Qualifier Exam Review

I. Qualifier Basics

* 40 multiple choice questions
* 3 hours
* Date & Time
* open book / notes ; no internet
* Topics: Thermo, Transport, kinetics, Statistics, Safety.
    - Concept list is provided online (Learning Suite)
* Questions are qualitative / conceptual. They can be tricky.
    - Example: water is boiling on a stovetop and one increases the rate of heat trasfer by turning up to power to the stove. This causes:

        (a) The water to increase temperature more quickly

        (b) Does not change the temperature of the water

        (c) causes the water to increase temperature more slowly.

        (d) It's effects cannot be known w/o more information.

* Questions include significant undergraduate material.

* Your score will be combined w/ your grades to determine one of 3 outcomes:

  - Fail : Dismissal from program
  - "Low" Pass : pass at M.S. level
  - "High" Pass : " " PhD level.

  • You can re-take courses or the exam, but this does not always result in a better score.

  • You can complete an MS and then do a PhD if "Low" pass. often very helpful.

* I suggest you take the exam seriously and study the material.

  - prioritize what you are/ are not comfortable with (comfortable, important → only study these)

  - Identify key concepts, equations, types of calculations.

II. Review of undergraduate fluid mechanics

A. Fluid Statics

* The key equation in fluid statics is the "static pressure equation":

$$\underline{\nabla} P = \rho \underline{g}$$

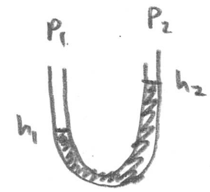
pressure gradient ↗ ↖ gravitational force

$$\underline{g} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$$

* Align w/ z-axis gives:

$$P(z) = P_0 - g \int_0^z \rho \, dz' = P_0 - \rho g z \quad \text{const } \rho.$$

* Can use this to relate height & pressures in manometers: $P_1 - P_2 = \rho g (h_2 - h_1)$



* Can calculate the force on surfaces

$$\underline{F}_P = -\int_S \underline{n} P \, dS$$

- leads to buoyancy : $F = \rho g V$

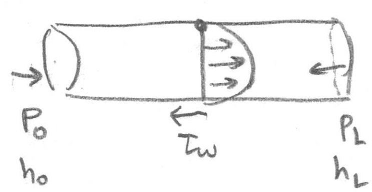
(*) Surface tension is an important concept in fluid statics that I will not review here.

* key concepts:
  - Pressure increases with the weight of the fluid above it.
  - Pressure is isotropic

B. Internal Flow (pipe flow)

* Flows that are bounded between walls, like pipe flow, are called internal flows.
* Internal flows are often driven by a combination of pressure & gravity.
* At steady state: pressure + gravity = wall shear stress

$$-\frac{D}{4} \frac{\Delta(P + \rho g h)}{L} = \tau_w \qquad \text{(force balance)}$$



$\Delta P = P_L - P_0$
$\Delta h = h_L - h_0$
$\mathcal{P} = P + \rho g h$

$$-\frac{D}{4} \frac{\Delta \mathcal{P}}{L} = \tau_w$$

* The friction factor is a dimensionless wall shear stress

$$f = \frac{\tau_w}{\frac{1}{2}\rho u^2} \quad \text{or} \quad f_D = \frac{8\tau_w}{\rho u^2} = 4f$$

(divide by inertial stress)

Fanning friction factor   Darcy friction factor

- Re-writing the above gives:

$$f = \frac{D_H}{2\rho u^2}\frac{|\Delta P|}{L}$$

generalized for non-circular pipes.

$$D_H = \frac{4A}{C}$$

A: cross-sectional area

C: wetted perimeter

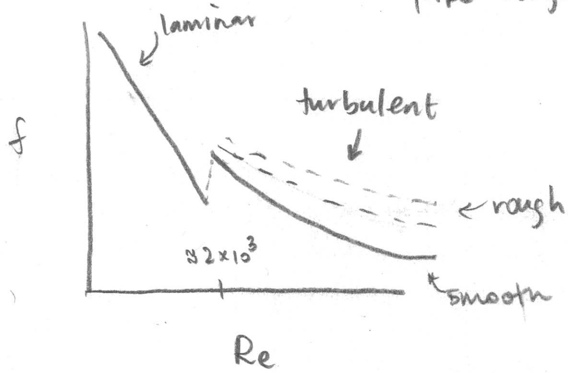
D_H: hydraulic diameter

* Pipe flow has two kinetic regimes: laminar & turbulent.

Dimensional analysis shows these are a function of the Reynolds number ($Re = \rho u D/\mu$, $u = Q/A$) and pipe roughness (k).

Q: volumetric flow rate

(☆) Moody Diagram.



rough idea of experimental data

correlations to data

$$f_{laminar} = \frac{16}{Re}$$

(get from theory, too.)

$$\underbrace{\frac{1}{\sqrt{f}} = -4\log\left(\frac{1.26}{Re\sqrt{f}} + \frac{k}{3.7D}\right)}_{\text{turbulent.}}$$

* Q: If f goes down as $u\uparrow$, does τ_w or $|\Delta P/L|$ go down or up as $u\uparrow$?

* A: ΔP & τ_w go up! u^2 in denominator of f.

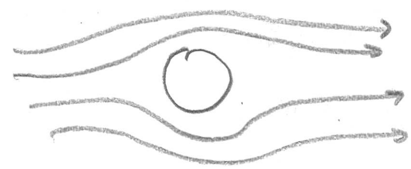
* key calculations: pressure drop in a pipe with length and gravity. will see again in design of pipelines.

* key concepts:

- laminar & turbulent flow
- dimensional analysis: f, Re, k/D
- "major losses": pressure drop consumes energy overcoming wall friction $\dot{E}_v = Q|\Delta P|$

C. External Flow

* External flows are unbounded flows with objects inside of them. The prototypical example is a sphere in steady flow:



- Object is stationary

  OR

- Object is moving & fluid is stationary (ref. frame)

* The steady-state force balance on an object in an external flow gives:

(terminal velocity)

$$F_D = \tau_w \cdot A_\perp$$

F_D: drag force; τ_w: shear stress @ wall; $A_\perp$: surface area.

- Note that $\tau_w \cdot A_\perp$ is a way to tabulate a complex integral.

$A_\perp$: "projected" area of object.

* The drag coefficient is a dimensionless wall shear stress (analogous to f!). (*)

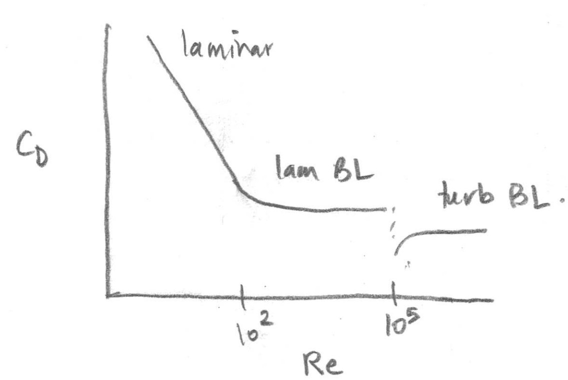
$$C_D = \frac{\tau_w}{\frac{1}{2}\rho u^2} = \frac{2F_D}{A_\perp \rho u^2}$$

combine w/ force balance.

(*) Caveat: for a flat plate only. For bluff objects force balance includes pressure drag too. This dominates at high Re.

$$F_D = (\tau_w + \Delta P) A_\perp$$

* External flows have three kinetic regimes: laminar, laminar BL / turbulent wake, turbulent BL / turbulent wake



$C_D = \frac{24}{Re}$ (sphere, laminar)

$C_D \simeq 0.445$ (sphere, lam BL)

$C_D \simeq 0.19$ (sphere, turb BL)

* Aside: why don't internal flows have BLers? They do. Fully developed flow happens when $\delta \sim D$ (BL size ~ pipe diameter).

* Flat plates are a bit weird b/c they are only a boundary layer (laminar region is not anything).

$$C_f = 1.328\, Re^{-1/2} \quad \text{(flat plate, laminar BL)} \quad 10^2 \lesssim Re \lesssim 10^5$$

$$C_f = \frac{0.455}{(\log Re)^{2.58}} - \frac{1050}{Re} \quad \text{(flat plate, turbulent BL)} \quad Re \gtrsim 10^5$$

* key calculations: Drag force, terminal velocity.

* key concepts:
  - Kinetic regimes, dimensional analysis
  - Friction drag & form drag

$$\text{Friction drag} \downarrow \tau_w \qquad \text{form drag} \downarrow \Delta P$$

* There are more complex combinations of internal & external flows. Porous media is an example that has elements of both. I'm not reviewing this.

D. Pipe Network Design

* Pipe Network Design is based on two integral balances. A mass balance and a mechanical energy balance:

mass flow rate

$$\frac{dm_{cv}}{dt} = \sum_i^{\text{inlets}} w_i - \sum_i^{\text{outlets}} w_i$$

mass balance (discrete inlets/outlets, uniform density at i/o)

$$\left(\frac{bv^2}{2} + \frac{P}{\rho} + gh\right)_{out} - \left(\frac{bv^2}{2} + \frac{P}{\rho} + gh\right)_{in} = \frac{1}{w}\left(w_m - E_v\right)$$

Mech. Energy Bal. (single i/o, unidirectional flow at i/o, fixed c.v, small stress @ i/o, steady)

$$v = u \uparrow \quad \frac{Q}{A} = \frac{w}{\rho A}$$

$$b = \begin{cases} 2, & \text{laminar} \\ 1.06, & \text{turbulent} \\ 1, & \text{plug flow} \end{cases}$$

- Need to do balance on a control volume.

* E_v: These are the viscous losses. E_v is in units of power.

often use units of "head"

$$h_L = \frac{E_v}{wg} \qquad \text{(head losses)}$$

$$E_v = \sum_i \frac{1}{2} k_i w u^2 \qquad k_i = \frac{E_{v,i}}{\frac{1}{2} w u^2}$$

- loss coefficient
- dimensionless viscous loss.

- major losses: from pressure drop

$$k_{major} = \frac{4Lf}{D}$$

- minor losses: bends, elbows, expansions, contractions, fittings, valves, etc.

K_i = tabulated (usually a number)

* W_m: This is the shaft work. Pumps or turbines that add or subtract mechanical energy from the fluid.

- Postive displacement vs. centrifugal pumps
  (fixed volume, high ΔP, low Q) (mod to low ΔP, high Q)

- Centrifugal pumps W_m depends on Q.

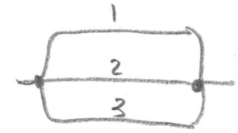
$$h_p = \frac{W_m}{wg} = h_{max} - \frac{h_{max}}{Q_{max}} Q \qquad \text{(ideal)}$$

$$\simeq h_{max} - BQ^n \qquad \text{(non-ideal, } n \approx 2\text{)}$$

- turbines do negative W_m

* Networks:

- In parallel configuration



h_L or h_p = equal in each branch.

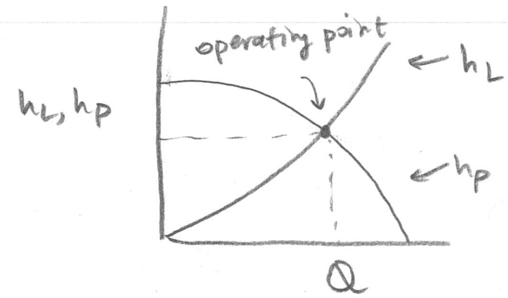
$Q_{tot} = \sum Q_i$ in each branch

- In series configuration

$h_{tot} = \sum_i h_{L,i}$ (or h_p)

$Q_1 = Q_2 = Q_3$

- Can put all of this together on a "system demand / system supply" curve.



$*$ usually $\Delta P = \Delta h = \Delta v = 0$

- Q: what way does h_L move for parallel, serial?
  " " " h_p " " " " ?
  (if adding two curves together)

  A: In parallel, curves add to the right (Q adds)
  In serial, curves add to the top (h_L, h_p adds).

* Net positive suction Head - pumps create a pressure minimum, this must be above the vapor pressure or the fluid will cavitate:

$$NPSH = h_{inlet} - \frac{P_{vap}}{\rho g}$$

$$NPSH_{req} = h_{inlet} - h_{min}$$ ← From manufacturer. Amount of h_{inlet} needed.

$$h_{min} > \frac{P_{vap}}{\rho g} \Rightarrow NPSH > NPSH_{req}.$$

* key problems: Find pump size, operating point, pipe diameter, flow rate, etc. Design pipe network.

* key concepts:
- Pumps increase & decrease pressure. operate at constant speed. Pumps don't modulate Q.
- Valves change h_L to change flow rate. Think about a faucet.

III. Review of undergraduate Heat & Mass Transfer

A. Overview

* There are 3 modes of heat transfer: conduction, convection, radiation.
* There are 3 "levels" of difficulty of problems:

  (a) Algebraic problems - simple heat transfer with linear gradients or lumped capacitance (more later)

  (b) Differential - moderately difficult geometries. 1D or 2D. ODEs or PDEs. This has been our focus in Transport this semester.

  (c) Numerical - harder geometries, 3D. David's numerical methods class.

• we will focus on (a). Undergrad class has a lot of (b), but we have done lots of that already. I will skip this.

B. Conduction

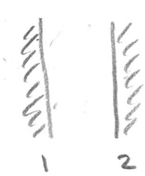
* The key equation for heat transfer is the rate of heat transfer:

$$q = -k A \frac{dT}{dx} \qquad \text{(cartesian)} \quad \text{Fourier's Law.}$$

heating rate (q)   thermal conductivity (k)

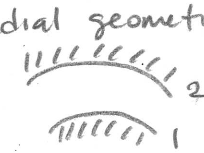
$$q = \frac{kA}{L}(T_1 - T_2)$$

For a planar geometry



$$q = \frac{2\pi L k}{\ln(r_2/r_1)}(T_1 - T_2)$$

For radial geometry



* Engineers like to write this as:

$$q = \frac{T_1 - T_2}{R} = UA(T_1 - T_2)$$

R: thermal resistance U: overall heat transfer coefficient

* The key equation for mass transfer:

total conc.

$$N_A = c\, D_{AB} \cdot A \cdot \frac{dx_A}{dx} \qquad \text{(cartesian)} \qquad \text{Fick's law}$$

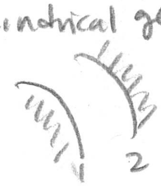
diffusivity.

rate of mass transfer

- For a planar geometry: $N_A = \frac{D_{AB} \cdot A}{L} (C_{A,1} - C_{A,2})$

1 2

- For cylindrical geometry (radial): $N_A = \frac{2\pi L D_{AB}}{\ln(r_2/r_1)} (C_{A_1} - C_{A_2})$



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(assumes stationary medium that it is diffusing through)

* Again, engineers like to write:

$$N_A = \frac{C_{A,1} - C_{A,2}}{R} = K_A A (C_{A_1} - C_{A,2})$$

overall mass transfer coefficient

- Note: Be careful at phase boundaries. Need a partition coefficient. Different than heat transfer.

C. Convection

* Heat transfer can aslo occur by convection. In this case heat (or mass) transfer is enhanced by bulk flow of a fluid (gas or liquid).

* Convection can be:

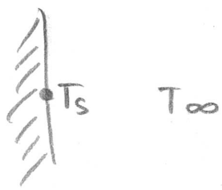
  - forced (e.g. wind, pipe flow)
  - natural (e.g. buoyancy driven)
  - driven by a phase change (evaporation, condensation)

* The rate of heat transfer by convection is described by Newton's law of cooling:

resistance: $R = \frac{1}{hA}$

$$q = h\,A\,(T_s - T_\infty)$$

h: heat transfer coefficient; A: area



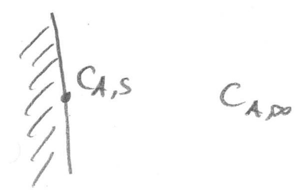
* h holds all the information about fluid flow. More on this later. (More a definition of h than anything else)

* Convective mass transfer can be similarly described:

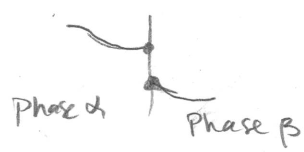
resistance: $R = \frac{1}{h_m A}$

$$N_A = h_m A\,(C_{A,s} - C_{A,\infty})$$

h_m: mass transfer coefficient



• Again, be careful with $C_{A,s}$ if this is a phase boundary. May need a partition coefficient:



$$C_{A,\alpha} = K\,C_{A,\beta}$$

D. Types of Algebraic Heat/Mass Transfer Problems

(i) Resistance Problems

* Treating the above equations like "thermal circuits", we can find heat (or mass) transfer accross multiple domains

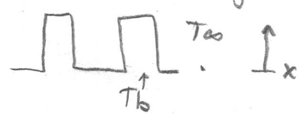
* These are combinations of thermal resistances in series or in parallel.

Series: $R_{tot} = \sum_i R_i$ ← come from conduction or convection conditions

parallel: $(R_{tot})^{-1} = \sum_i (R_i)^{-1}$

(ii) Fin problems (quasi-2D)

* We want to increase the surface area, A, to increase heat transfer. That often results in geometries with extended surfaces:



* We can solve this geometry with an effective 1D solution. Most common geometries are solved and tabulated. → get $T = T(x)$

* These are then converted to a fin efficiency

$$\eta_f = \frac{q_{fin}}{q_{max}}$$

fin efficiency

← from analytical solution

← max theoretical heat transfer from fin surface (assumes surface is at T_b for whole fin)

$$q_{max} = h A (T_b - T_\infty)$$

fin area

$$q_{fin} = \eta_f h A (T_b - T_\infty)$$

tabulated for different geometries (see example)

Example: $\eta_f = \dfrac{\tanh(mL_c)}{mL_c}$, rectangular fin (L, w, t)

$$L_c = L + t/2$$

$$A = 2wL_c$$

$$m = \left(\frac{2h}{kt}\right)^{1/2}$$

(iii) 2D problems (shape factor)

* kind of like fin problems, engineers prefer to keep the simple form:

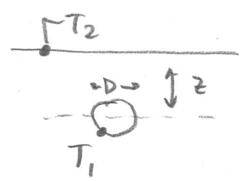
$$q = \underline{\quad\quad} \cdot (T_1 - T_2)$$

even as the problem gets complex. So, for 2D, they add a "shape factor", S.

$$q = S k \, (T_1 - T_2)$$ ← conduction only.

* S holds all the information about diffusion for a given shape. These are then tabulated. (other people have solved for S using analytical or numerical methods.)

* Example:



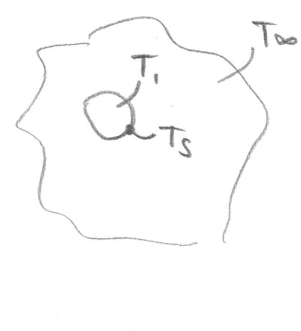
sphere in a semi-infinite medium

$$S = \frac{2\pi D}{1 - D/4z}$$

(iv) Transient Conduction

* A transient energy balance of an object gives:

$$m \hat{C}_p \frac{dT}{dt} = -hA(T - T_\infty)$$



* There are two relevant cases:

$$\frac{kA}{L}(T_1 - T_s) \overset{>}{\underset{<}{\text{or}}} hA(T_s - T_\infty)$$

↑ internal conduction ↑ external convection

* $Bi = \frac{hL}{k}$ $Bi \gg 1$ external convection is faster

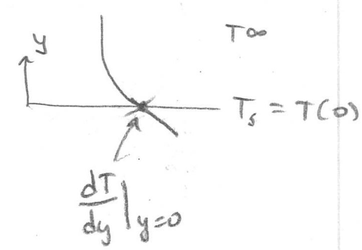
  $Bi \ll 1$ internal conduction is faster

- if $Bi \gg 1$, no internal gradients
  ↳ "0D"-heat transfer. Object has one temperature. "Lumped capacitance"
- If $Bi \ll 1$, yes internal gradients.
  ↳ Need to solve harder problem.

E. Heat/Mass Transfer Coefficients

* How do we get h (h_m) for convection problems?
* we know at high Re, surfaces will have boundary layers. We can look at heat/mass transfer at a boundary layer:

$$\underset{\text{heat flux}}{q''} = \underbrace{-k \left.\frac{dT}{dy}\right|_{y=0}}_{\text{conduction at surface}} = \underbrace{h(T_s - T_\infty)}_{\text{convection at surface}}$$

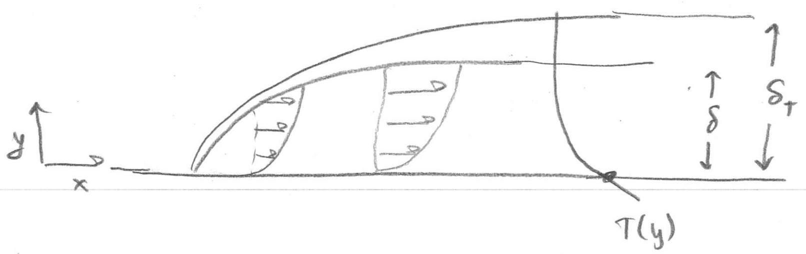


$$h = \frac{-k \left.\frac{dT}{dy}\right|_{y=0}}{T_s - T_\infty}$$

(for mass)

$$h_m = \frac{-D_{AB} \left.\frac{dC_A}{dy}\right|_{y=0}}{C_{A,s} - C_{A,\infty}}$$

* For a boundary layer, the math problem (Boundary layer equations) are the same for momentum, mass, and heat transfer!



* Boundary layers only differ by ratios of ν, α, D_{AB}

$Pr = \nu/\alpha$

$Sc = \nu/D_{AB}$

* It is easy to measure a drag coefficient

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho u^2} = \frac{\mu \, \partial v_x/\partial y \big|_{y=0}}{\frac{1}{2}\rho u^2}$$

← gradient still!

← note different units.

* Using B.L. equations and dimensional analysis, can show that the following are equal:

$$C_f \cdot \frac{Re_L}{2} = \underbrace{\frac{hL}{k}}_{Nu = \frac{hL}{k}} Pr^{-n} = \underbrace{\frac{h_m L}{D_{AB}}}_{Sh = \frac{h_m L}{D_{AB}}} Sc^{-n} = f(Re_L, \text{shape})$$

$$\boxed{C_f \frac{Re_L}{2} = \frac{Nu}{Pr^n} = \frac{Sh}{Sc^n}}$$

Heat/Mass Transfer Analogy.

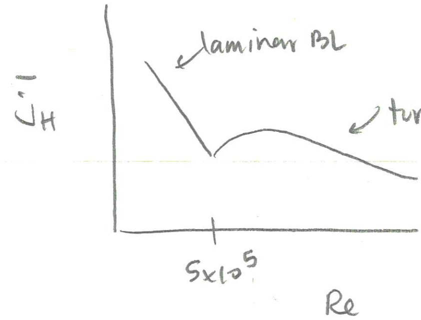
* Get correlations for one using whatever is easiest as a function of Re, shape. Then use to get h or h_m.

* Reynolds analogy: Assume $Pr = Sc = 1$

* Chilton-Colburn analogy: $n = 1/3$

$$\frac{C_f}{2} = \underbrace{\frac{Nu}{Re\, Pr^{1/3}}}_{j_H} = \underbrace{\frac{Sh}{Re\, Sc^{1/3}}}_{j_M}$$

→ colburn factors.



flat plate

Recall: $C_f = 1.328\, Re^{-1/2}$ (laminar BL)

$Nu = 0.664\, Re^{1/2} Pr^{1/3}$

$Sh = 0.664\, Re^{1/2} Sc^{1/3}$

* Notice the above definitions of h & h_m (and C_f) are local. For engineering, we need properties over the whole surface, so we average. Plot of $\bar{j}_H$ is average.

* This works for both internal and external flows. Numerous correlations are available. For internal flows, we use the friction factor, not C_f.

Ex: turbulent pipe flow

$$f = 0.092\, Re^{-1/5}$$

$$Nu = 0.023\, Re^{4/5} Pr^{1/3} \quad \text{(Colburn Eq.)}$$

* Skipping Free (Natural) Convection.

F. Heat Exchangers

Out of time to lecture on this topic & to prep it. Finish this next year.

(To be continued)