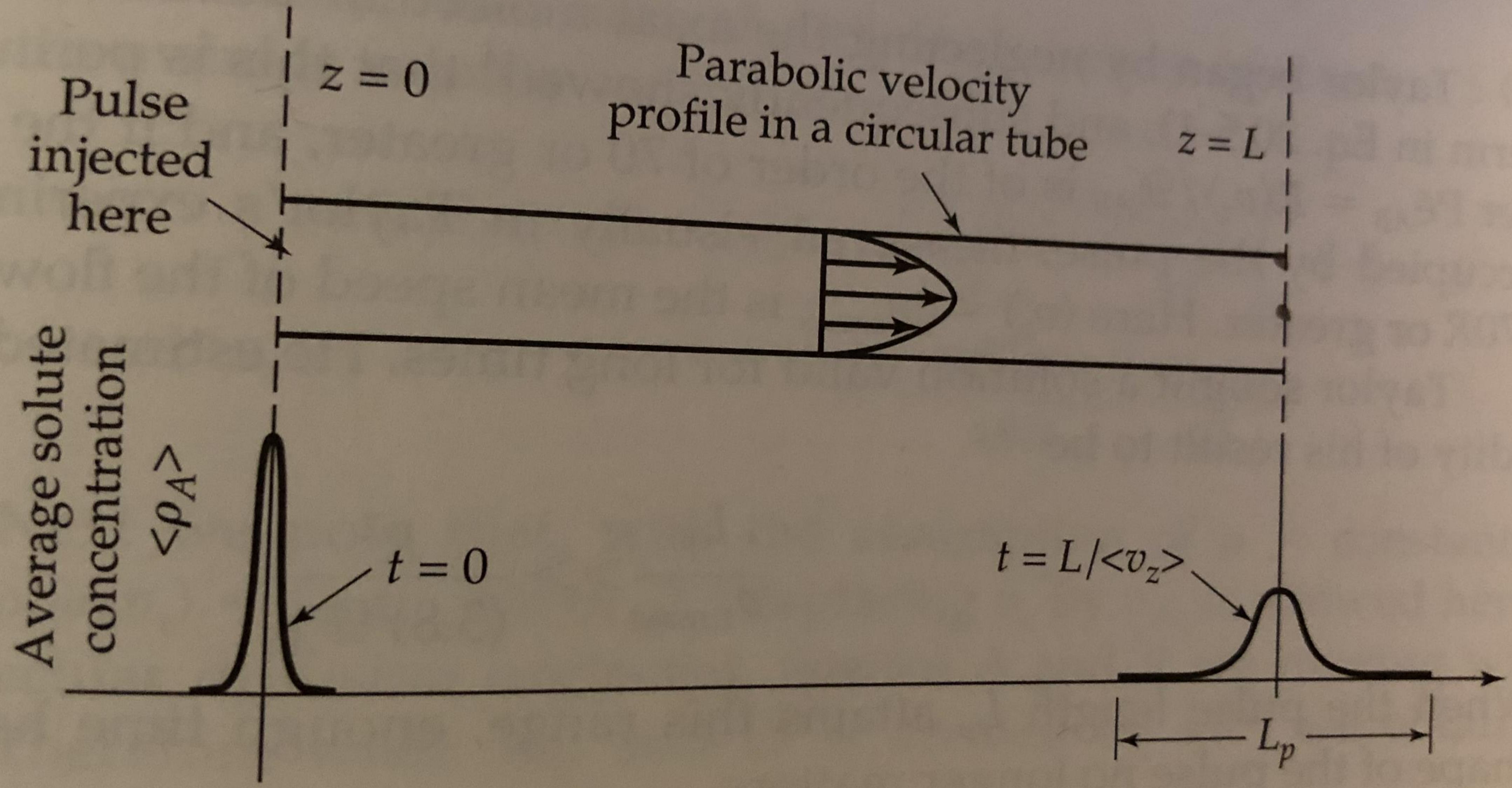
TAYLOR DISPERSION
"DISPERSION OF SOLUBLE MATTER IN SOLVENT FLOWING SLOWLY THROUGH A TUBE" – SIR GEOFFREY TAYLOR



ChEn 533 – Transport Phenomena

December 14, 2020



Pulse
injected
here
$z = 0$
Parabolic velocity
profile in a circular tube
$z = L$
Average solute
concentration
$\langle \rho_A \rangle$
$t = 0$
$t = L/\langle v_z \rangle$
L_p

IMPORTANCE OF TAYLOR DISPERSION

- Process Control
- Medical Diagnostic Procedures
  - “The results may be useful to physiologists who may wish to know how a soluble salt is dispersed in blood streams.” Taylor
- Environmental Applications
- Simple Method to Determine Diffusion Coefficient

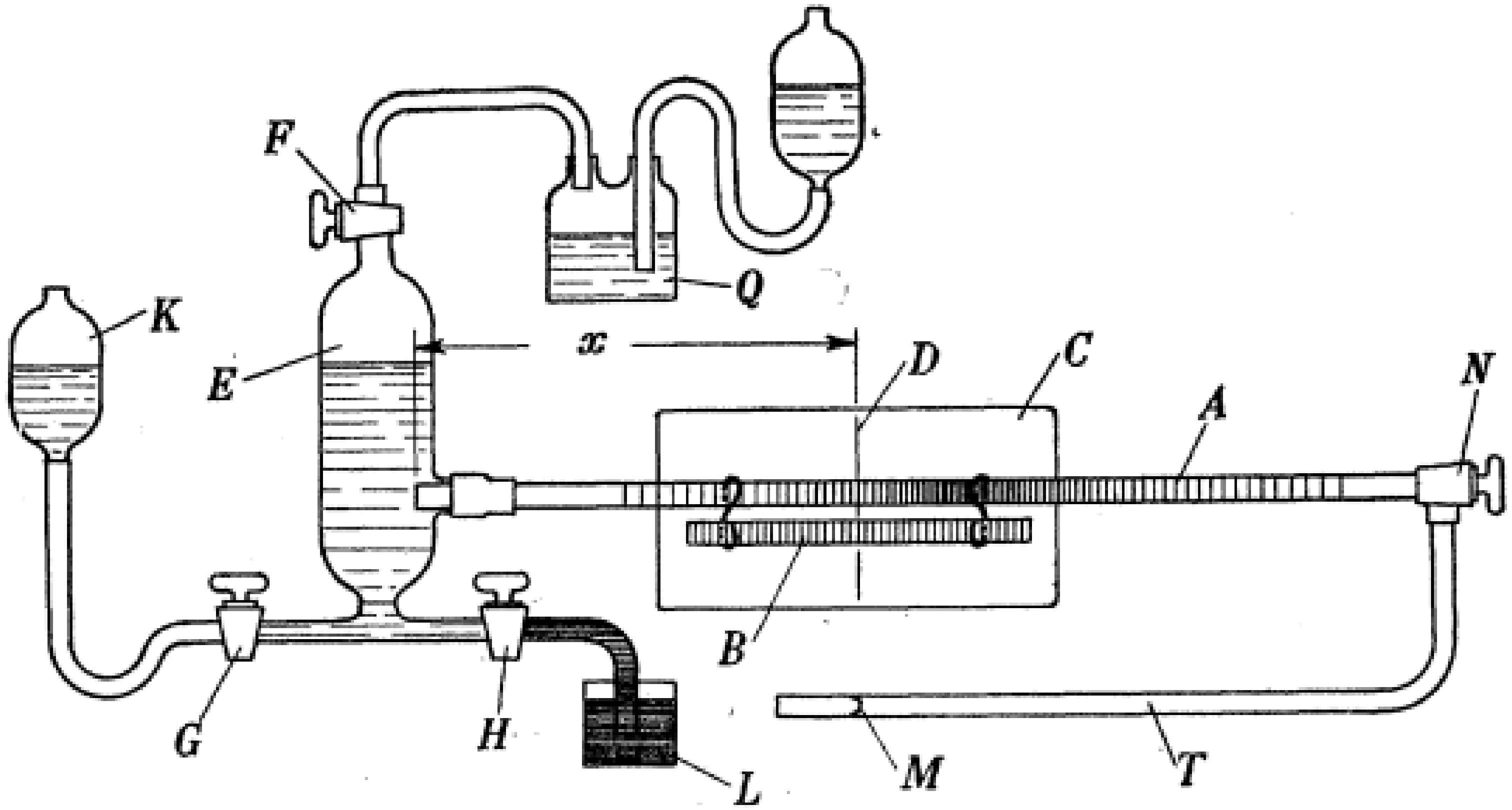


FIGURE 2. Apparatus set up with experimental tube horizontal.

METHOD

- Apply assumptions
- Introduce shifted axial coordinate and dimensionless radial coordinate
- Solve for average profile over the cross section
- Solve for averaged mass flux
- Derive expression for axial dispersion coefficient
- Insert mass flux expression to continuity equation, then use Similarity Method to get a concentration profile as a function of z and t.

DERIVATION

- $\frac{dw_A}{dt} + v_{z,max}\left[1 - \left(\frac{r}{R}\right)^2\right]\frac{dw_A}{dz} = D_{AB}\left(\frac{1}{r}\frac{d}{dr}\left(r\frac{dw_A}{dr}\right) + \frac{d{w_A}^2}{dz^2}\right)$ B.C.: $\frac{dw_A}{dr} = 0$ at r=0 and r=R
  - Can't solve this analytically
- Can neglect $\frac{d{w_A}^2}{dz^2}$ if Pe>70 and L>170R
- $\acute{z} = z - \langle v_z \rangle t$ Shifted axial coordinate
- $\xi = \frac{r}{R}$
- $\cancel{\frac{dw_A}{dt}} + v_{z,max}[0.5 - \xi^2]\frac{dw_A}{d\acute{z}} = \frac{D_{AB}}{R^2}\frac{1}{\xi}\frac{d}{d\xi}(\xi\frac{dw_A}{d\xi})$ Can neglect the time derivative term

DERIVATION CONT.

- $w_A(\xi, \acute{z}, t) = \langle w_A \rangle(\acute{z}, t) + w_A'(\xi, \acute{z}, t)$
  - $w_A'(\xi, \acute{z}, t) \ll \langle w_A \rangle(\acute{z}, t)$
- $\frac{1}{\xi}\frac{d}{d\xi}\left(\xi \frac{dw_A}{d\xi}\right) = \frac{R^2 v_{z,max}}{D_{AB}}[0.5 - \xi^2]\frac{d\langle w_A \rangle}{d\acute{Z}}$ Integrate with B.C.s to get
- $w_A(\xi, \acute{z}) = \frac{R^2 v_{z,max}}{8D_{AB}}\left[\xi^2 - \frac{1}{2}\xi^4\right]\frac{d\langle w_A \rangle}{d\acute{Z}} + w_A(0, \acute{z})$
- $w_A - \langle w_A \rangle = \frac{R^2 \langle v_z \rangle}{4D_{AB}}\frac{d\langle w_A \rangle}{d\acute{Z}}(-\frac{1}{3} + \xi^2 - \frac{1}{2}\xi^4)$
- Total mass flow of A through a plane of constant $\acute{z}$:
- $\Pi R^2 \rho \langle w_A(v_z - \langle v_z \rangle)\rangle = \frac{\Pi R^4 \rho \langle v_z \rangle^2}{D_{AB}}\frac{d\langle w_A \rangle}{d\acute{Z}}\int_0^1 \left(-\frac{1}{3} + \xi^2 - \frac{1}{2}\xi^4\right)(\frac{1}{2} - \xi^2)\xi d\xi = \frac{-\Pi R^4 \rho \langle v_z \rangle^2}{48D_{AB}}\frac{d\langle w_A \rangle}{d\acute{Z}}$

DERIVATION CONT.

- Because ρ=const, $\rho\left\langle w_A\langle v_z\rangle\right\rangle = \langle\rho_A\rangle\langle v_z\rangle$
- $\rho\langle w_A v_z\rangle \sim \langle\rho_A v_{Az}\rangle = \langle n_{AZ}\rangle$
- Divide equation on last slide by ΠR^2 to get:
- $\langle n_{AZ}\rangle = \langle\rho_A\rangle\langle v_z\rangle - K\frac{d\langle\rho_A\rangle}{dz}$
- $K = \frac{R^2\langle v_z\rangle^2}{48D_{AB}} = \frac{1}{48}D_{AB}Pe^2$ K is an axial dispersion coefficient

DERIVATION CONT.

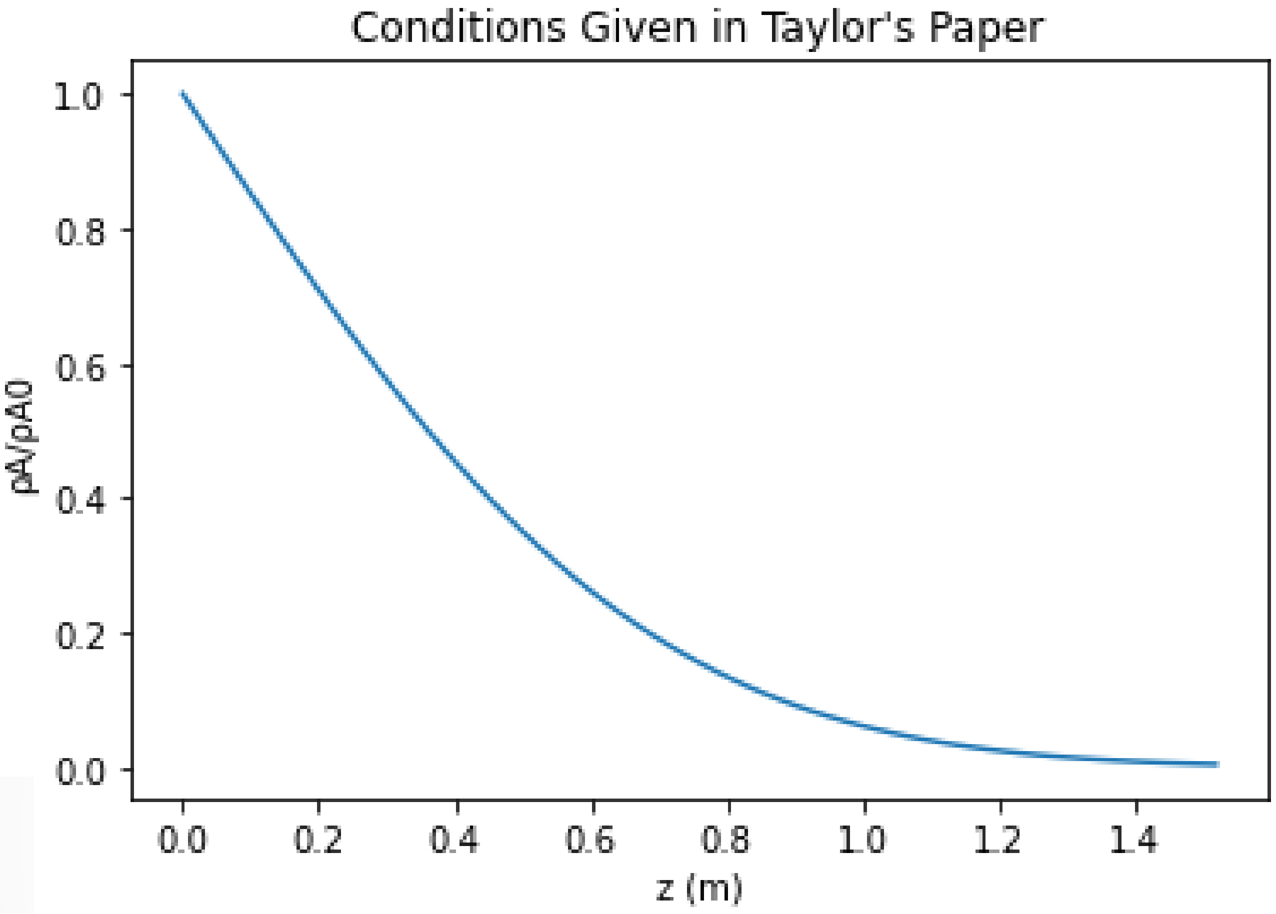
- Continuity Equation averaged over tube cross section:
- $\frac{d}{dt}\langle \rho_A \rangle = -\frac{d}{dz}\langle n_{AZ} \rangle$
- Put in expression for $\langle n_{AZ} \rangle$ to get
- $\frac{d}{dt}\langle \rho_A \rangle = \mathrm{K}\frac{d^2}{dz^2}\langle \rho_A \rangle$
- Use Method of Similarity

METHOD OF SIMILARITY

- $\phi = \frac{\rho_A}{\rho_{A,0}}, \tilde{z} = \frac{z}{L}, \tilde{t} = \frac{t}{\tau}$
- B.C. $\rho_{A(0,t)=\rho_{A,0}}, \rho_{A(\infty,t)=0}, \rho_{A(z,0)=0}$ -> $\phi(0)=1$, $\phi(\infty)=0$
- $\eta = \frac{z}{\sqrt{cK\tau}}$
- $\phi = 1 - \mathrm{erf}(\eta)$

ANALYTICAL RESULTS

- $K = \frac{1}{48} D_{AB} Pe^2$
- $\frac{\rho_A}{\rho_A 0} = 1 - \mathrm{erf}(\frac{z}{2\sqrt{Kt}})$
  - Note: Taylor got different solutions for different cases. This solution is for when there is a steady stream of solvent entering the system.
- BSL suggests that the best method of quickly getting liquid diffusivities is to measure the concentration profile experimentally, then calculating D_{AB}.



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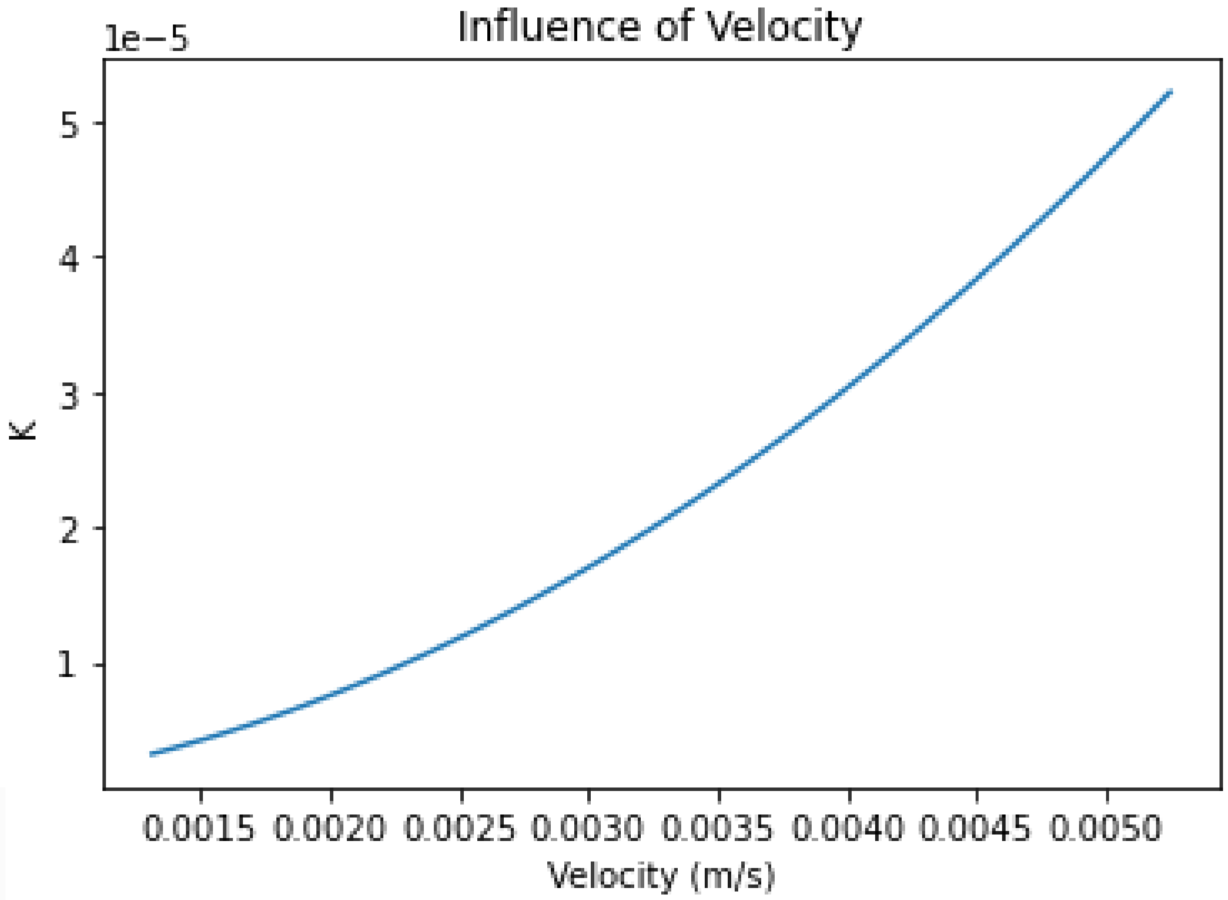

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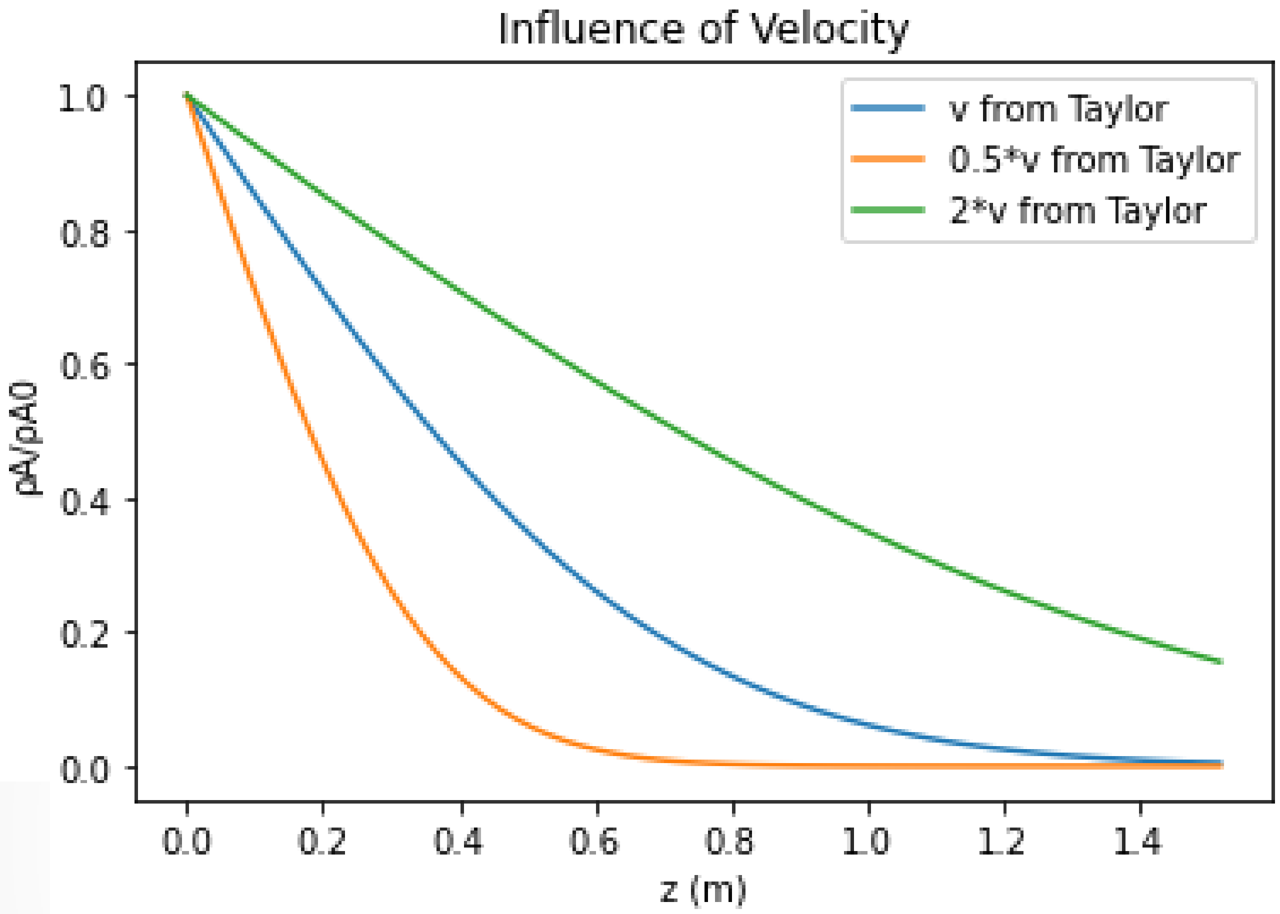

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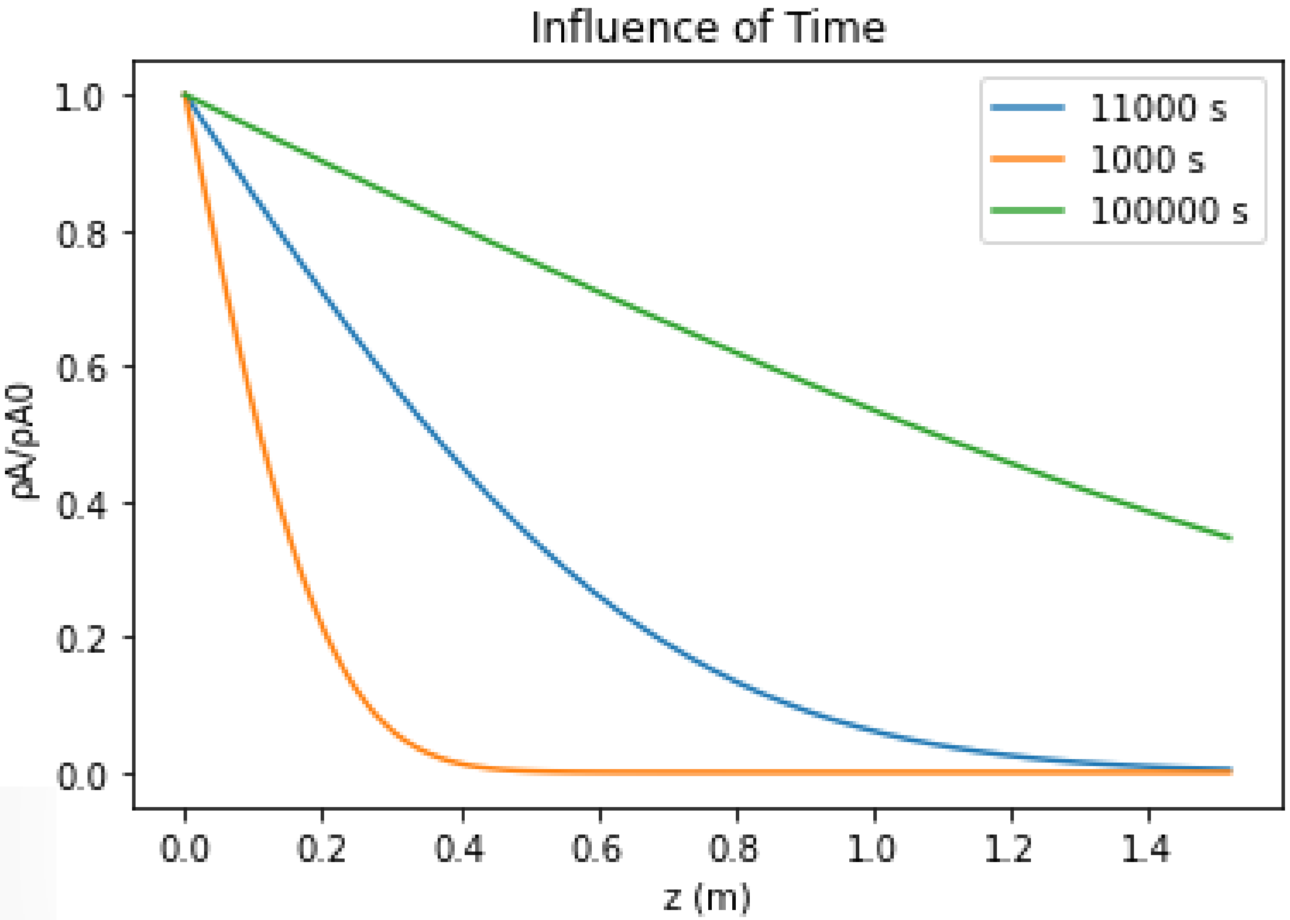

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$$\frac{\rho_A}{\rho_{A}0} = 1 - \mathrm{erf}\left(\frac{z}{2\sqrt{Kt}}\right)$$

SUMMARY

- Similarity Method was used to find concentration profile for Taylor Dispersion.
- If concentration profile was measured, the diffusion coefficient can be estimated.
- An increase of velocity increases the axial dispersion coefficient.
- Further work could be done deriving the expressions used for the other cases Taylor discusses.