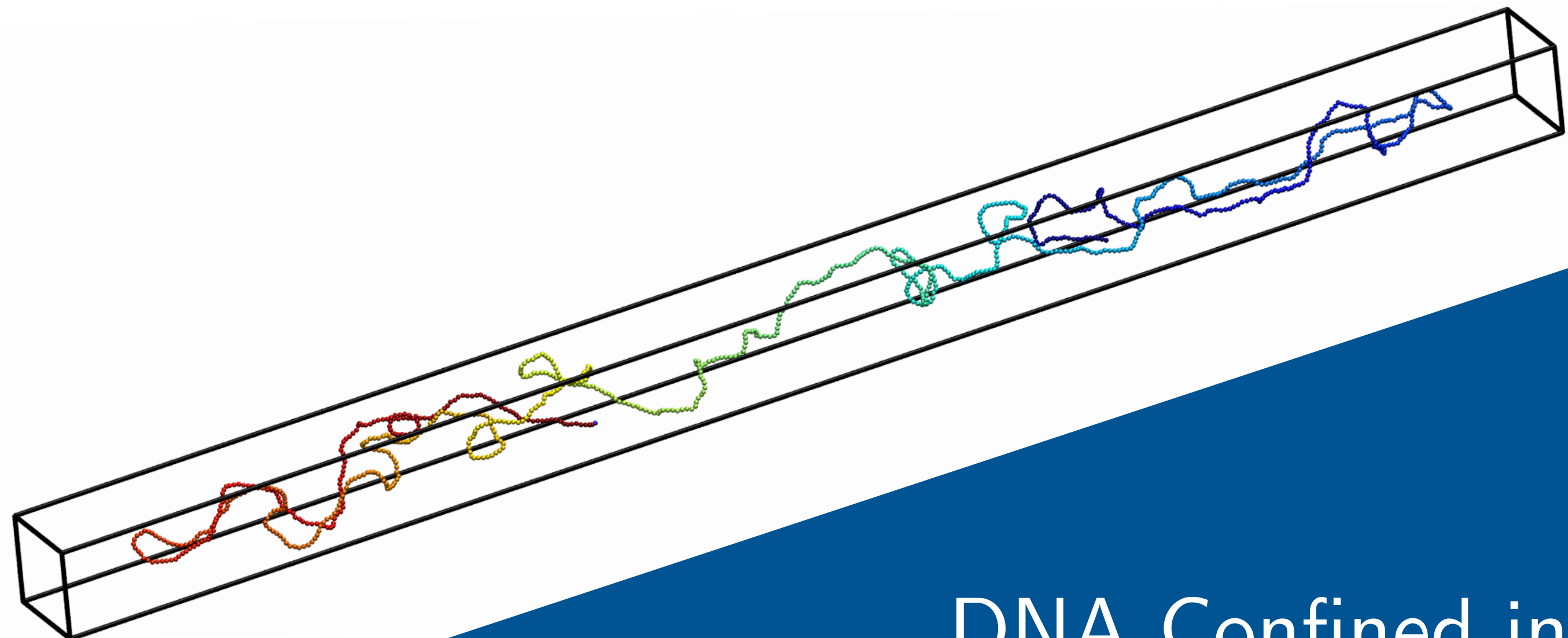




# DNA Confined in Nanochannels and Nanoslits

Douglas Tree

Dissertation Defense  
University of Minnesota  
Thursday May 1, 2014

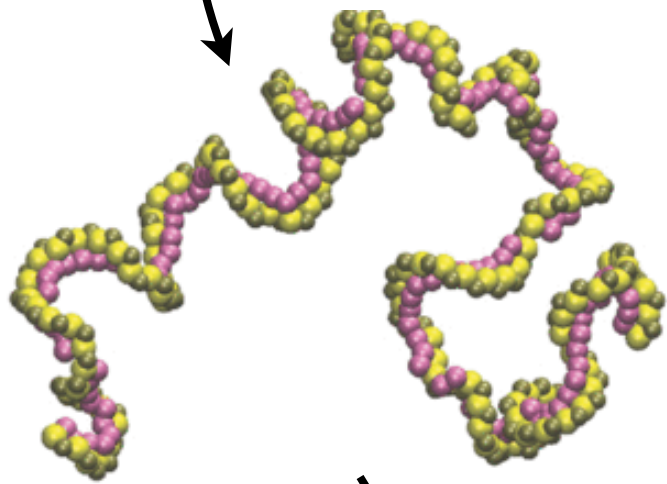
# Connecting genotype to phenotype

## Central Dogma in Molecular Biology

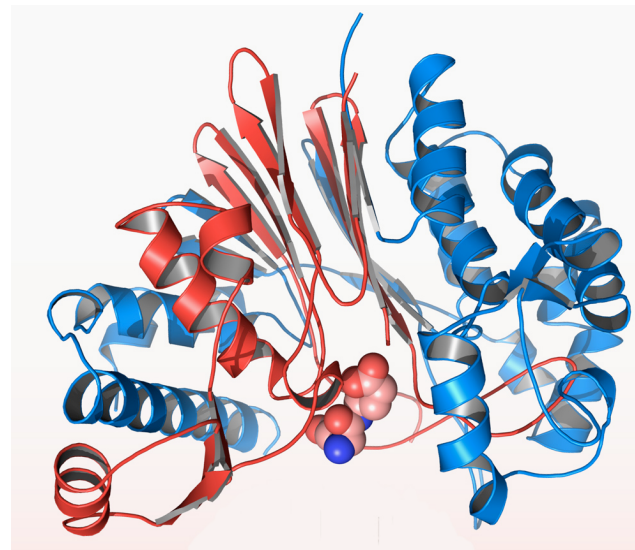
DNA (Genotype)



RNA



Protein



Applications (Phenotype)

Human Pathology

Agriculture

Environment

Speicher and Carter. *Nat. Rev. Genetics*. 6, 782 (2005)

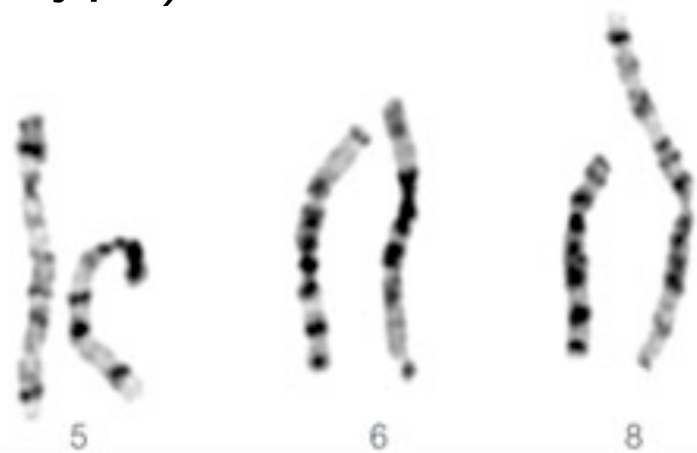
Hinckley et al. *J. Chem. Phys.* 139, 144903 (2013)

[http://en.wikipedia.org/wiki/Bacillus\\_anthraxis](http://en.wikipedia.org/wiki/Bacillus_anthraxis)

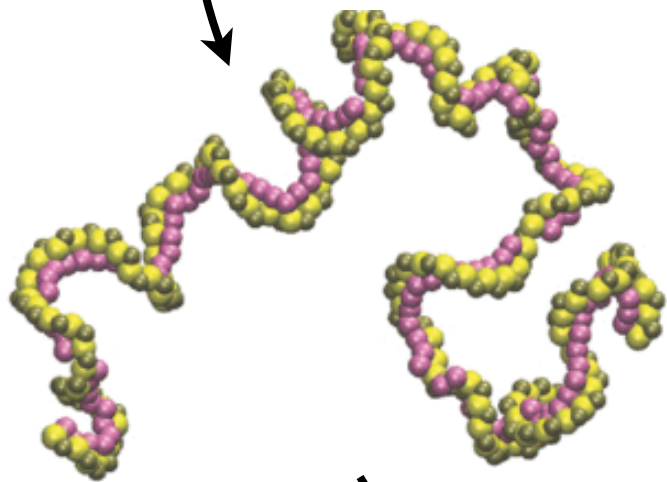
# Connecting genotype to phenotype

## Central Dogma in Molecular Biology

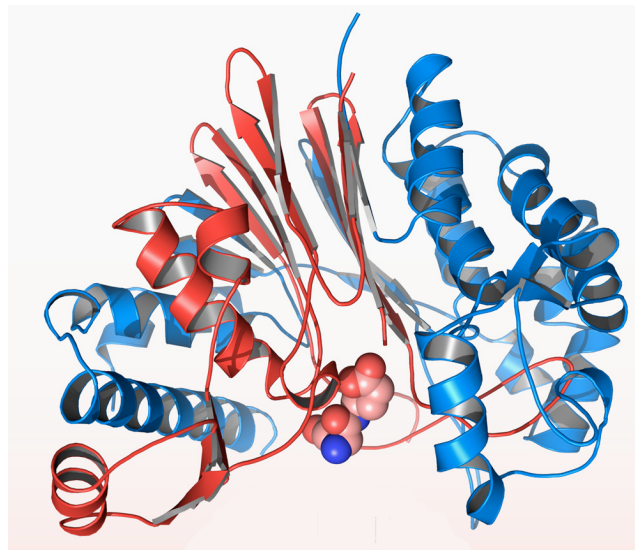
DNA (Genotype)



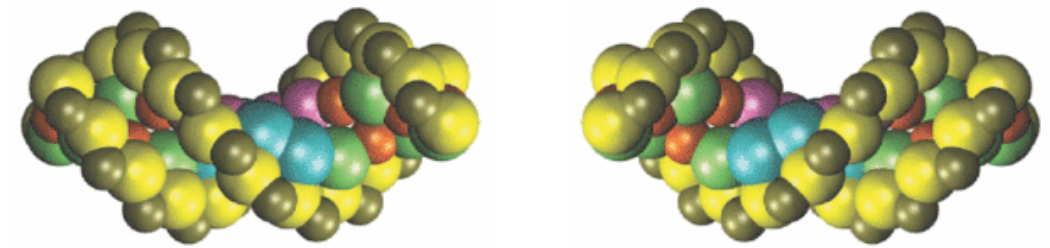
RNA



Protein



## Genomic Variability



...ACTGACGA... ...ACCAGTGA...

## Applications (Phenotype)

Human Pathology

Agriculture

Environment

Speicher and Carter. *Nat. Rev. Genetics*. 6, 782 (2005)

Hinckley et al. *J. Chem. Phys.* 139, 144903 (2013)

[http://en.wikipedia.org/wiki/Bacillus\\_anthraxis](http://en.wikipedia.org/wiki/Bacillus_anthraxis)

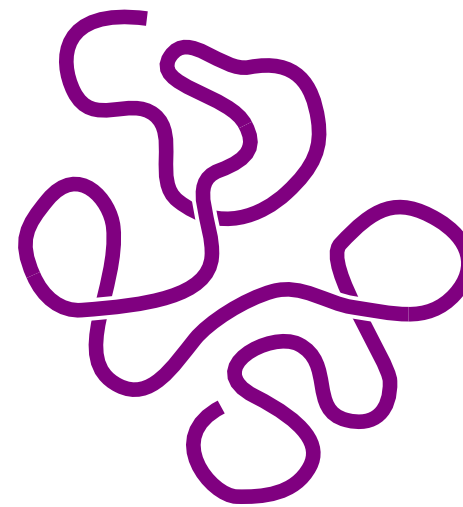
# Genomic variability

Variability is a multi-scale phenomena

Chromosome

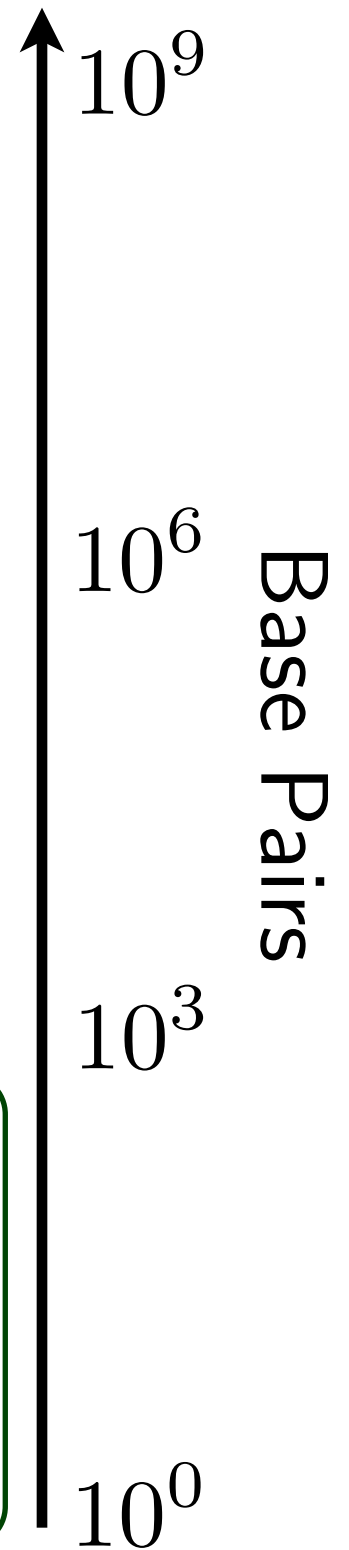


Gene



Sequencing

Nucleotide



# Genomic sequencing

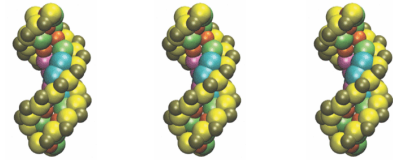
## Modern Sequencing

Chromosomal  
DNA



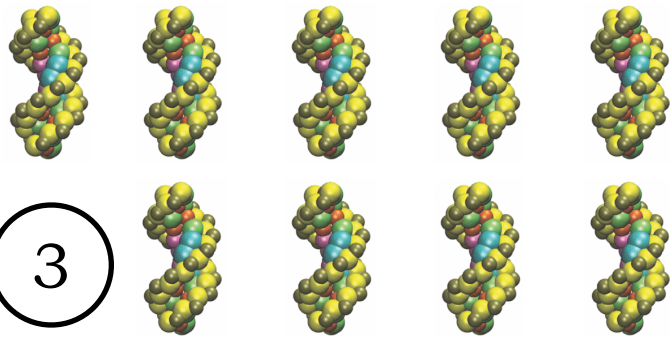
1

Shear into  
Fragments

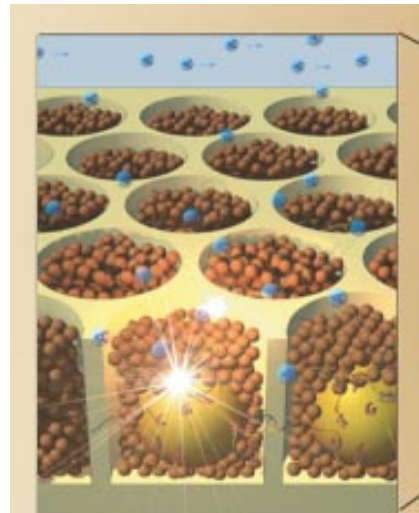


2

PCR Amplification



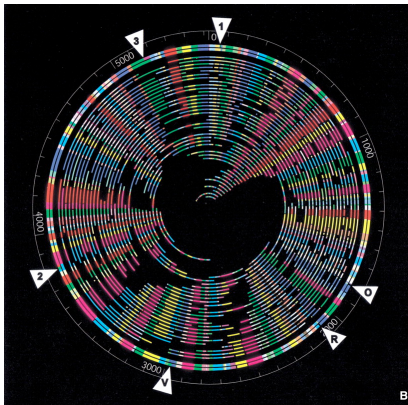
3



Fragment  
Sequencing

4

Sequence  
Assembly



5

Margulies et al. *Nature*. 437, 376 (2005)  
Speicher and Carter. *Nat. Rev. Genetics*. 6, 782 (2005)  
Hinckley et al. *J. Chem. Phys.* 139, 144903 (2013)  
Zhou et al. *Appl. Environ. Microbiol.* 68, 6321 (2002)

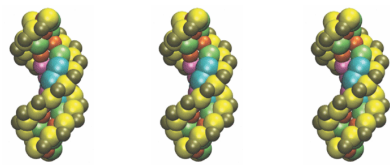
# Genomic sequencing

## Modern Sequencing

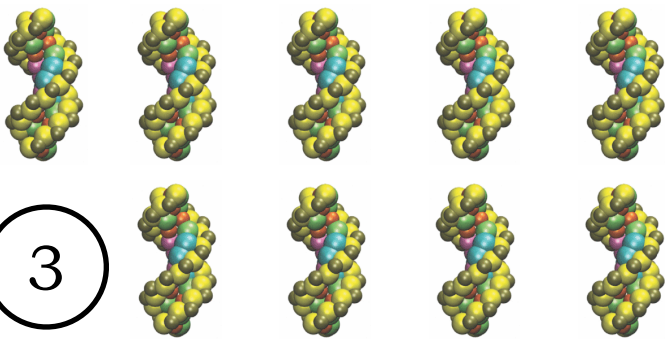
Chromosomal DNA



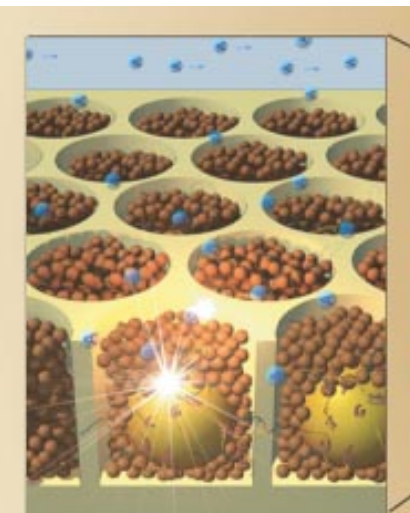
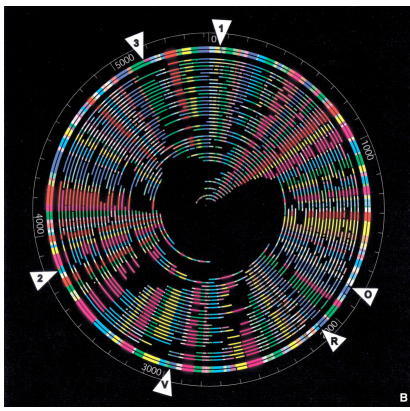
Shear into Fragments



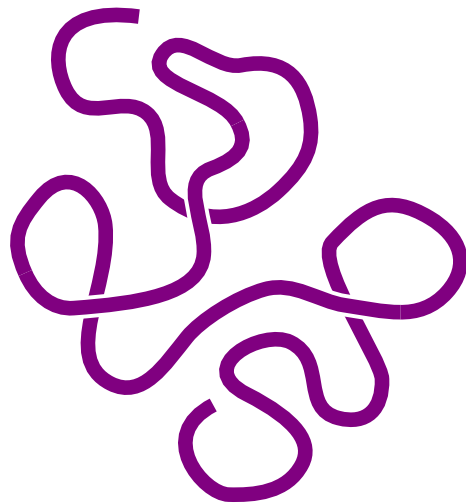
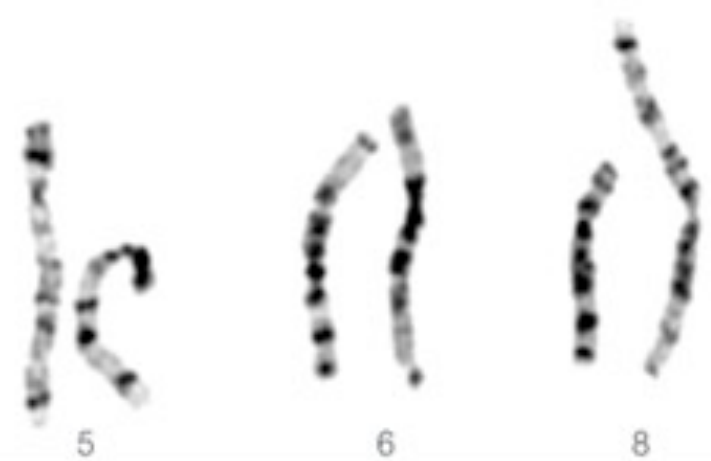
PCR Amplification



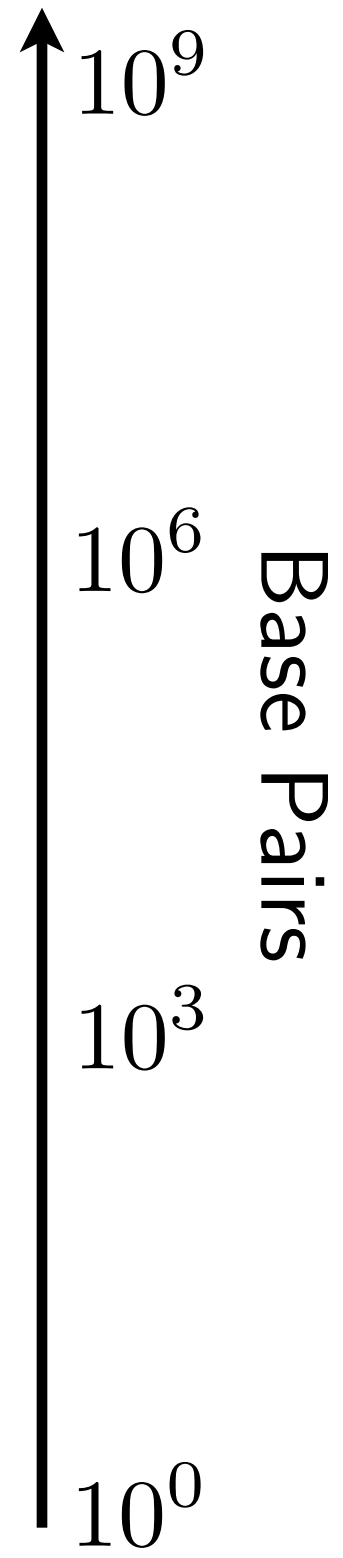
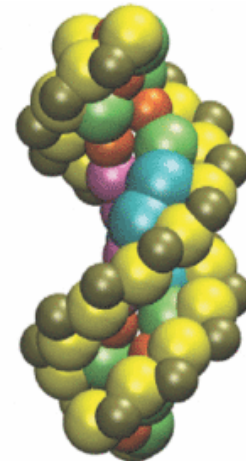
Sequence Assembly



Fragment Sequencing



..... Sequencing Read Lengths .....



Margulies et al. *Nature*. 437, 376 (2005)  
Speicher and Carter. *Nat. Rev. Genetics*. 6, 782 (2005)  
Hinckley et al. *J. Chem. Phys.* 139, 144903 (2013)  
Zhou et al. *Appl. Environ. Microbiol.* 68, 6321 (2002)

# The genome puzzle



<http://www.turbosquid.com/FullPreview/Index.cfm/ID/438424>

<http://www.gotchocolate.com/2011/02/1000-piece-chocolate-jigsaw-puzzle/>

# Genomic variability

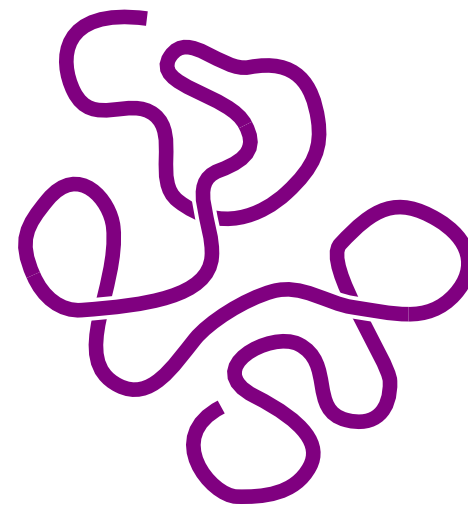
Variability is a multi-scale phenomena

Chromosome



Mapping

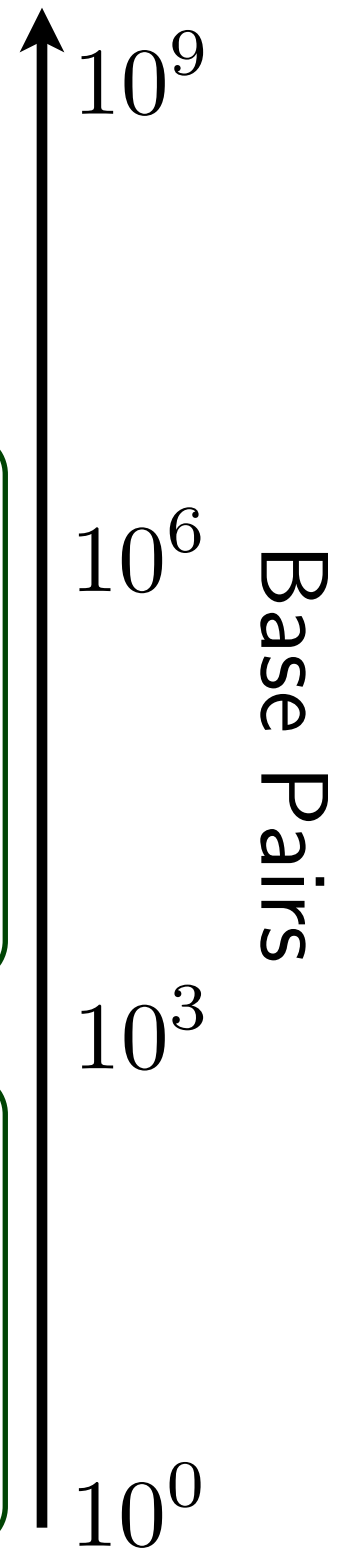
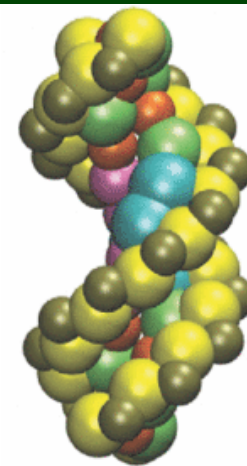
Gene



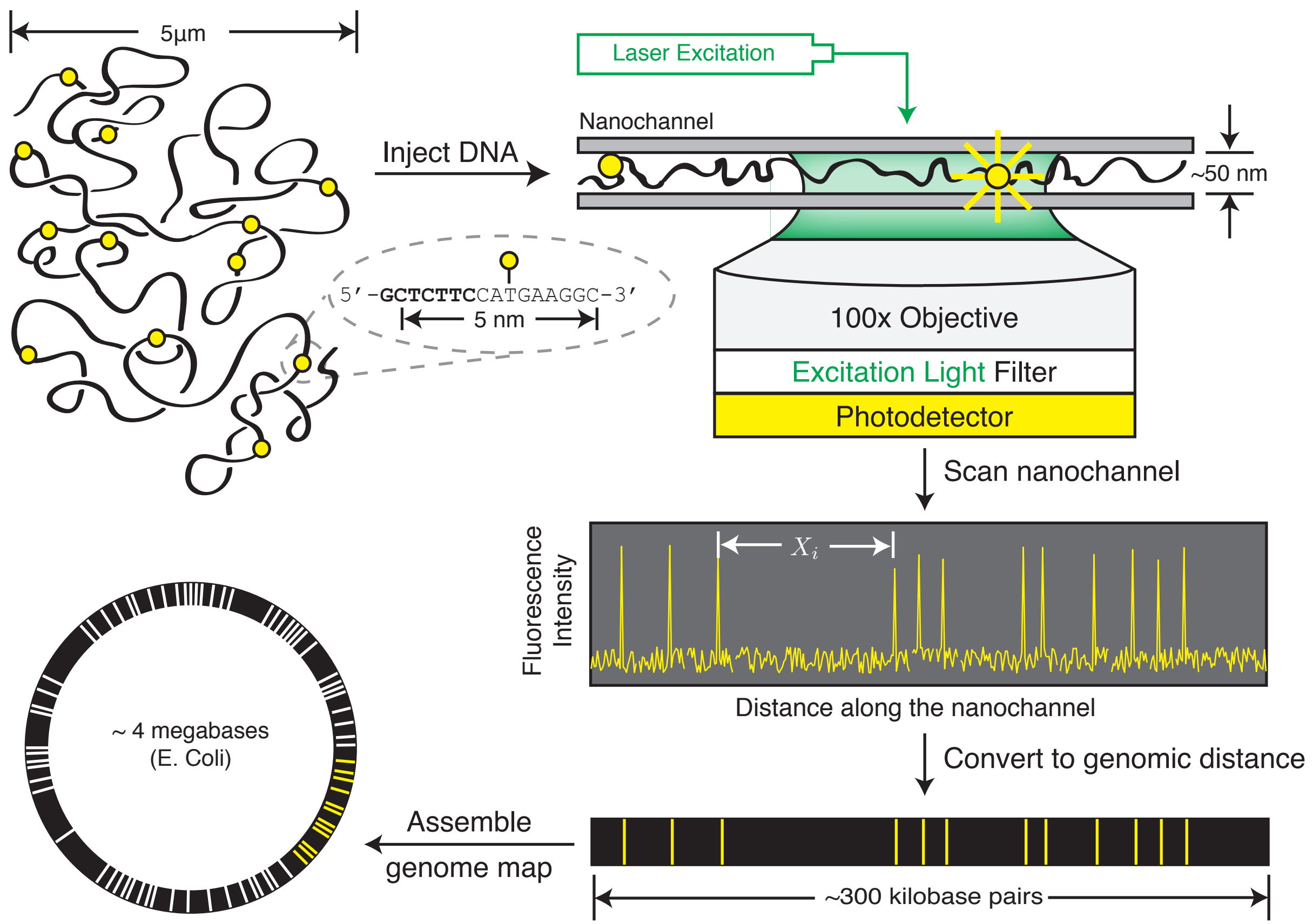
..... Sequencing Read Lengths .....

Sequencing

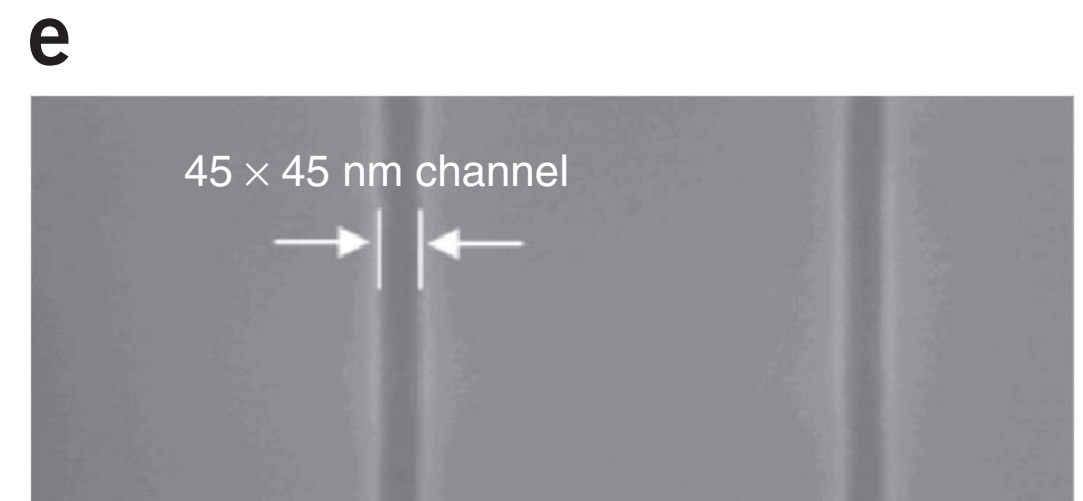
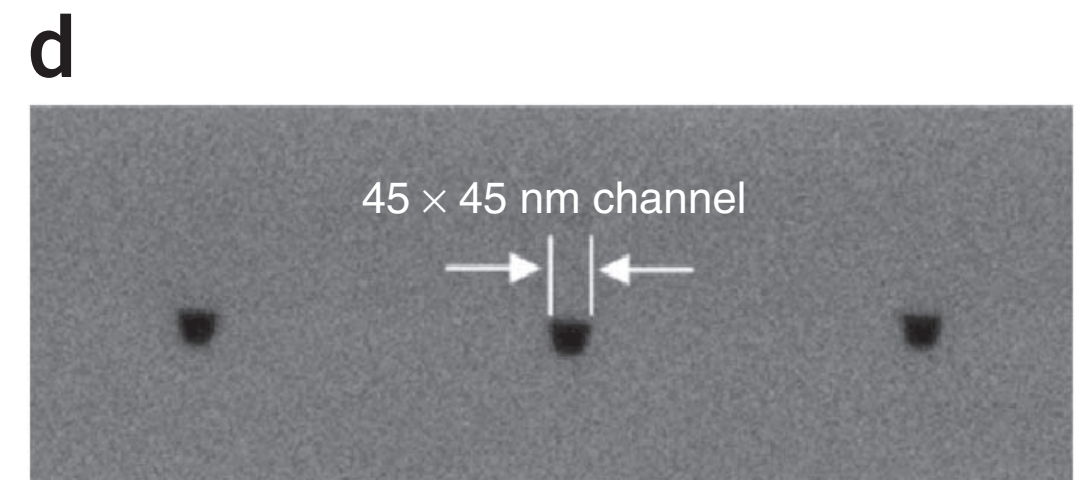
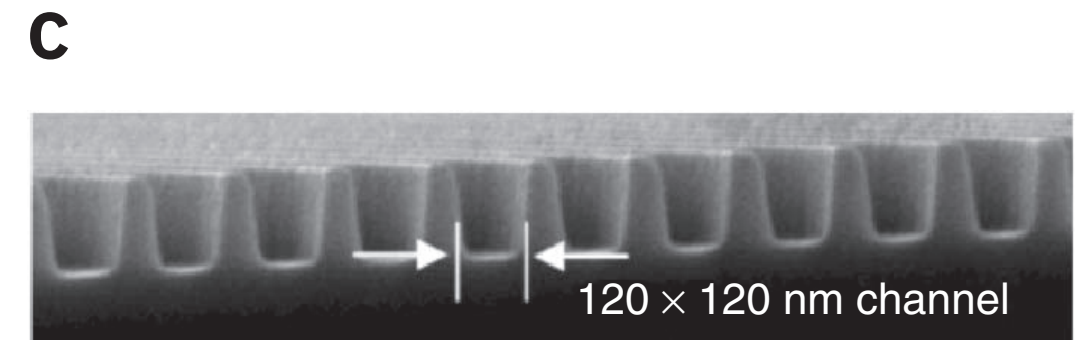
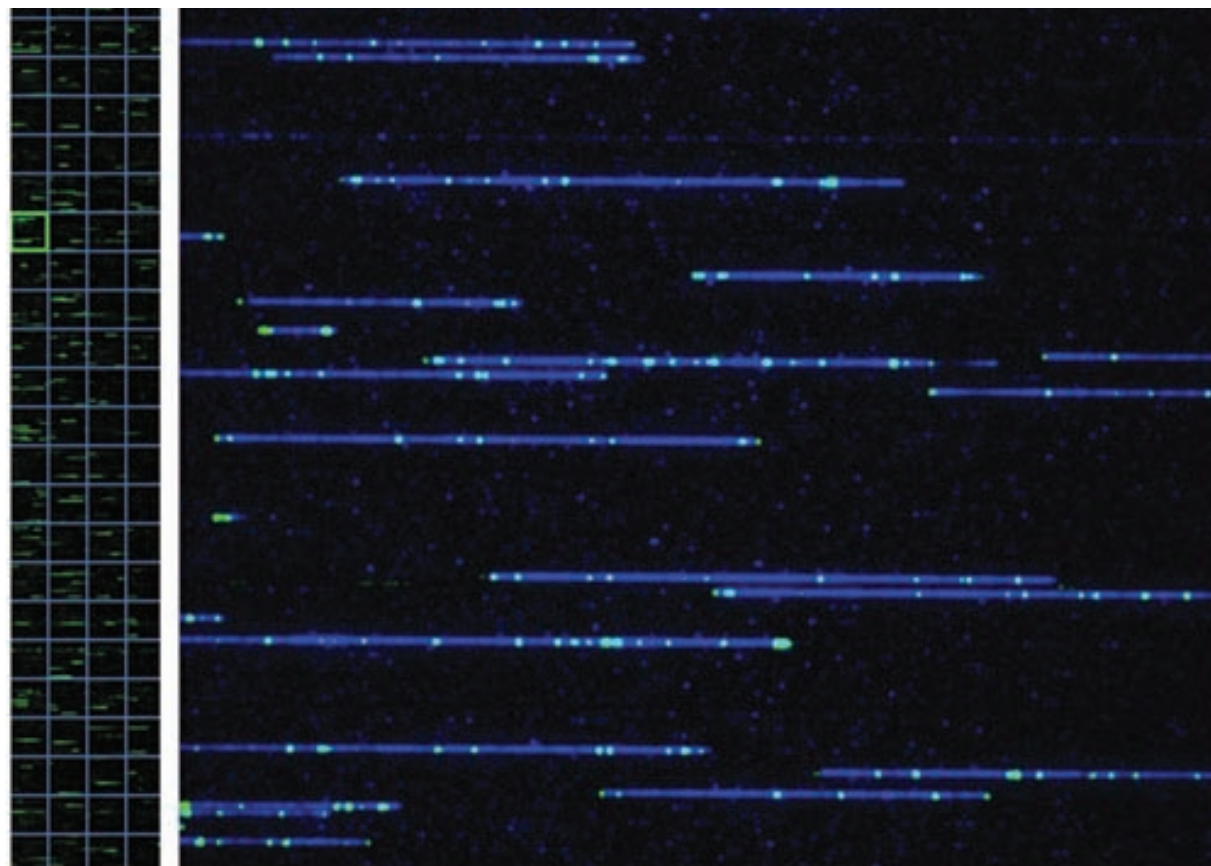
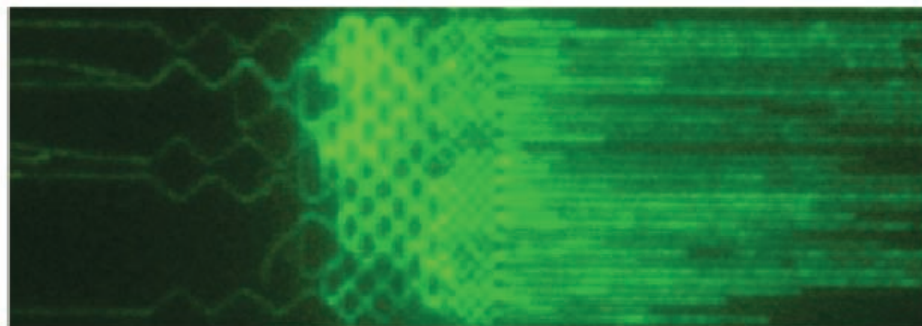
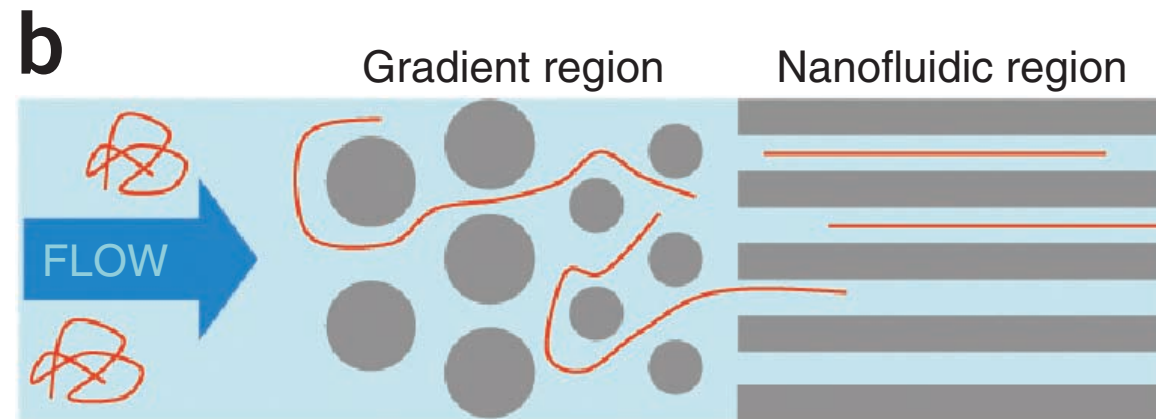
Nucleotide



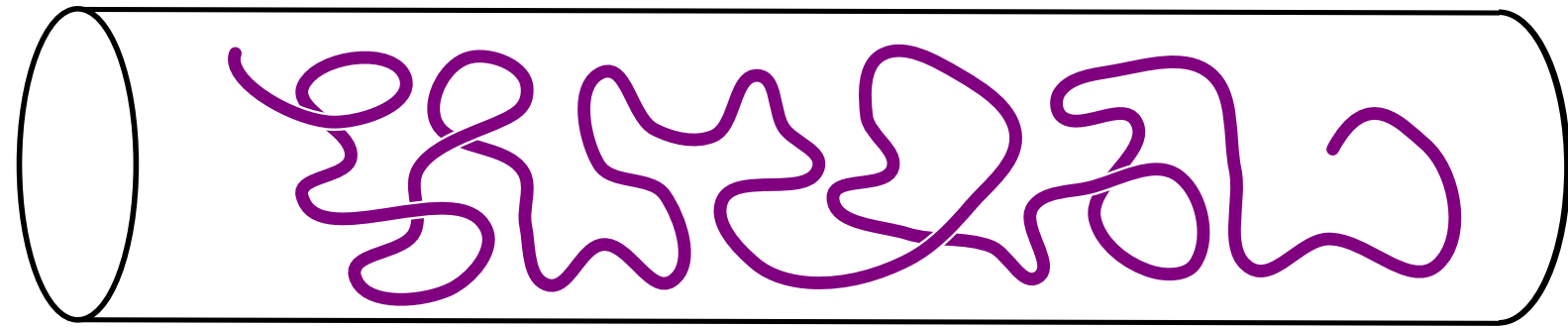
# Genomic mapping



# DNA linearization by confinement

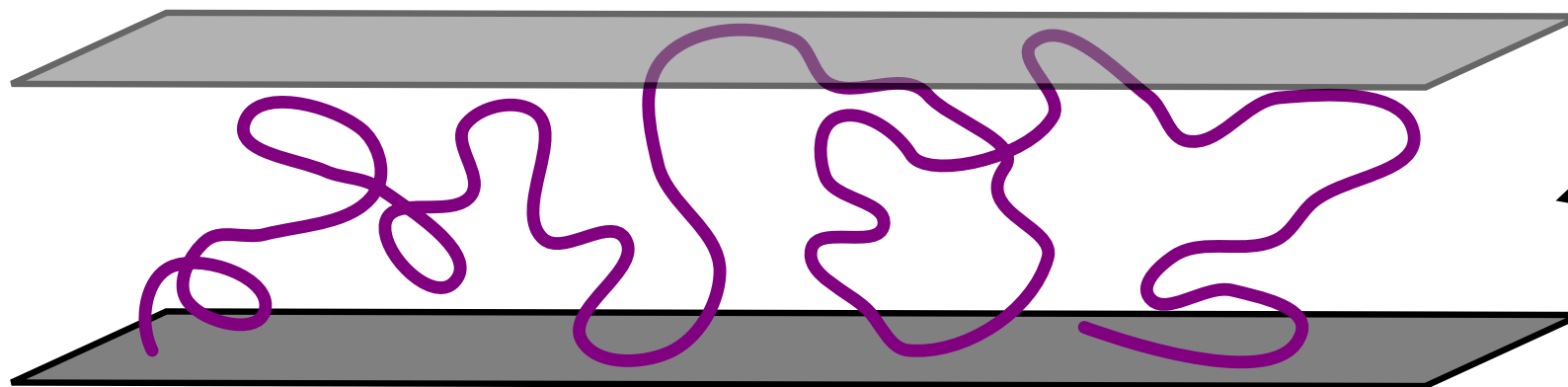
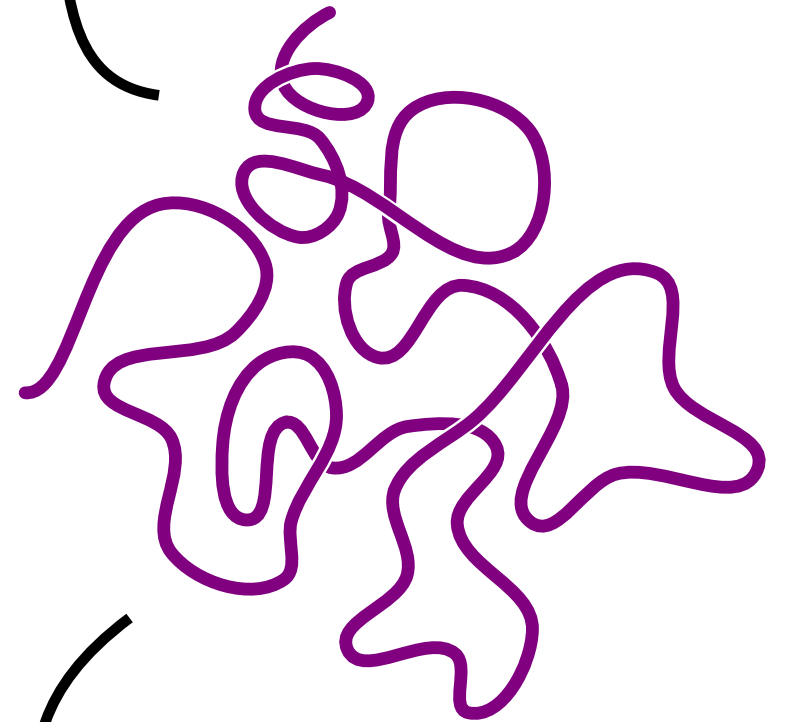


# DNA confinement is complicated



Nanochannel

- Polymer Physics
- Hydrodynamics
- Electrostatics
- Geometry
- Surface Chemistry



Nanoslit

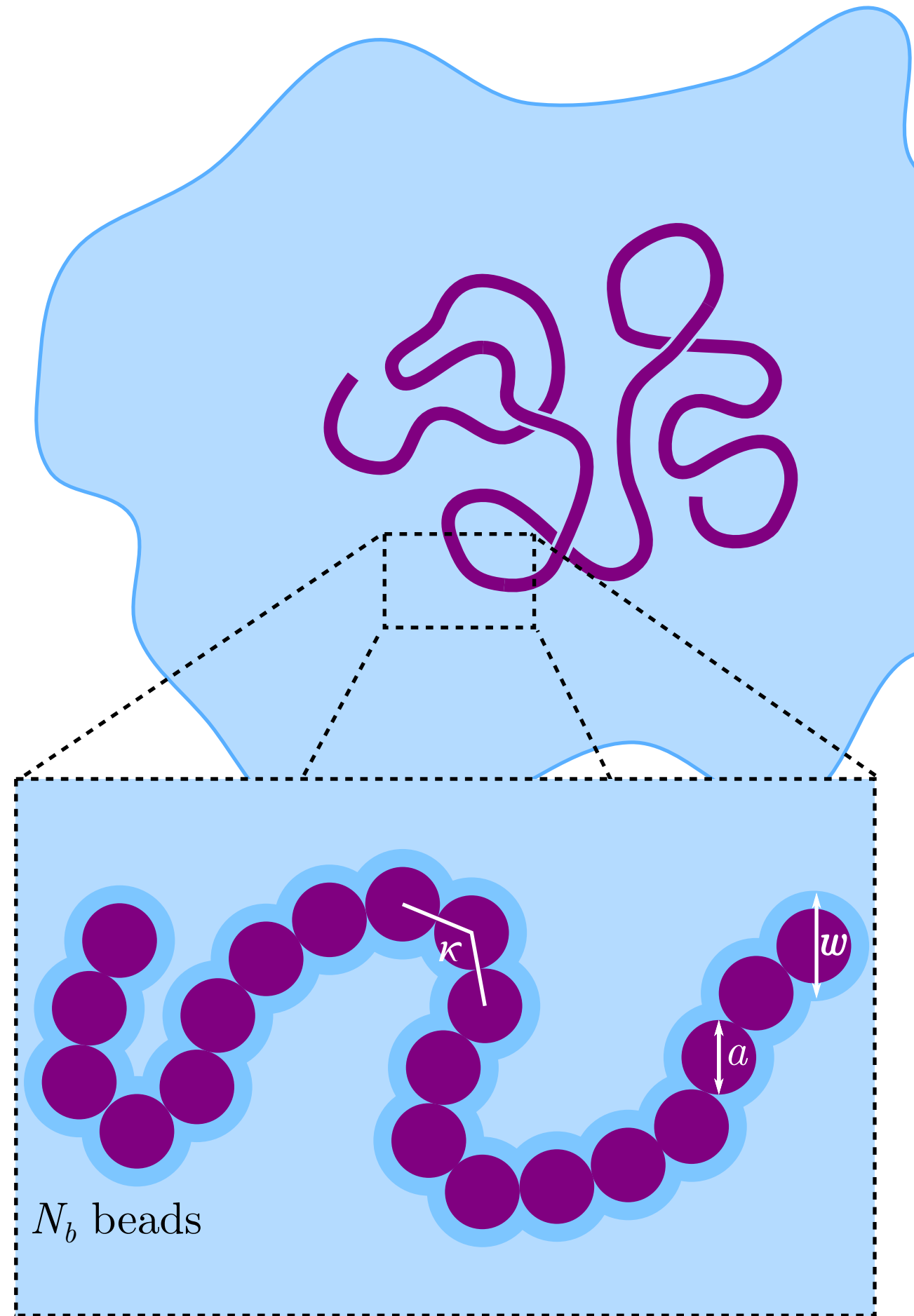
# A Numerical Model

DNA as a wormlike chain:

- sequence independent
- locally stiff
- short-range excluded volume interactions
- neutral polymer

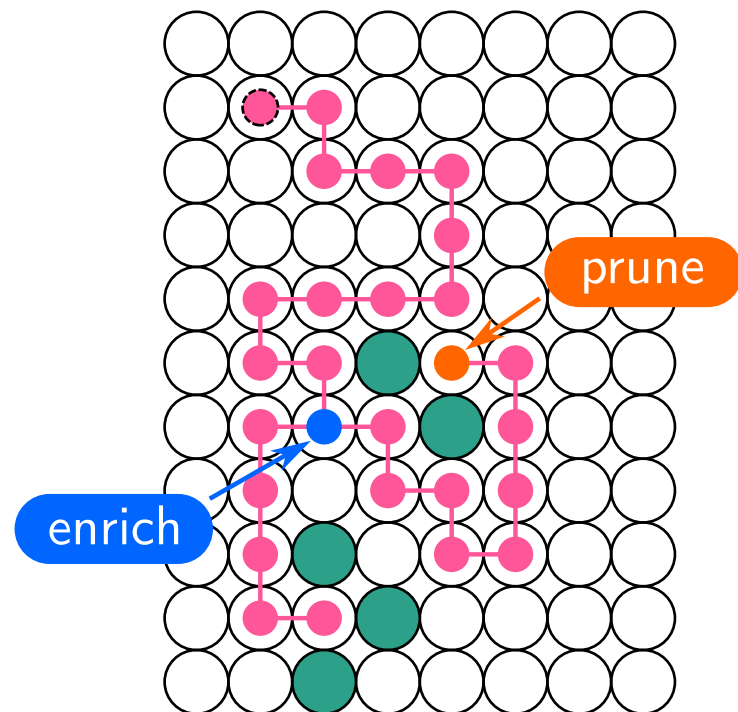
Appropriate length scale:

- Small enough to model confinement
- Large enough to reach experimental molecular weights

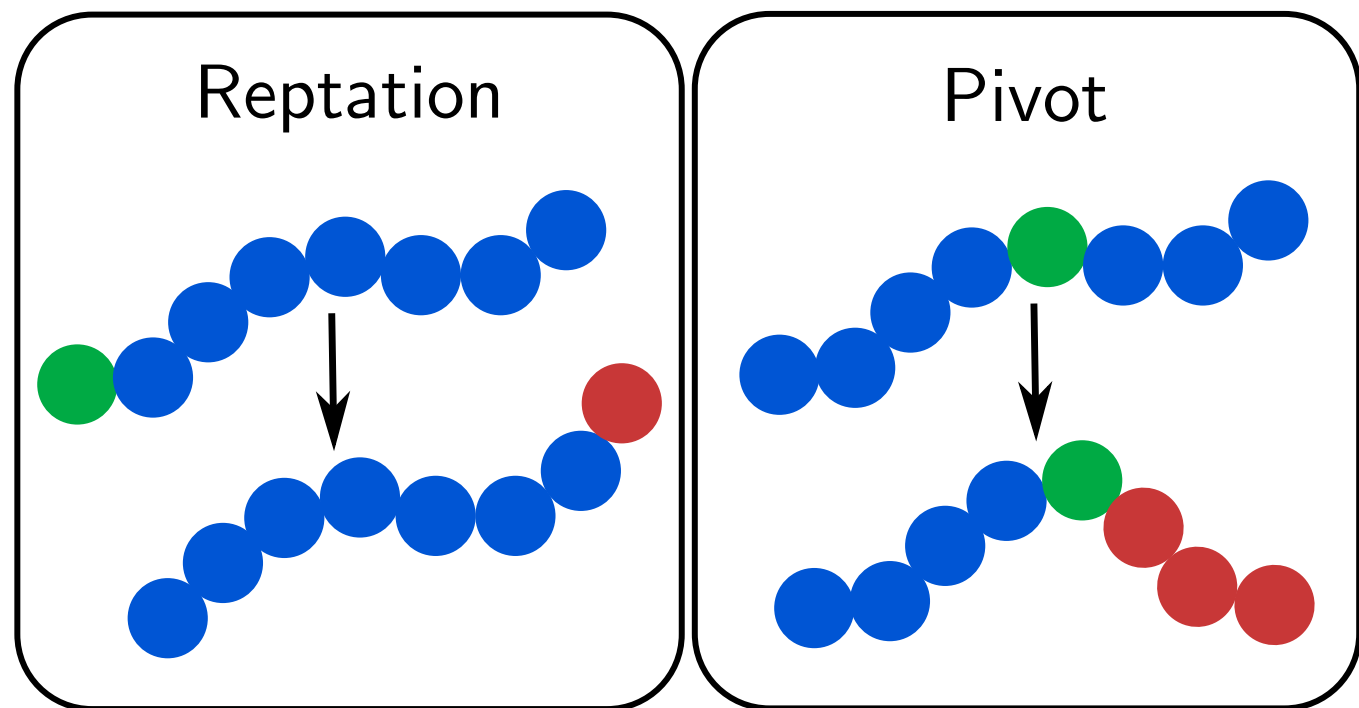


# Advanced Monte Carlo Simulations

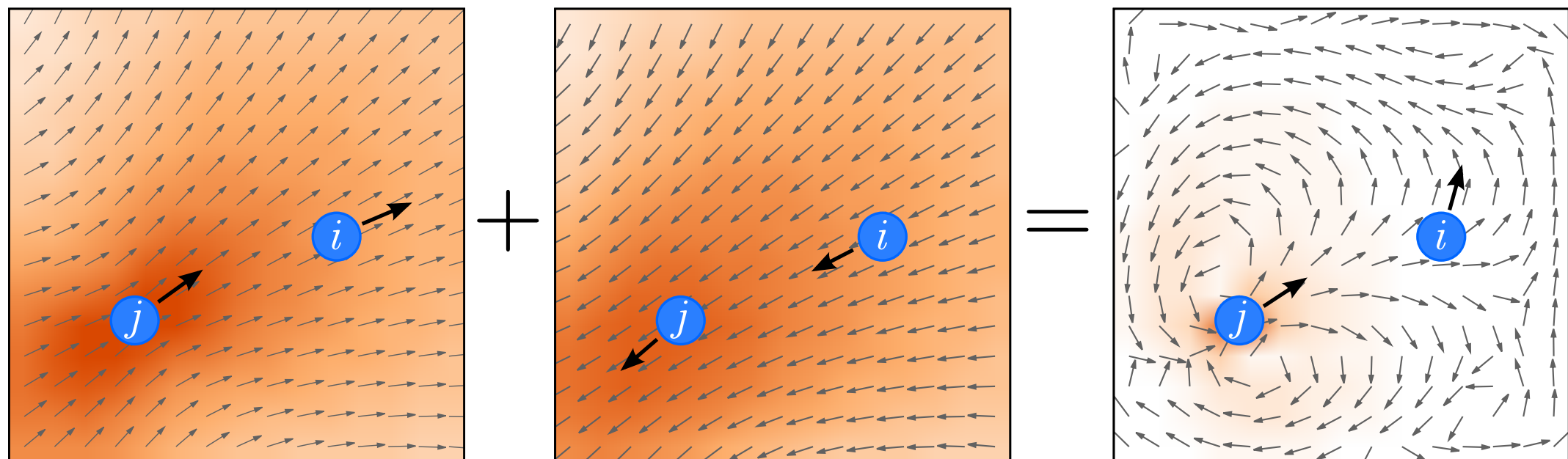
(a) PERM



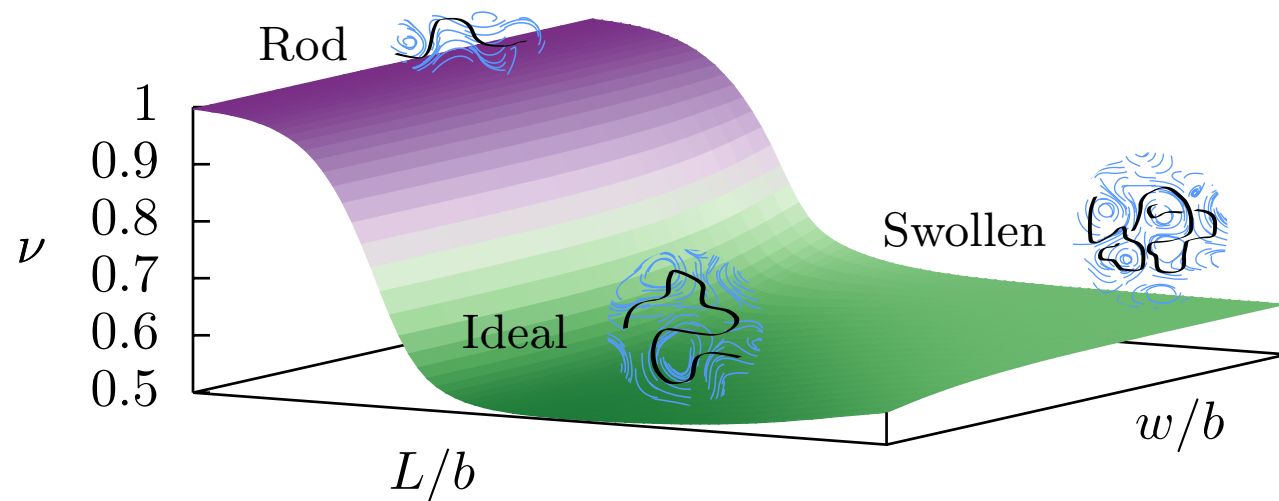
(b) Metropolis



## CFD for Stokes Hydrodynamics

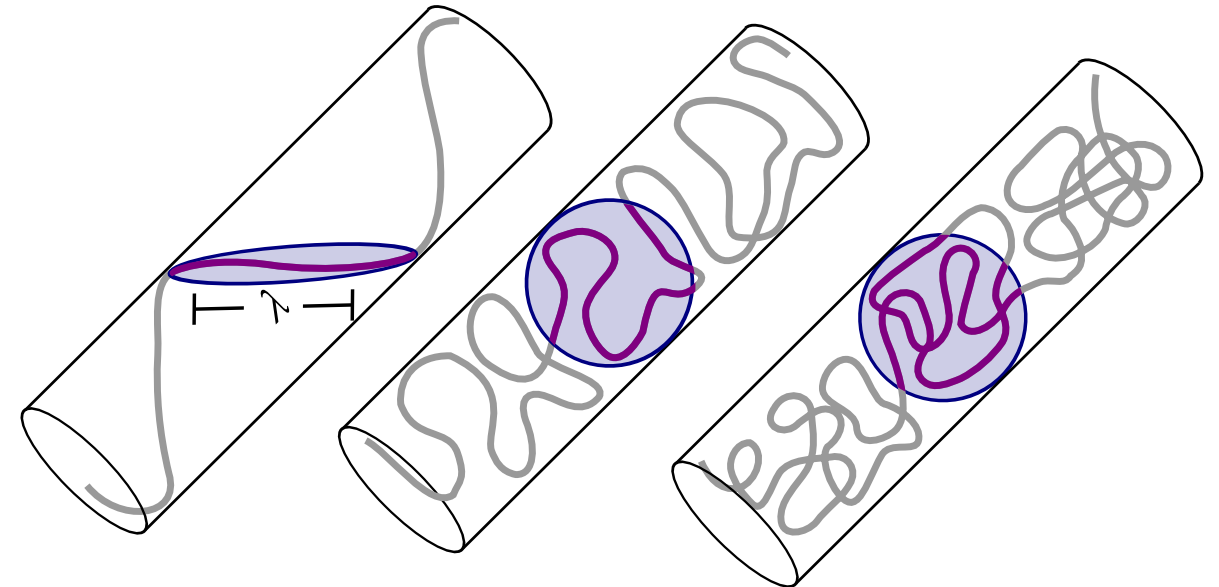


## DNA in Free Solution



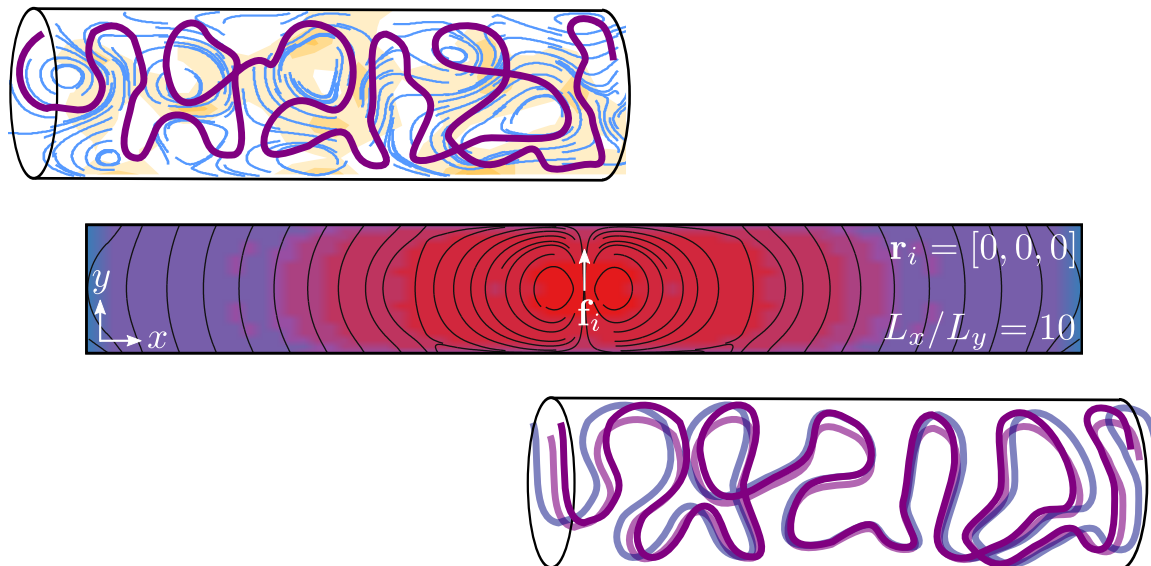
Tree et al. *Macromolecules* 46, 8369 (2013)

## Regimes of Confined DNA



Tree et al. *Phys. Rev. Lett.* 110, 208103 (2013)

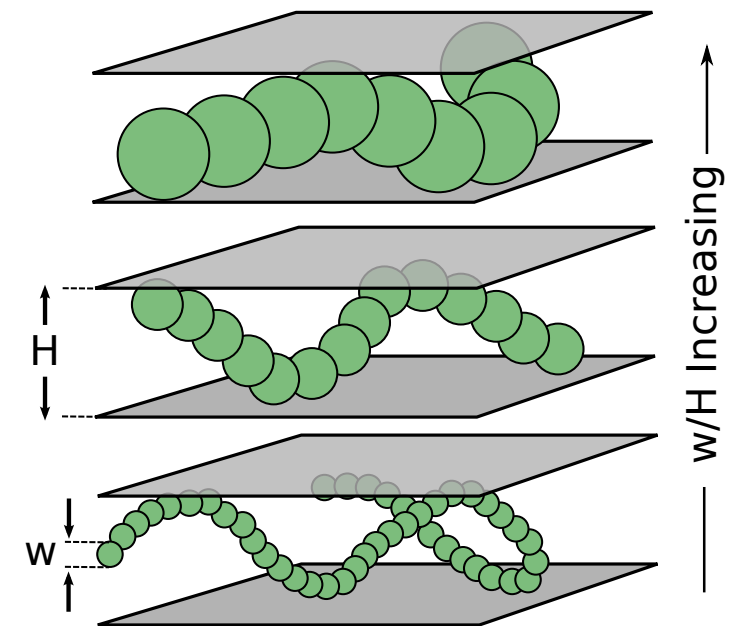
## Dynamics of Confined DNA



Tree et al. *Phys. Rev. Lett.* 108, 228105 (2012)

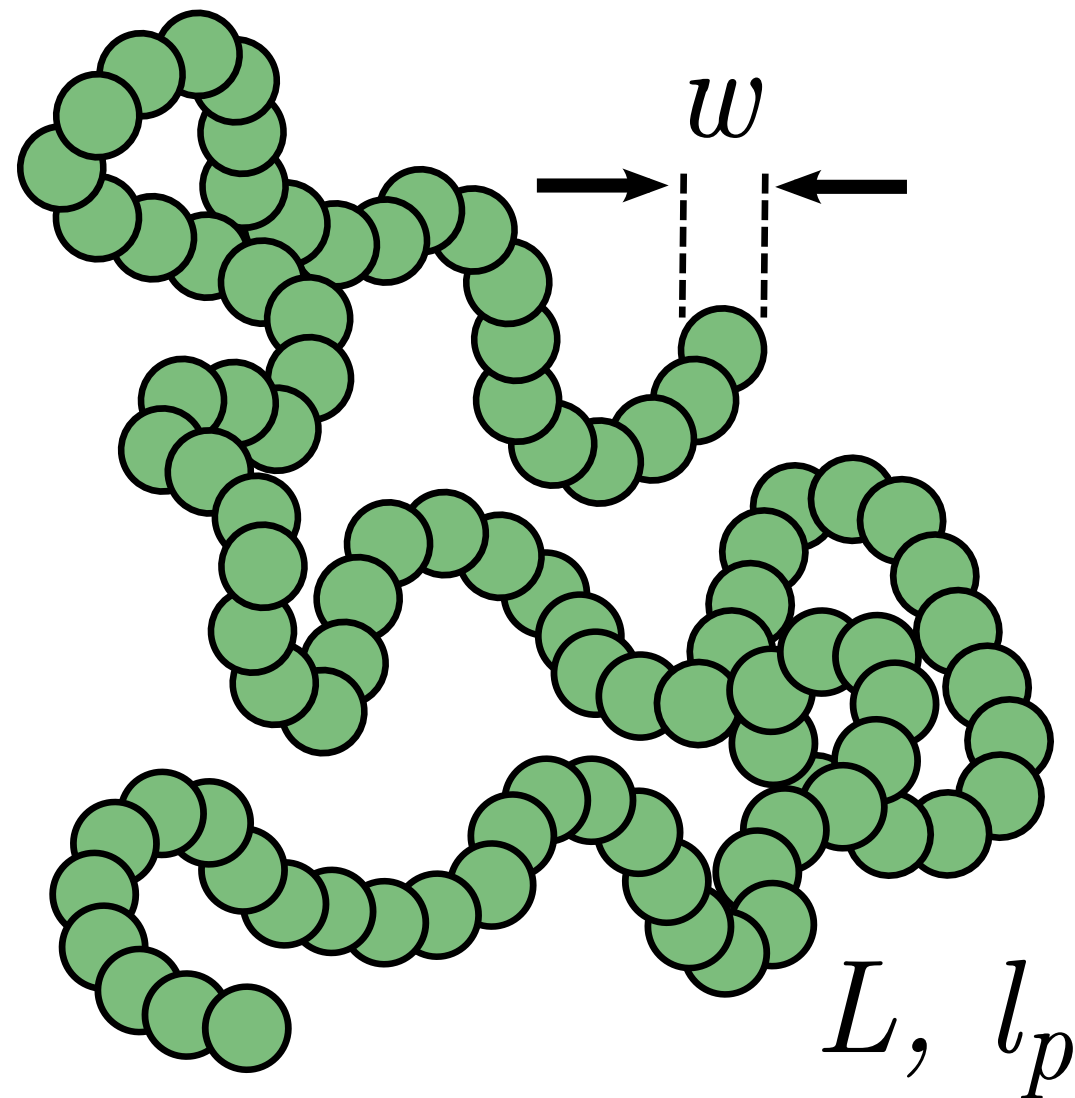
Tree et al. *Biomechanics* 7, 054118 (2013)

## DNA in Slits

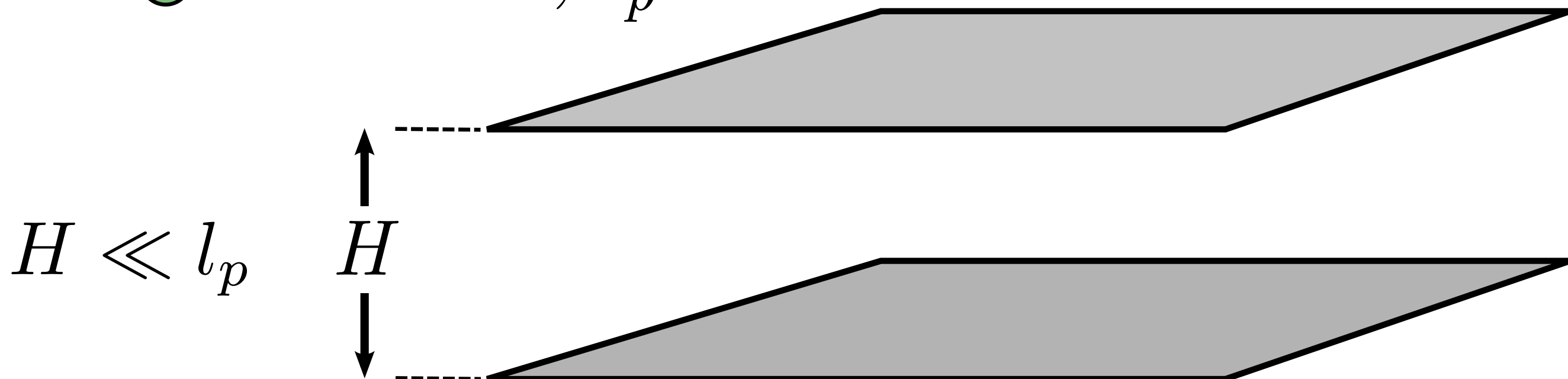


Tree et al. *In Review*

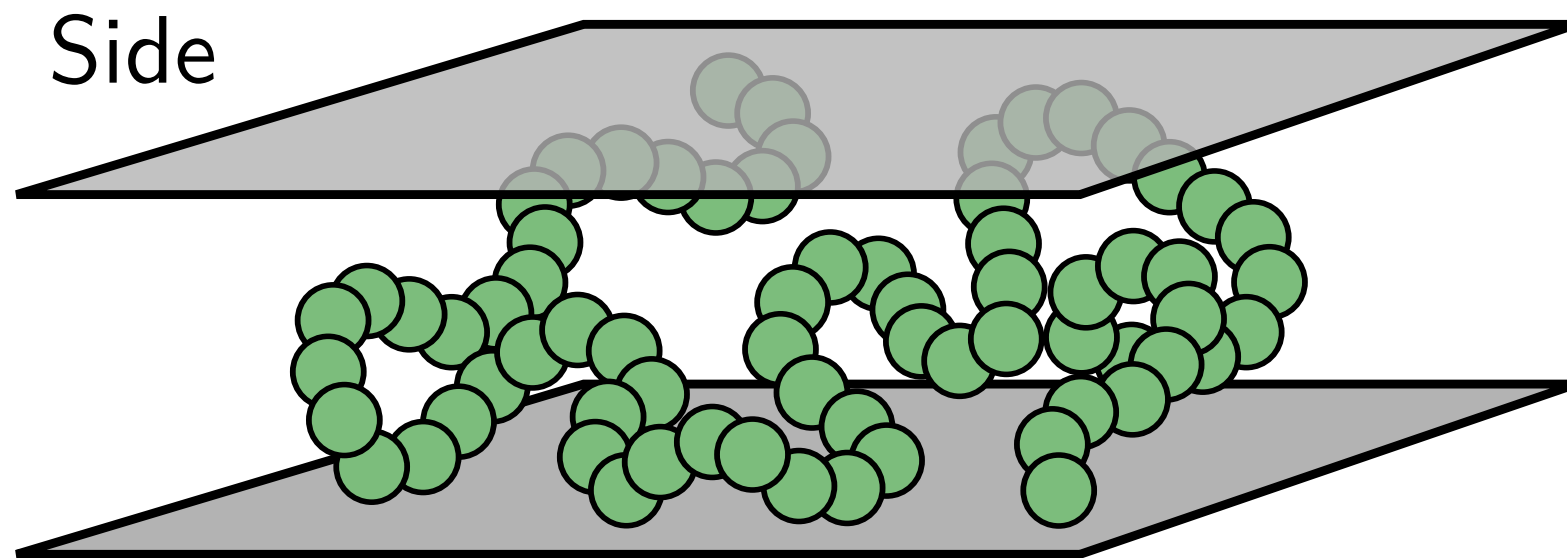
# DNA strongly confined in a slit



What are the equilibrium properties of a wormlike chain confined in a slit much smaller than its persistence length?



# Weakly confined chains

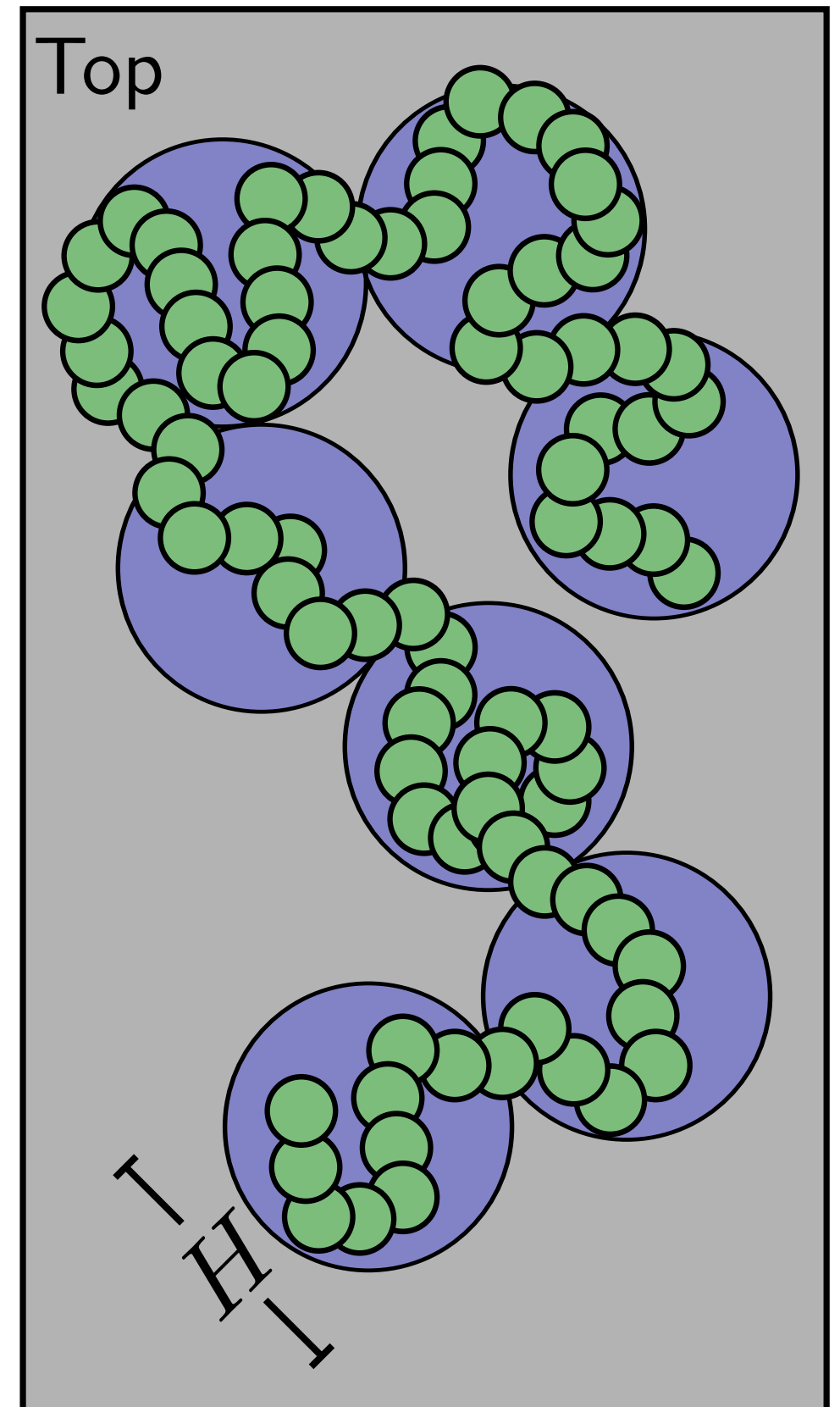


$$H \gg l_p$$

Blob Theory (de Gennes)

$$R \approx L^{\frac{3}{4}} l_p^{\frac{1}{4}} \left( \frac{w}{H} \right)^{\frac{1}{4}}$$

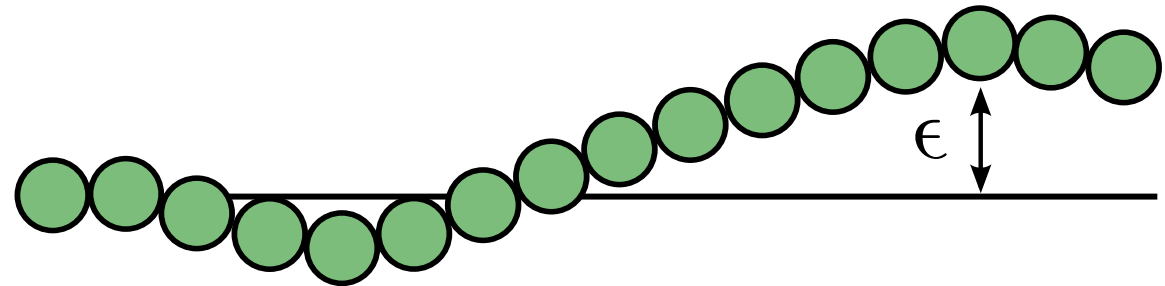
$$F \approx k_B T L l_p H^{-\frac{5}{3}} w^{-\frac{1}{3}}$$



# Deflection Segments (Odijk)

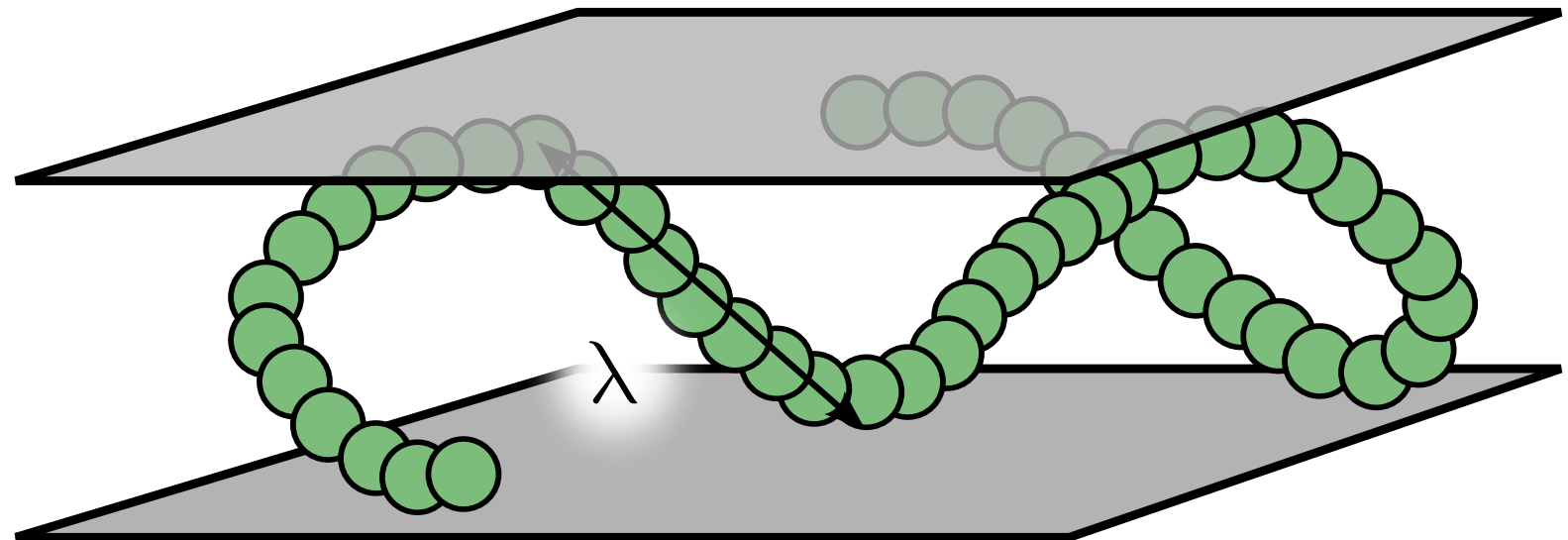
Mean square deviation  
of a free solution WLC

$$\langle \epsilon^2 \rangle \approx \frac{\lambda^3}{l_p}$$



Confinement  
Length Scale

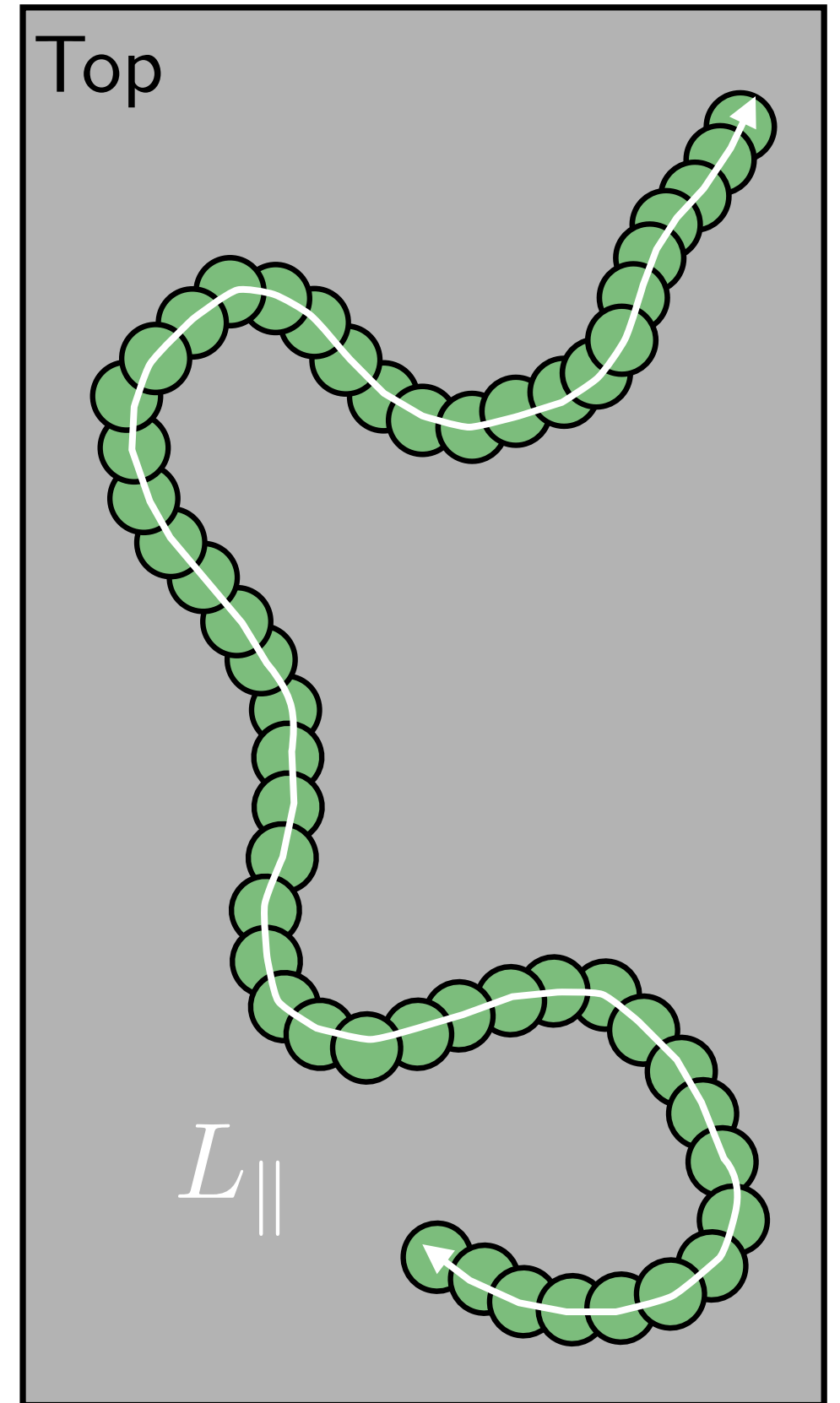
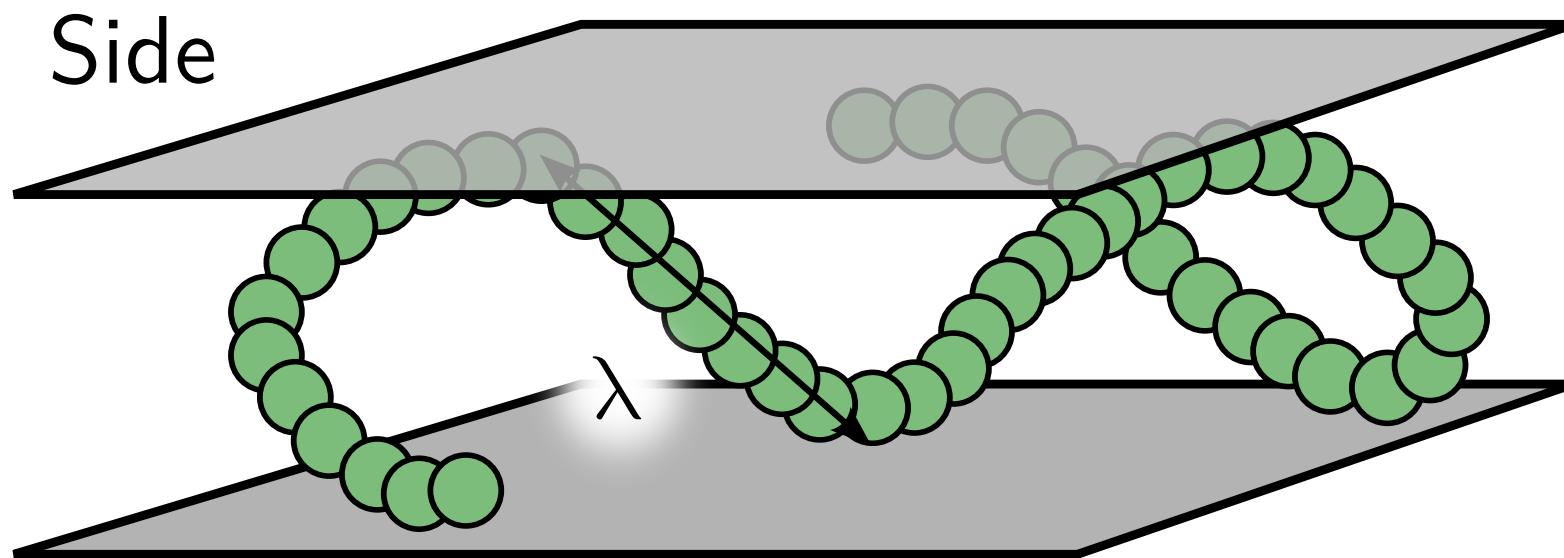
$$\langle \epsilon^2 \rangle \approx H^2$$



Deflection Length

$$\lambda \approx l_p^{\frac{1}{3}} H^{\frac{2}{3}}$$

# Ideal Chains



$$\frac{L_{||}}{L} = 1 - 0.09137 \frac{\lambda}{l_p}$$

$$\frac{F}{k_B T} = 1.1036 \frac{L}{\lambda}$$

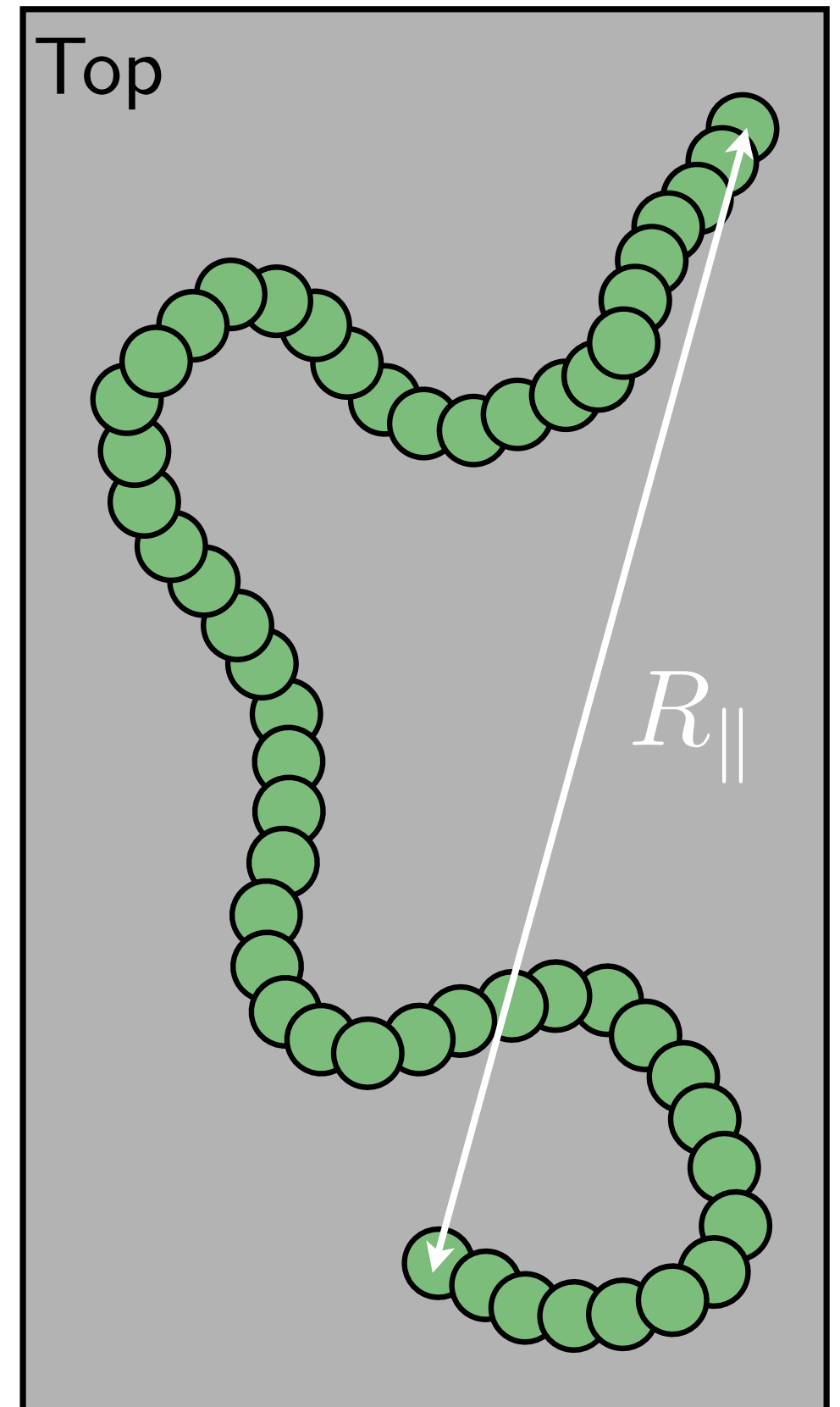
# Real Chains

- Incorporate excluded volume
- Hypothesis of Dai *et al.*: DNA is a 2D self-avoiding walk of a virtual projected chain.

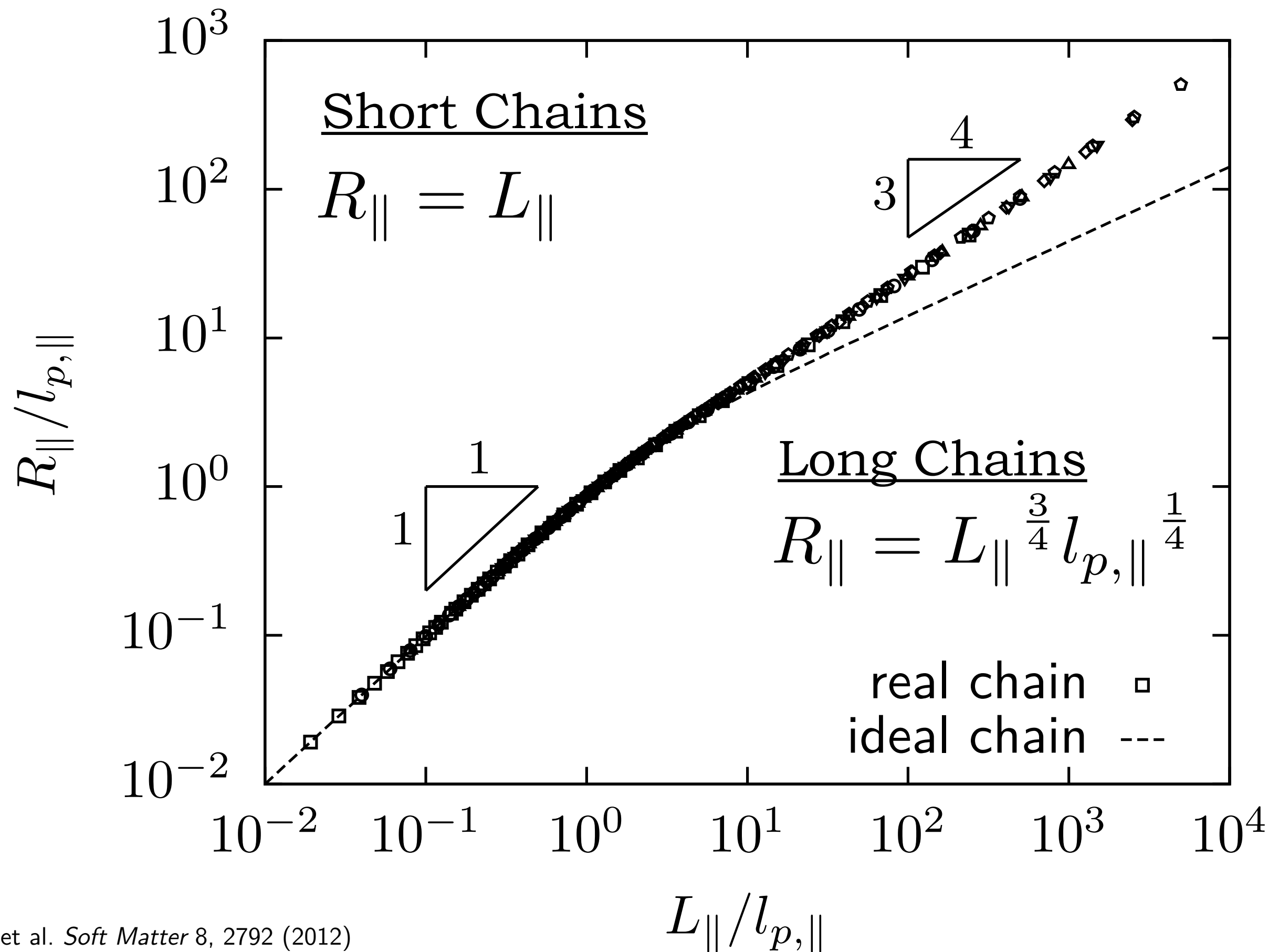
$$a_{\text{excluded}} = l_{p,\parallel}^2$$

- Parameters for virtual chain are obtained by mapping back to 3D chain

$$l_{p,\parallel} = 2l_p$$



# 2D Wormlike Chains



# Pruned Enriched Rosenbluth Method



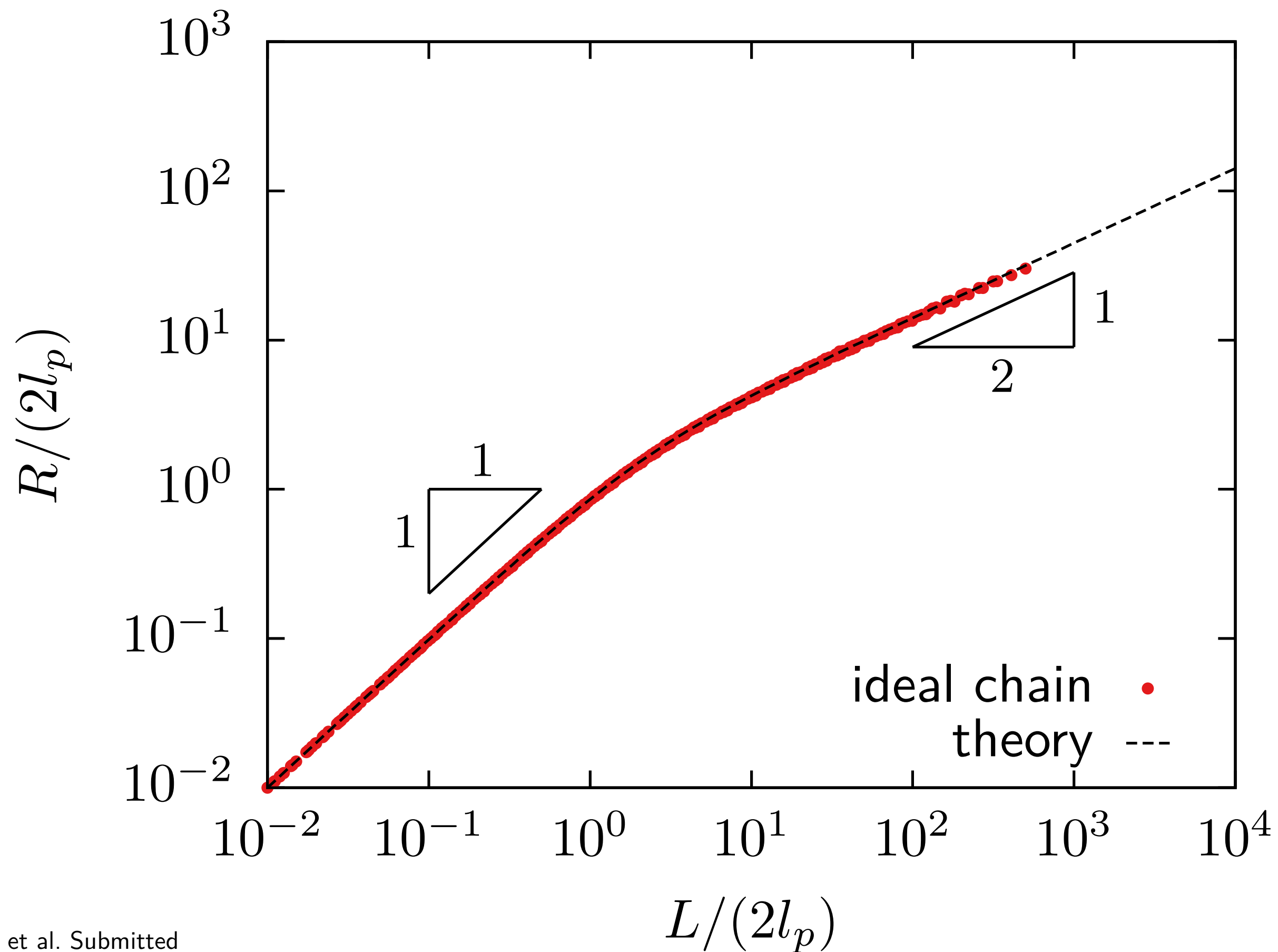
- ▶ Chain-growth Monte Carlo technique
- ▶ Efficient for  $O(10^4-10^5)$  beads
- ▶ Applicable to confined and unconfined systems
- ▶ Off-lattice
- ▶ Can estimate free energies

Tree et al. *Macromolecules*. 46, 8369 (2013)

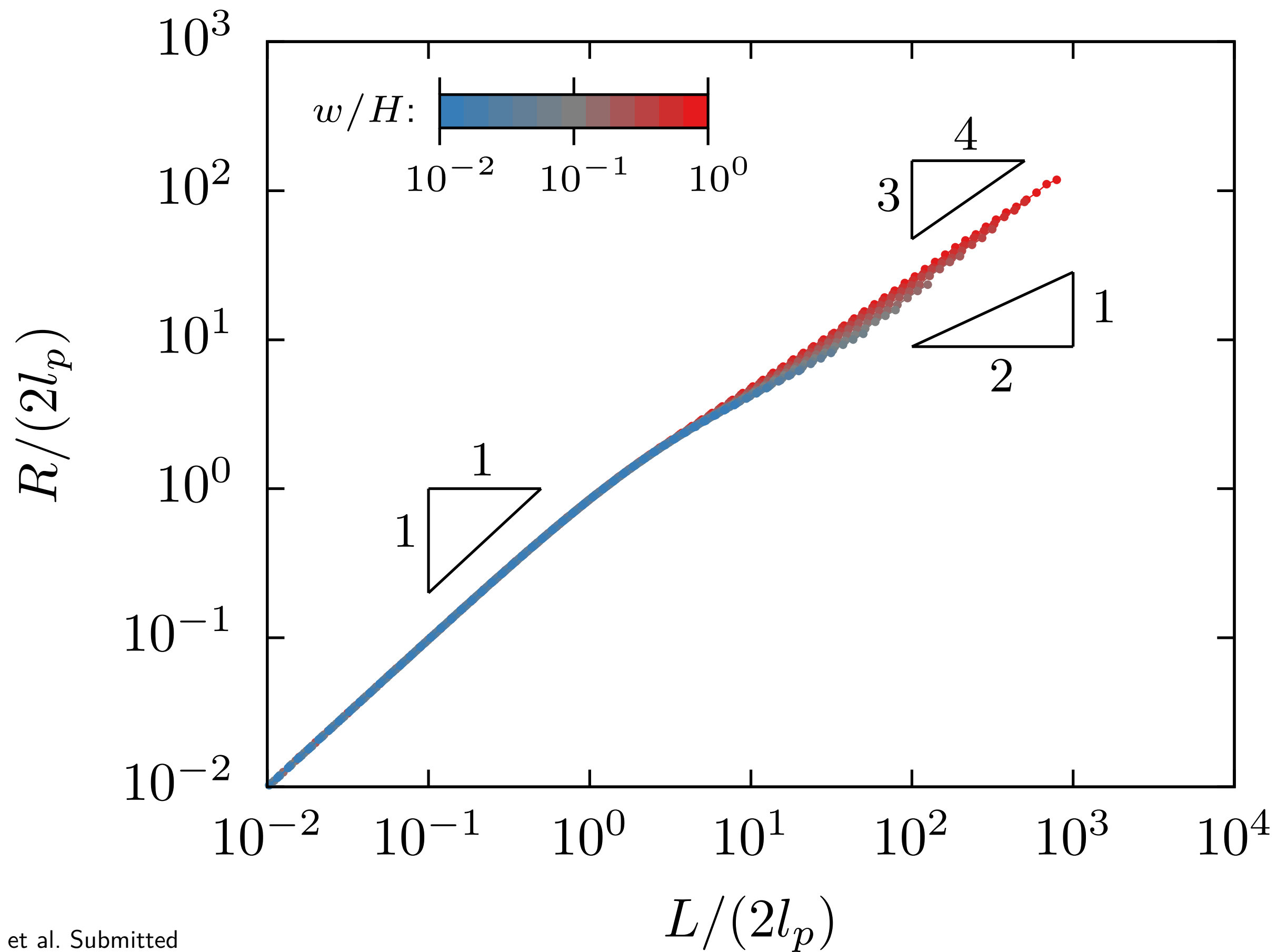
Grassberger. *Phys. Rev. E*. 56, 3682 (1997).

Prellberg and Krawczyk. *Phys. Rev. Lett.* 92, 120602 (2004).

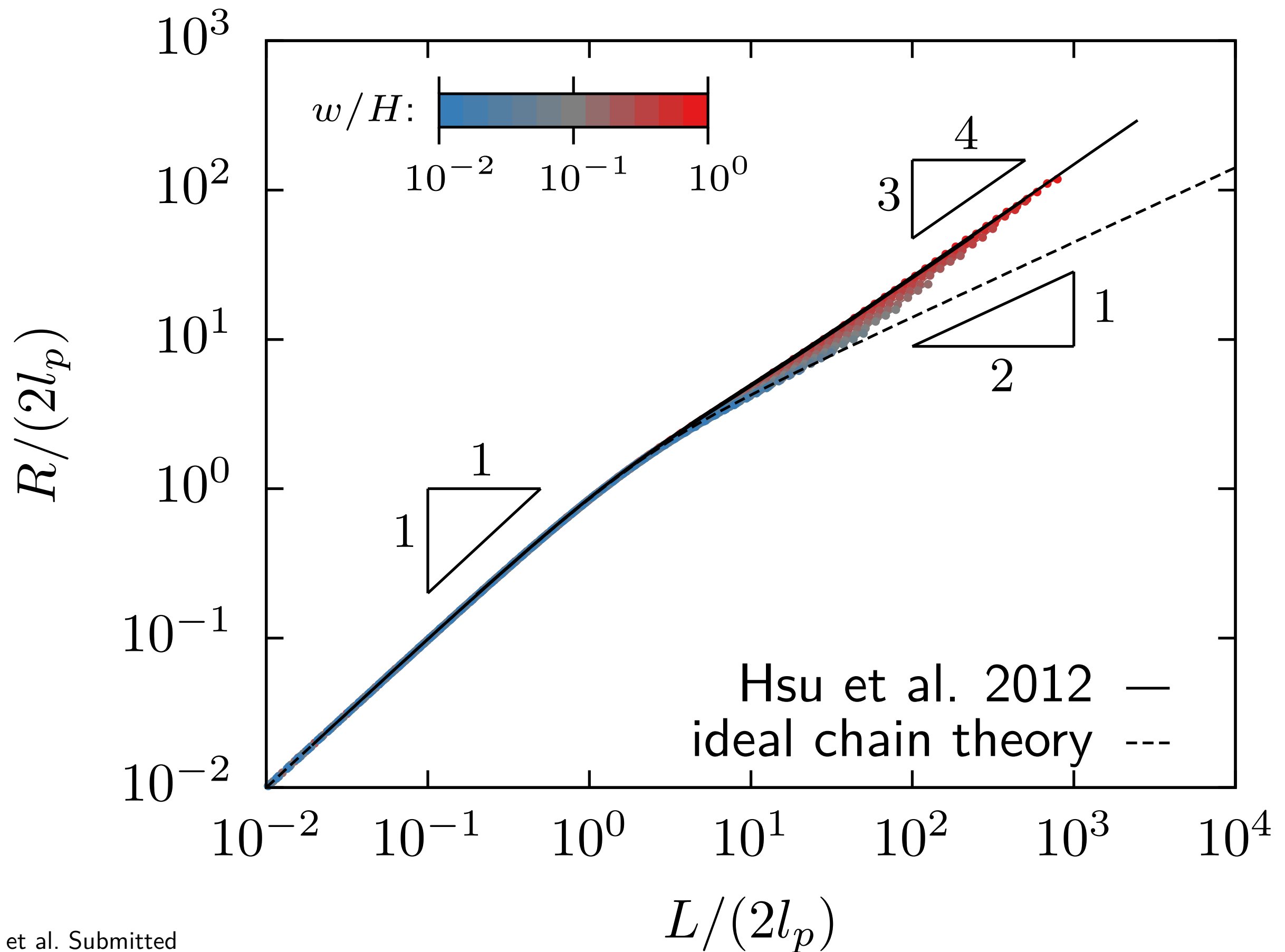
# PERM Results - Ideal Chains



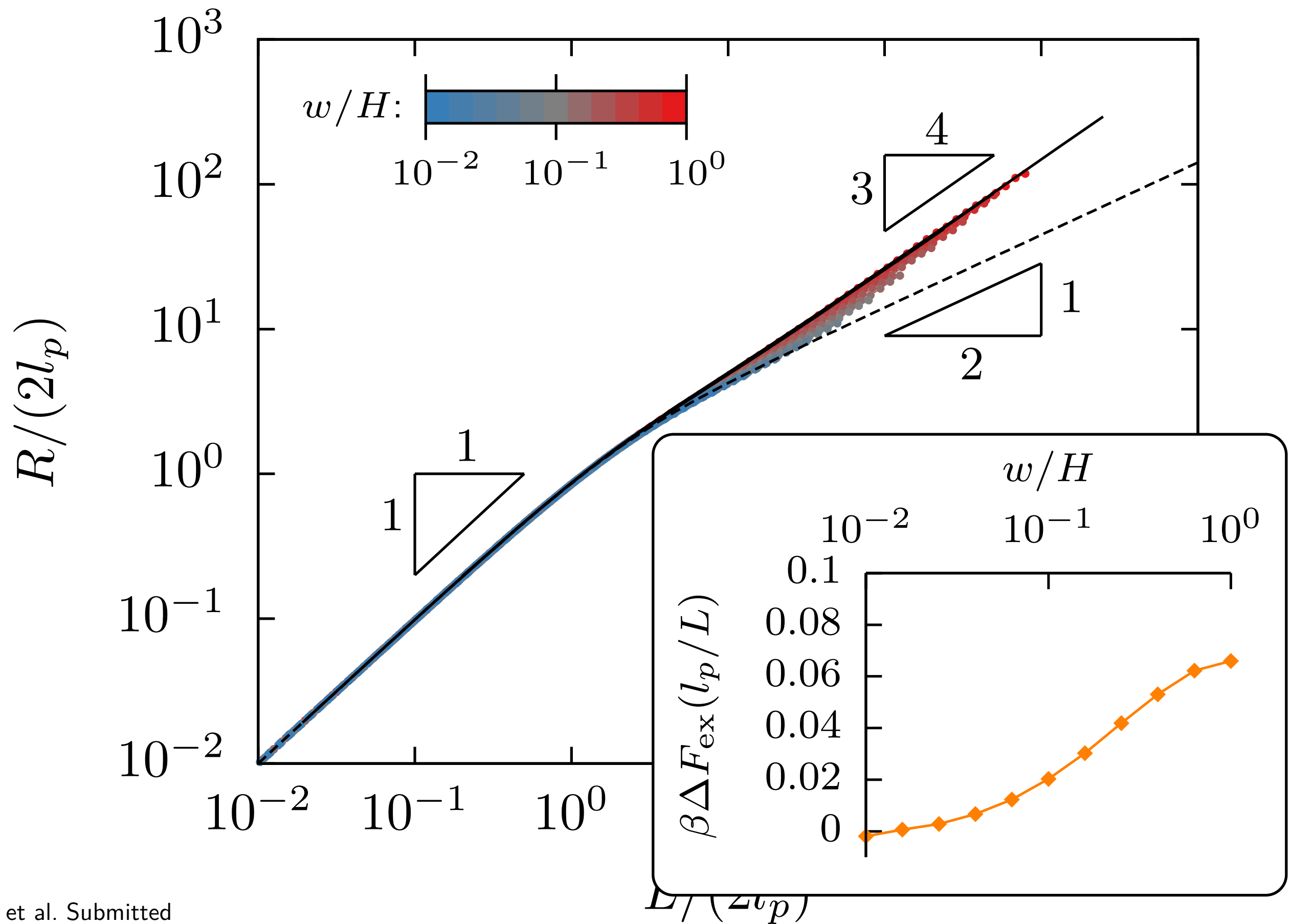
# PERM Results - Real Chains



# PERM Results - Real Chains

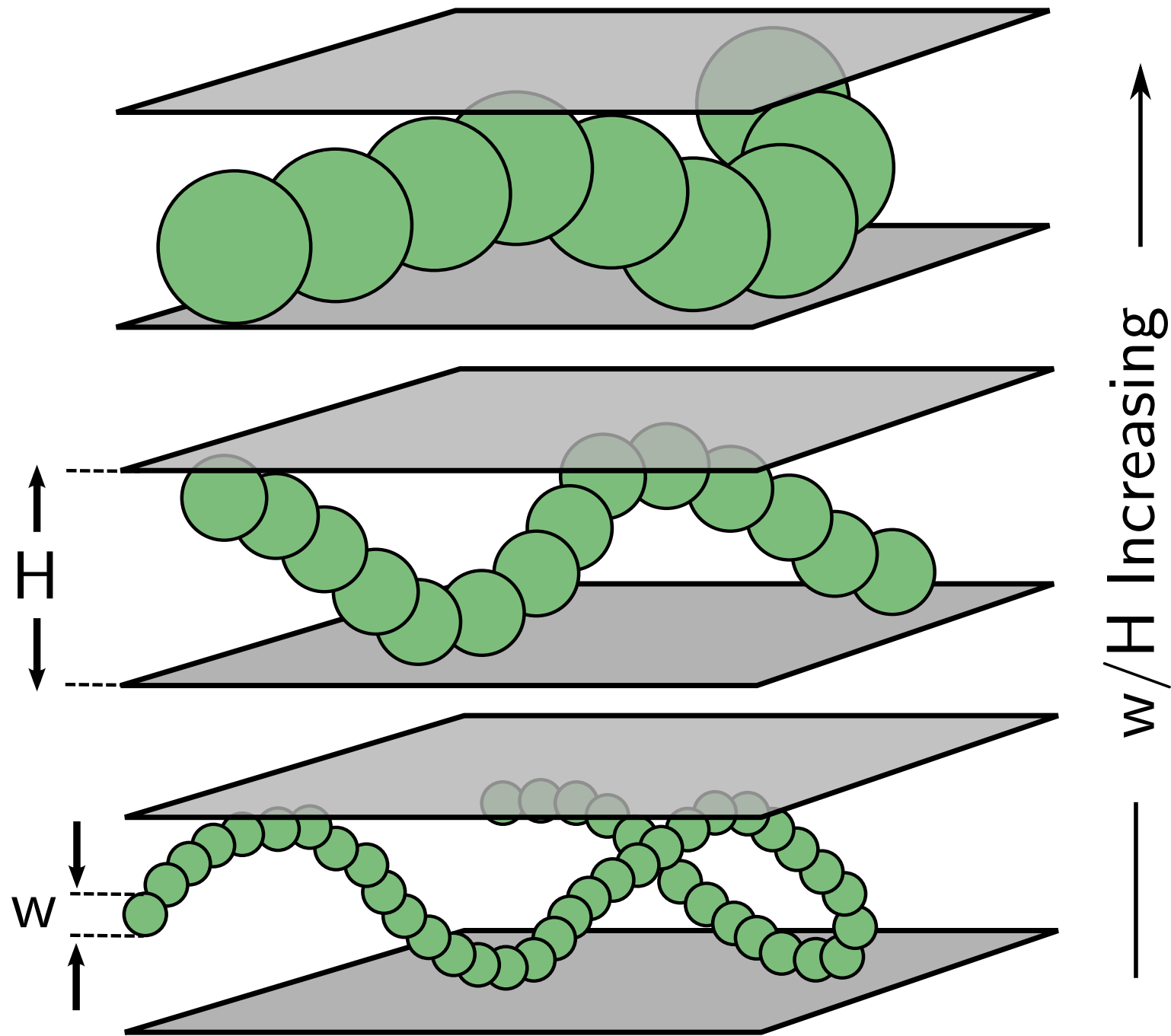


# PERM Results - Real Chains



# What is wrong?

- ▶ Projected chain theory doesn't treat excluded volume correctly!



- ▶ But, it has the right idea about a nearly 2D chain.

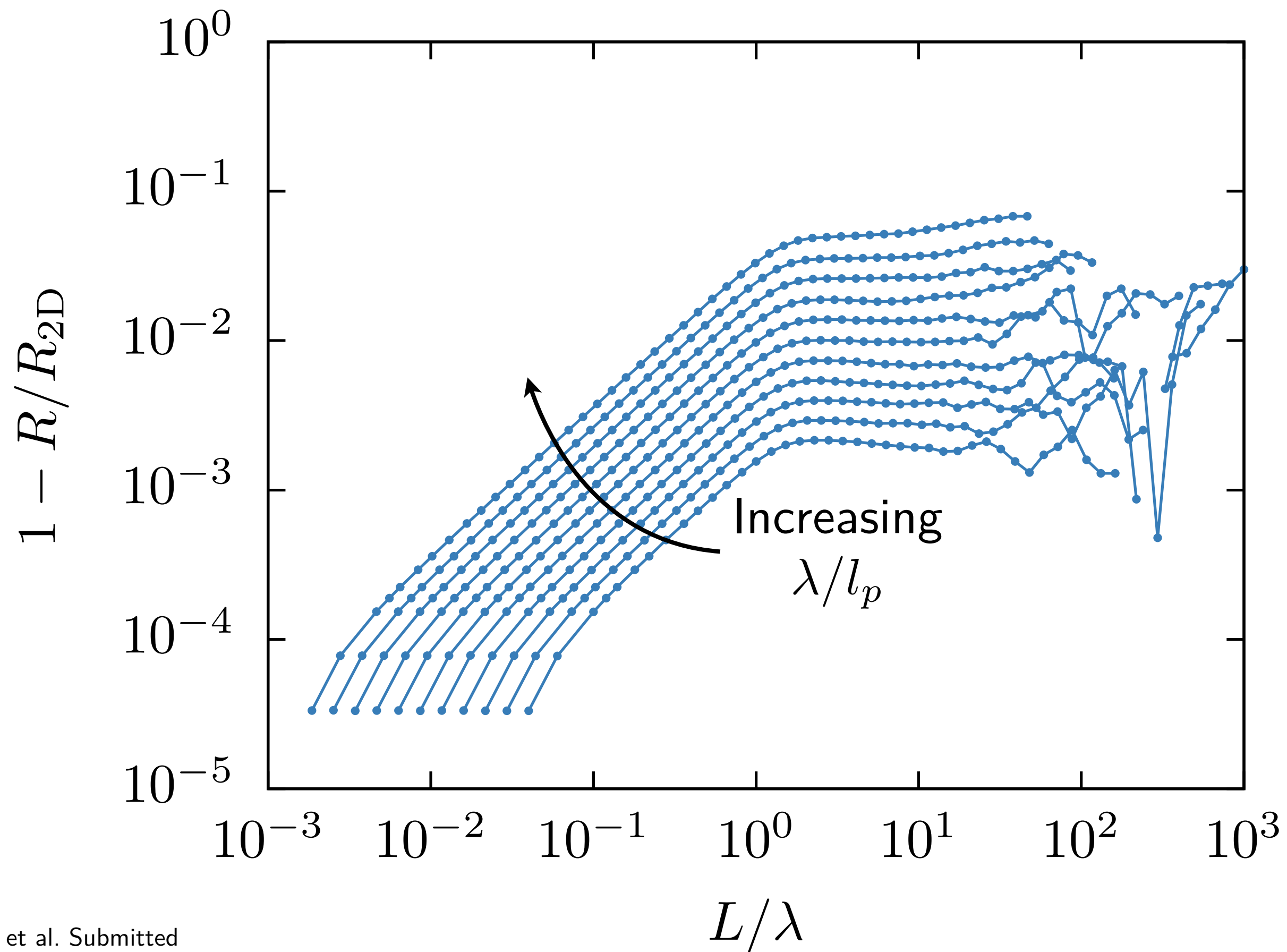
# Following De Gennes - Renormalize

- ▶ Chain is a two-dimensional walk of deflection segments
- ▶ As such we need to incorporate the correct excluded volume...

$$v_{\text{excluded}} = l_p^2 w$$

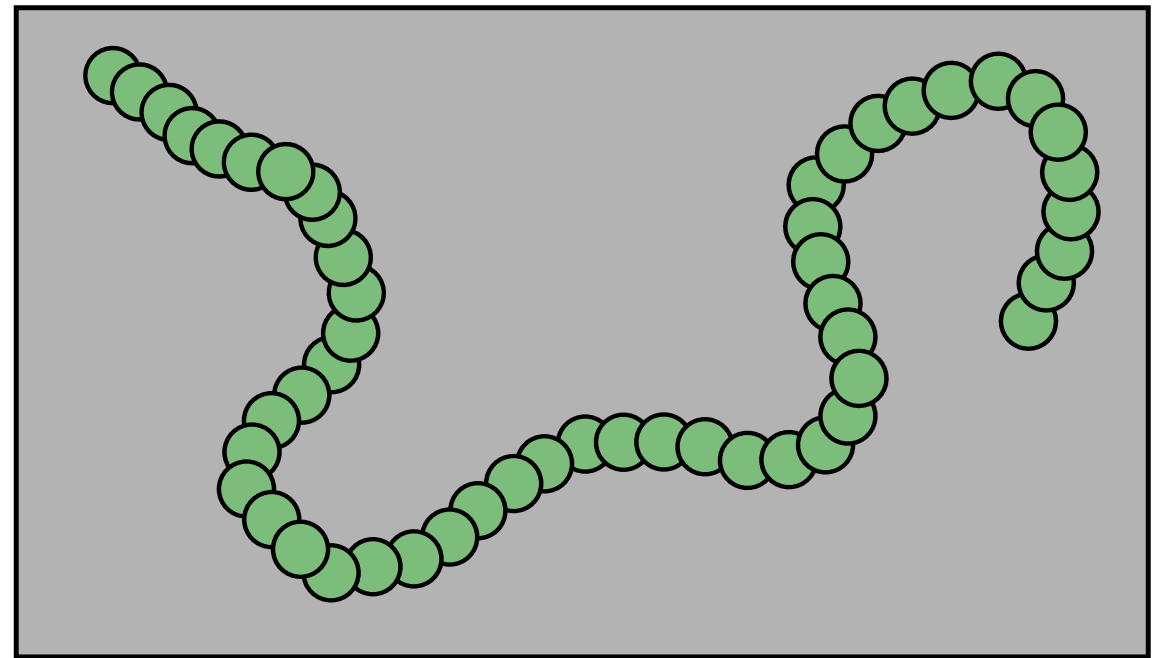
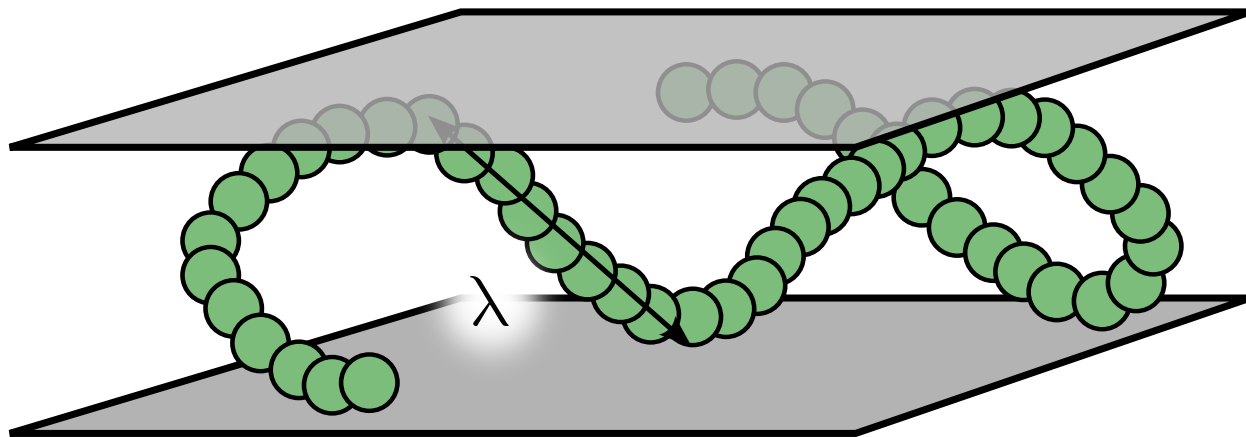
... and the dependence on the deflection segment length

$$R = R_{2\text{D}} h \left( \frac{\lambda}{l_p} \right)$$



# Conclusion

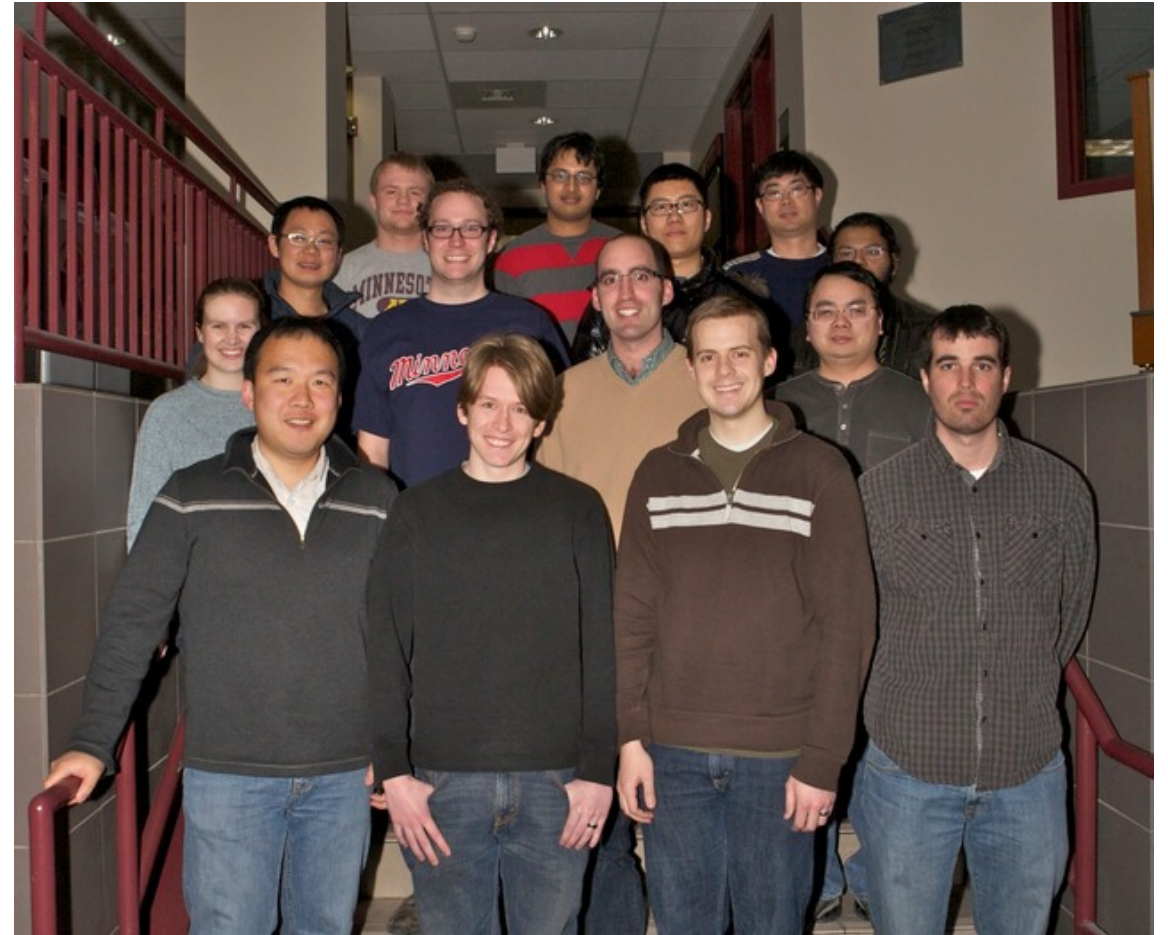
Odijk regime in slits is best described as a nearly two-dimensional walk of deflection segments.



This accounting gives three sub-regimes in strong confinement:

- ▶ Rod-like chains when contour length is small
- ▶ Nearly ideal chains when chain width is small
- ▶ Self-avoiding chains otherwise

# Acknowledgments



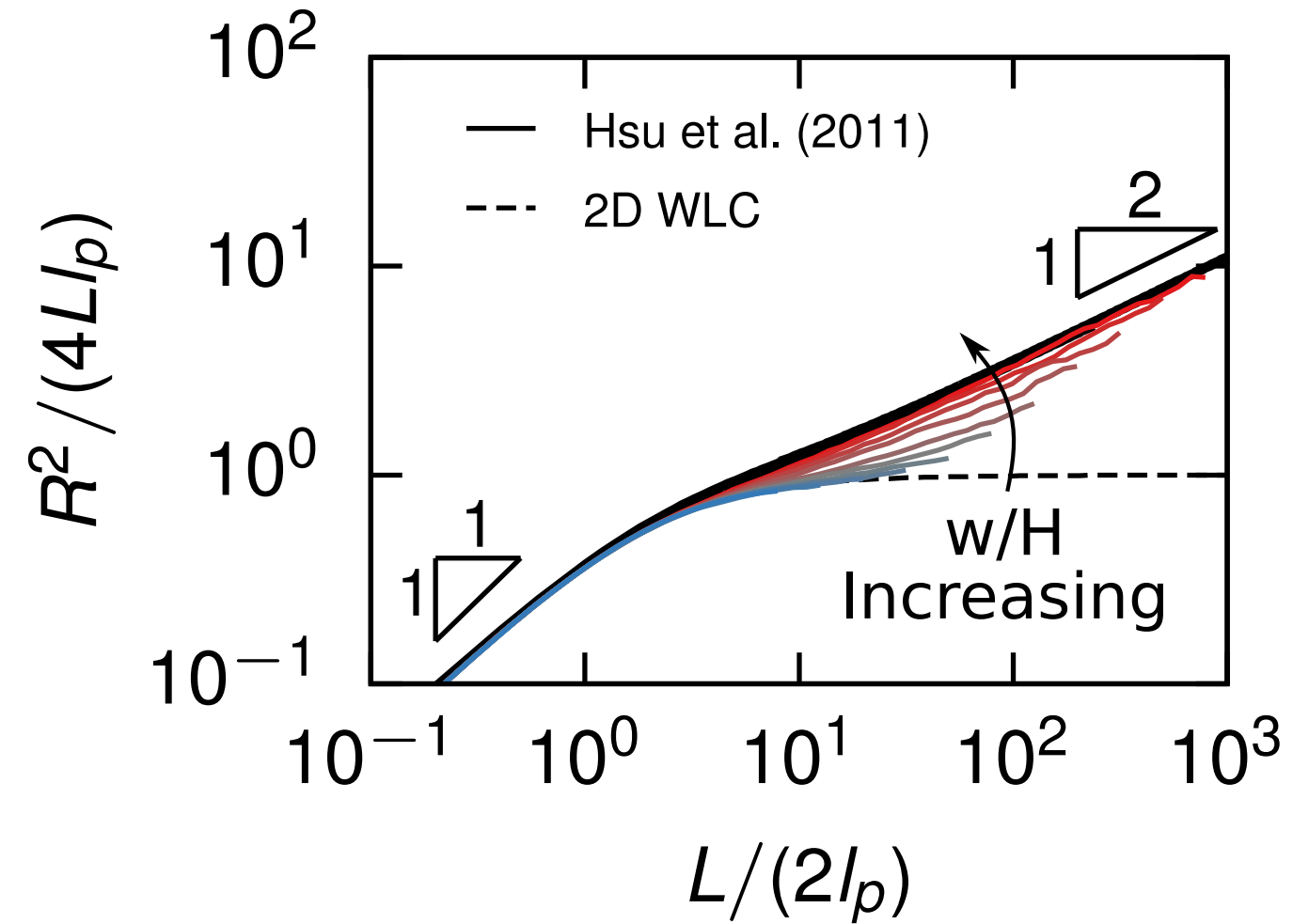
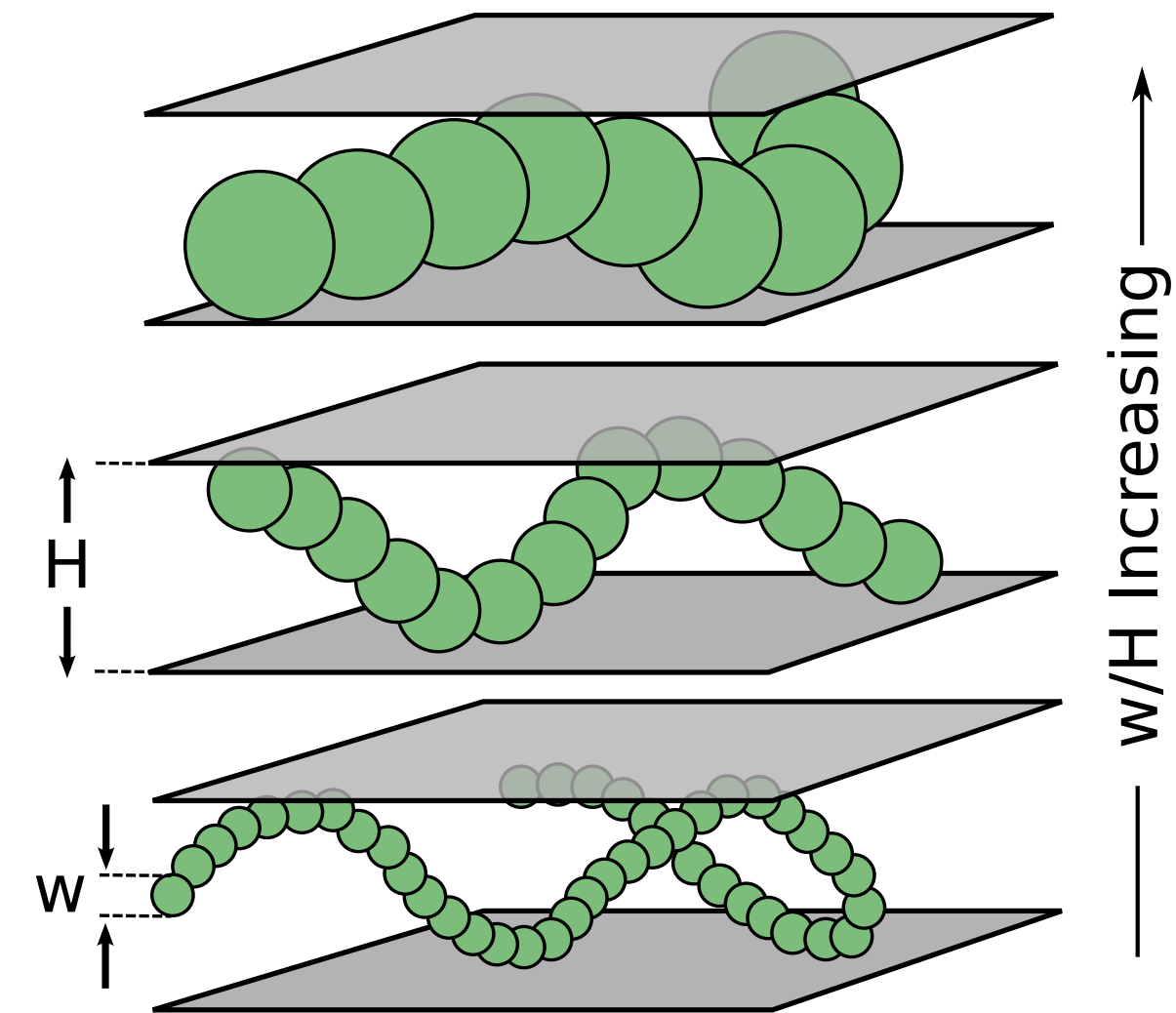
UNIVERSITY OF MINNESOTA

Supercomputing Institute

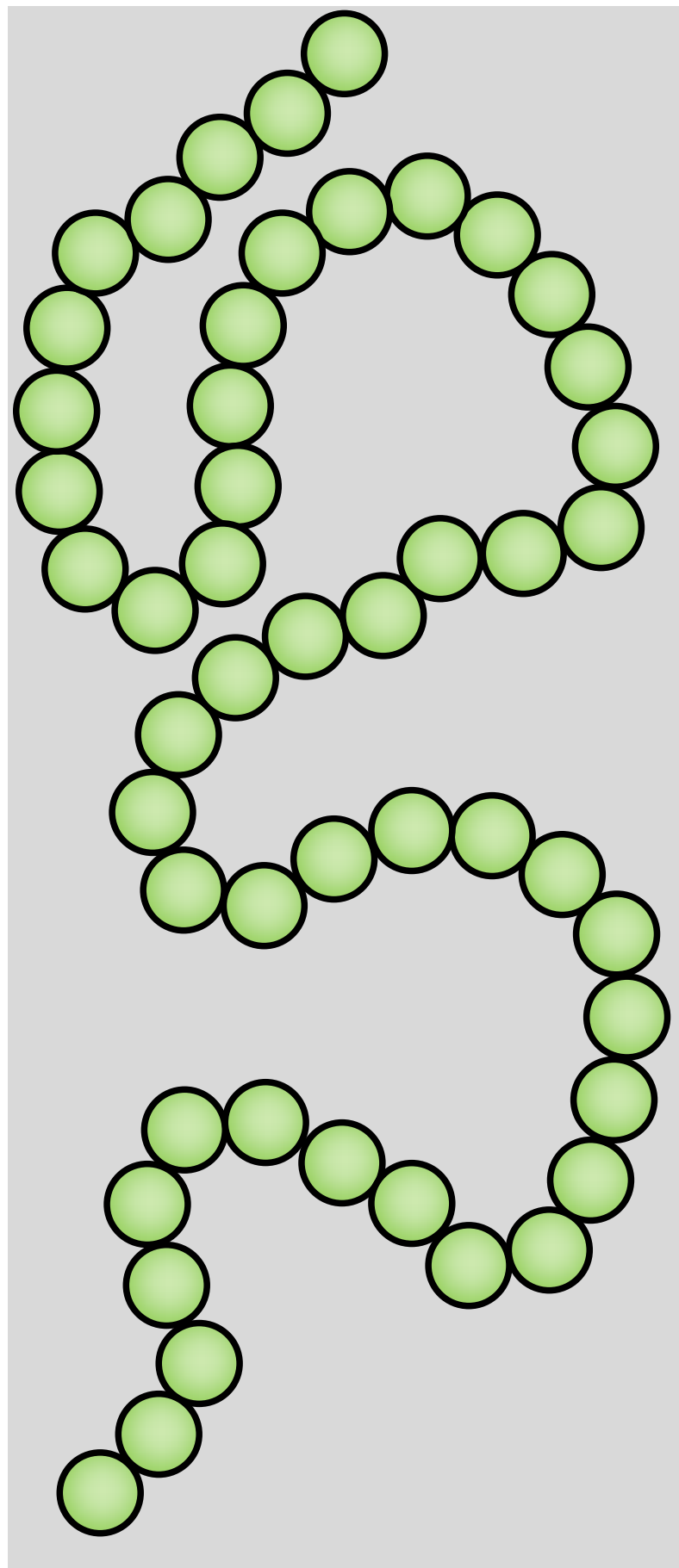
UNIVERSITY  
OF MINNESOTA



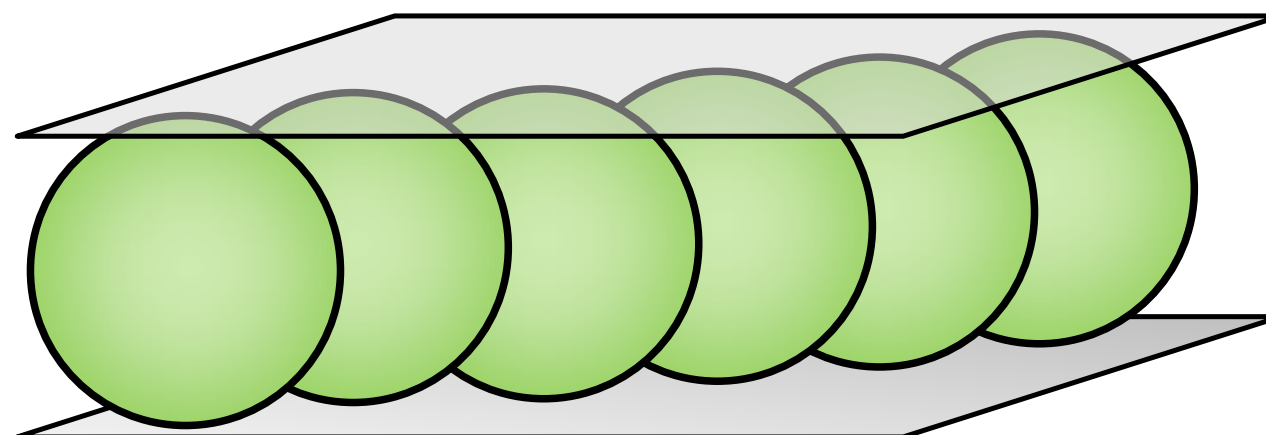
# Odnj Regime in Slits



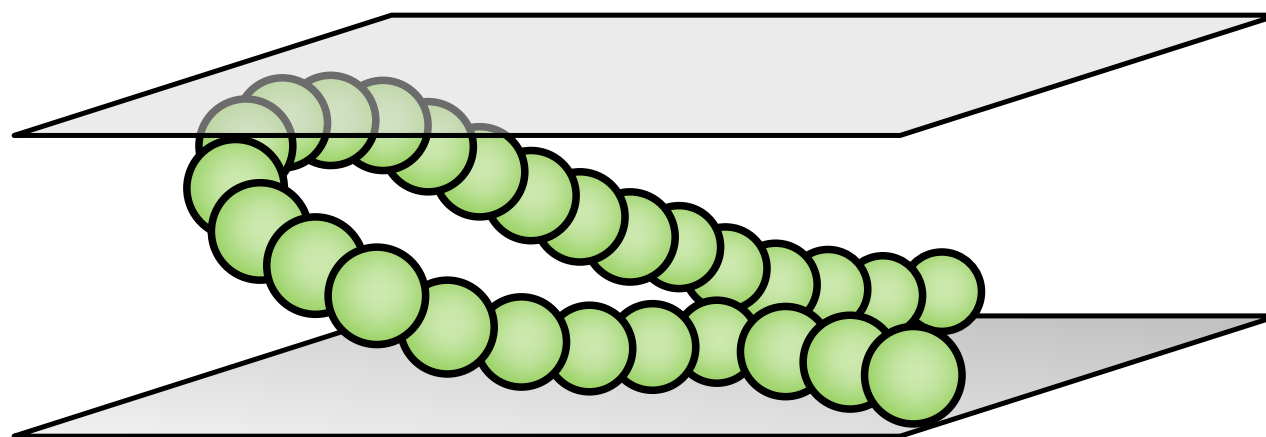
Top View



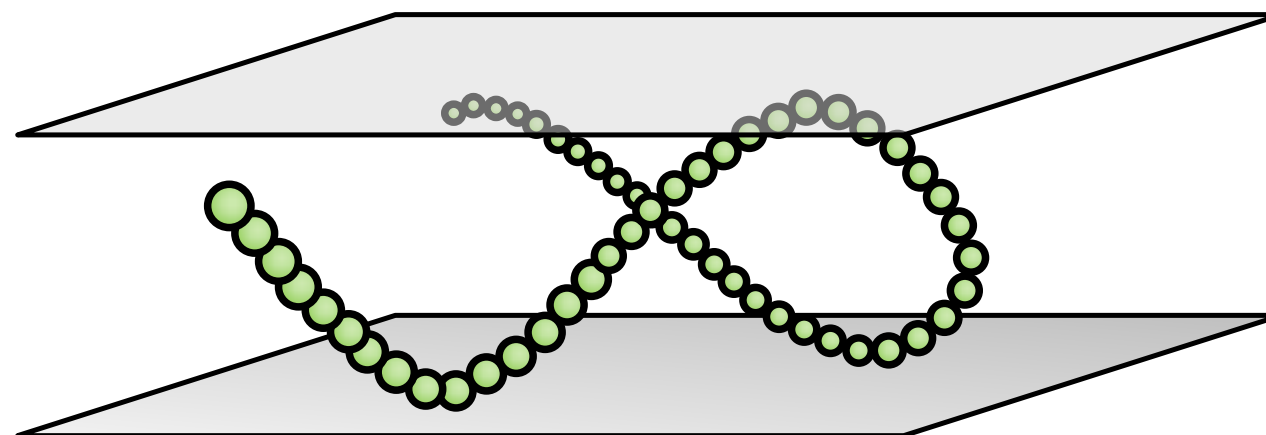
Side View



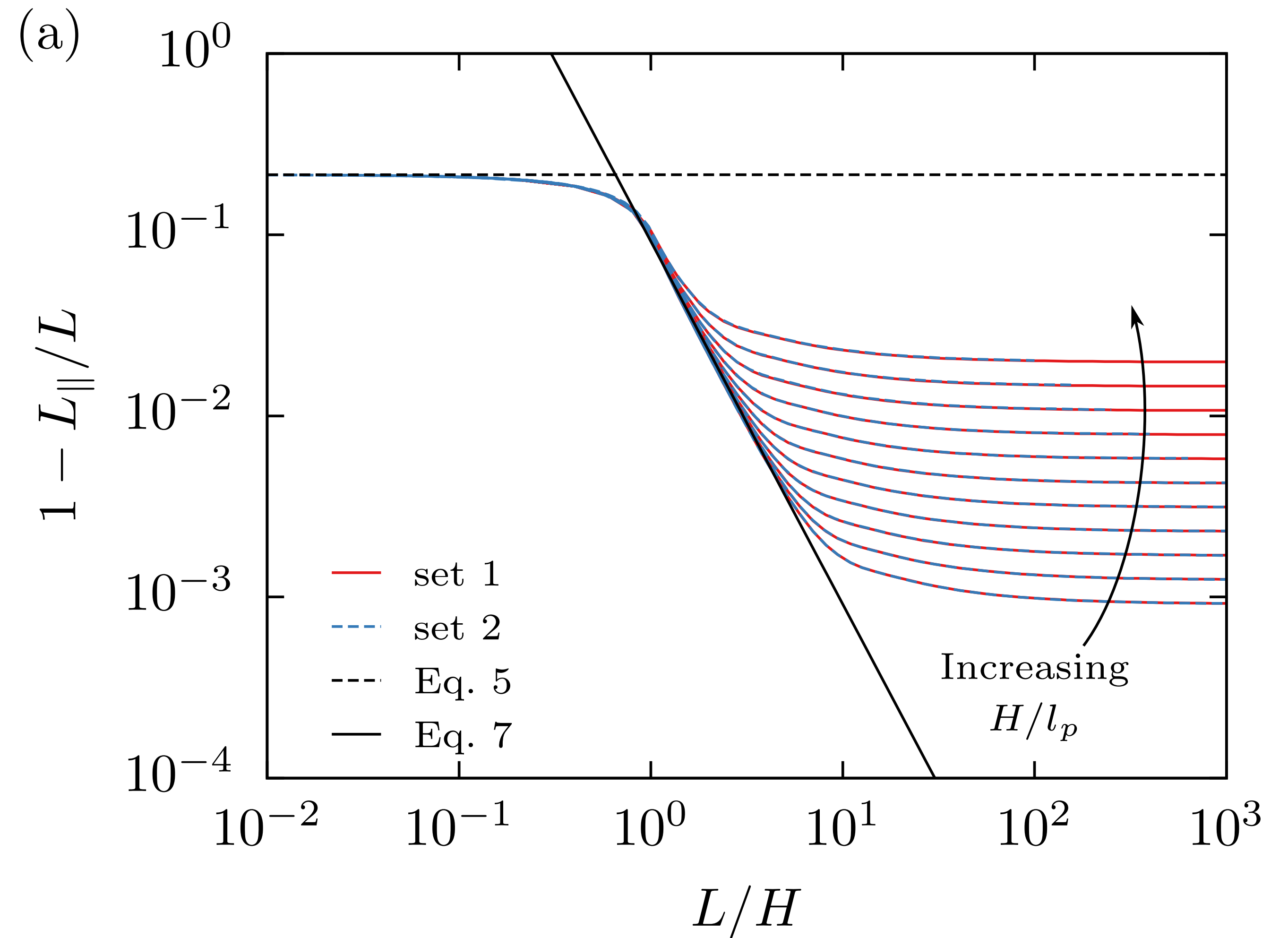
$$H = w$$



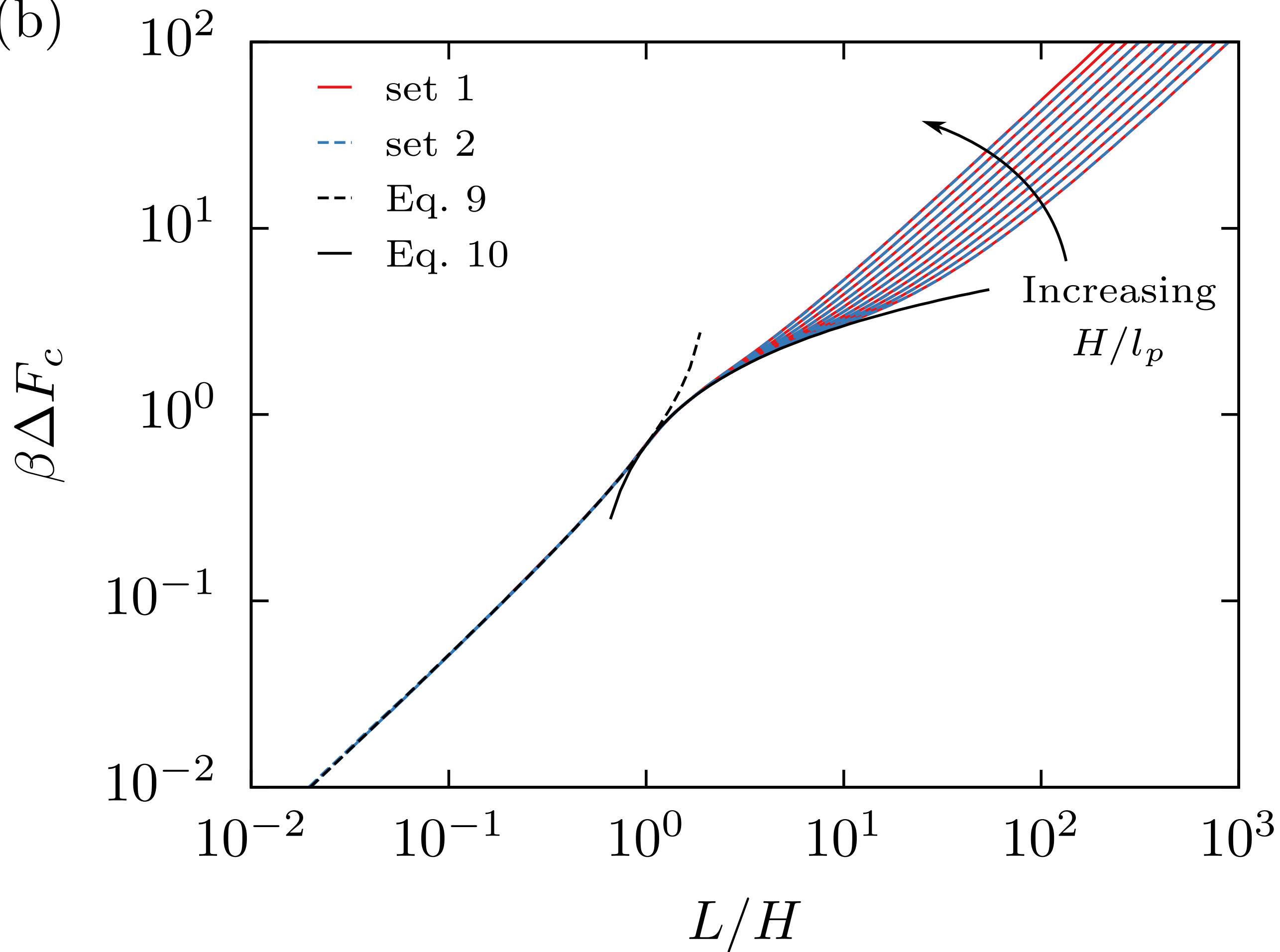
$$H > w$$

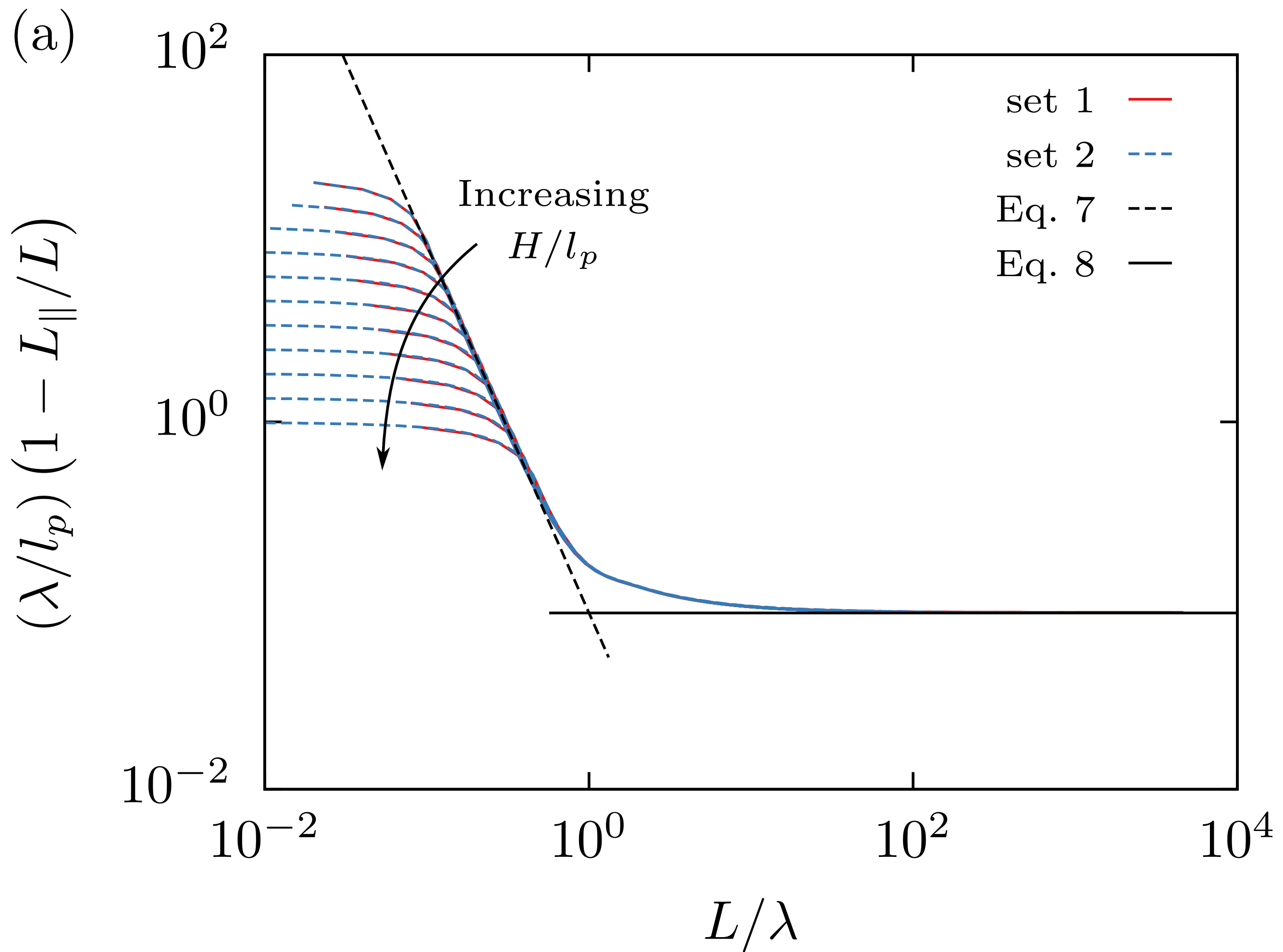


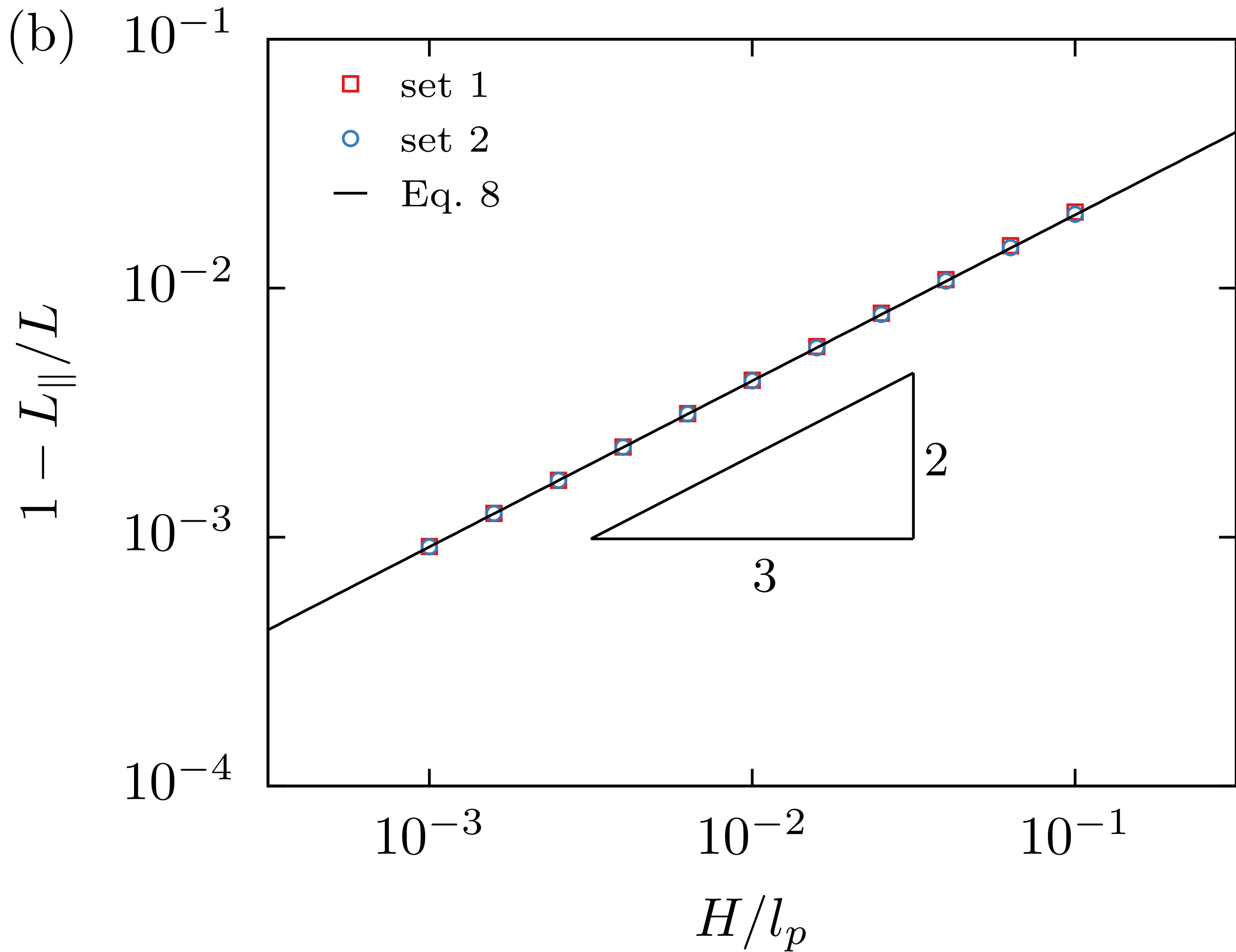
$$H \gg w$$



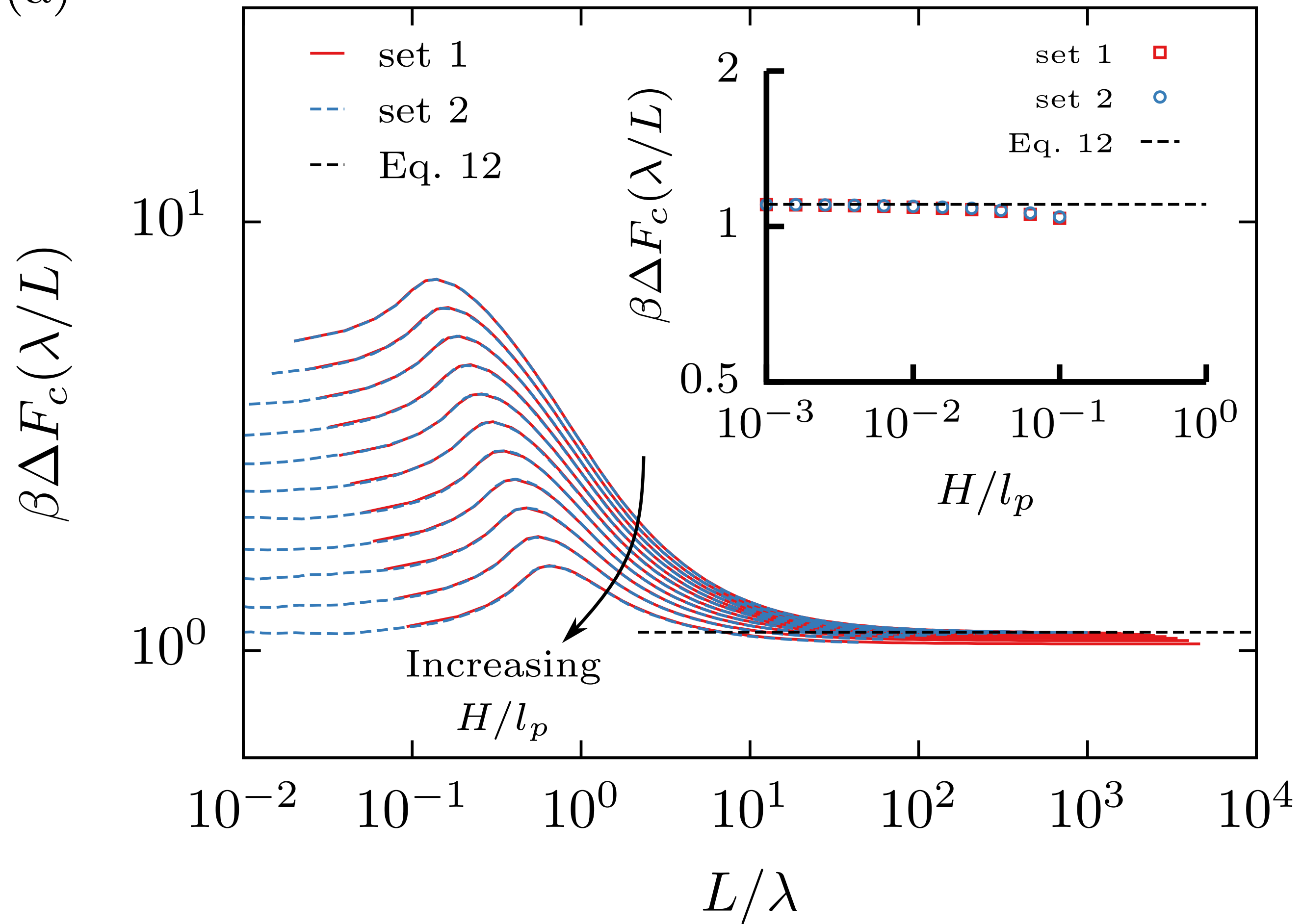
(b)



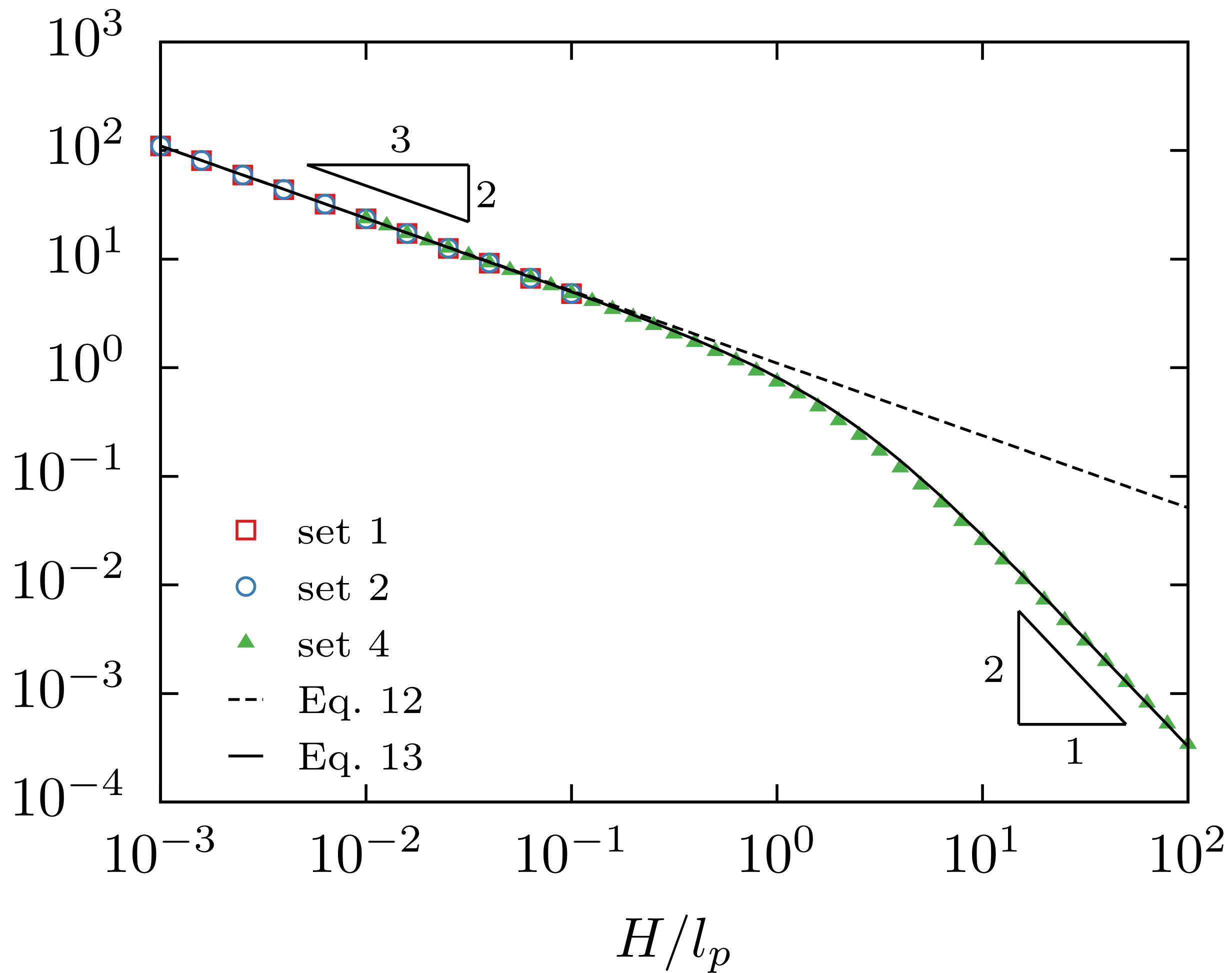


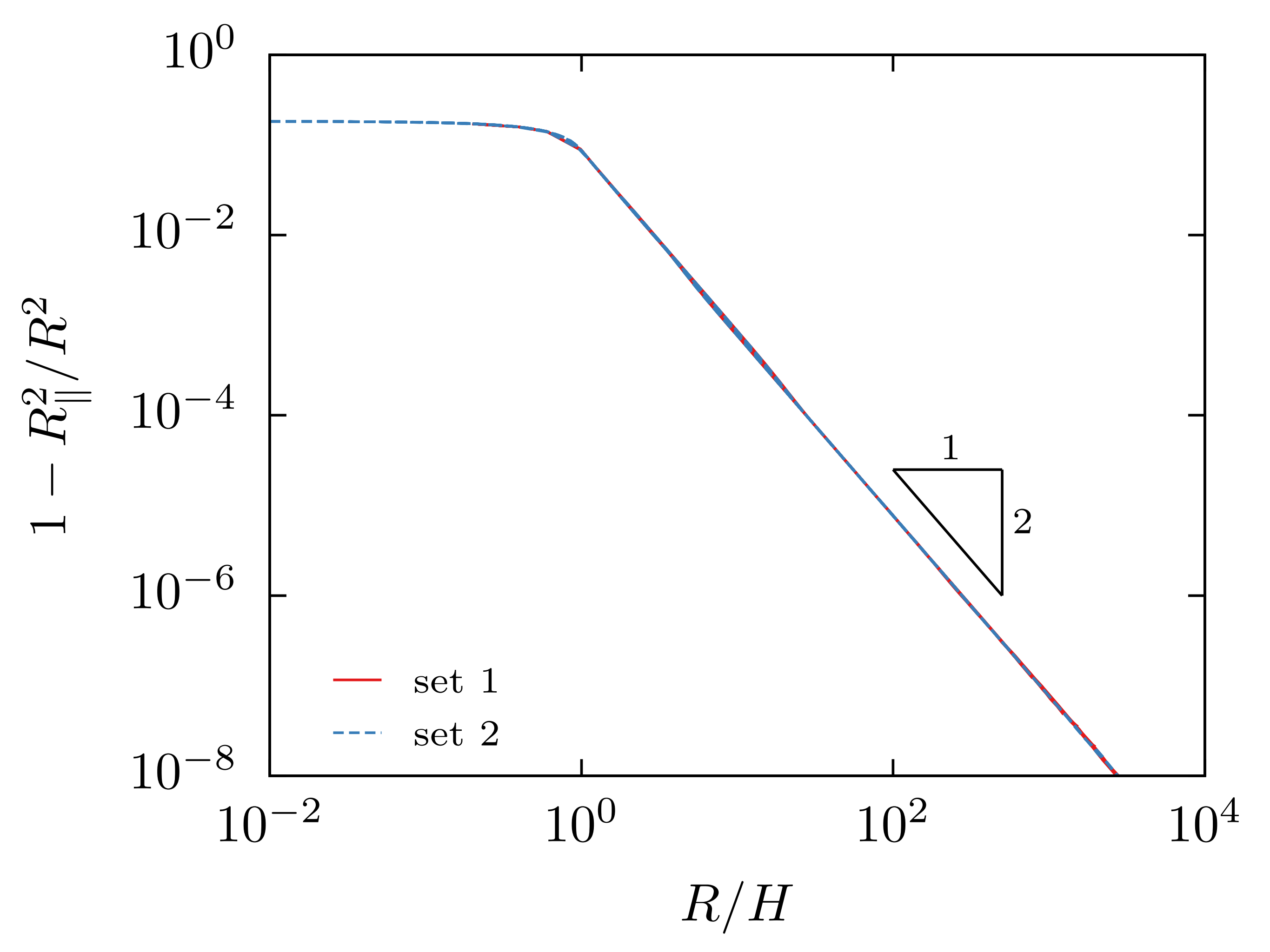


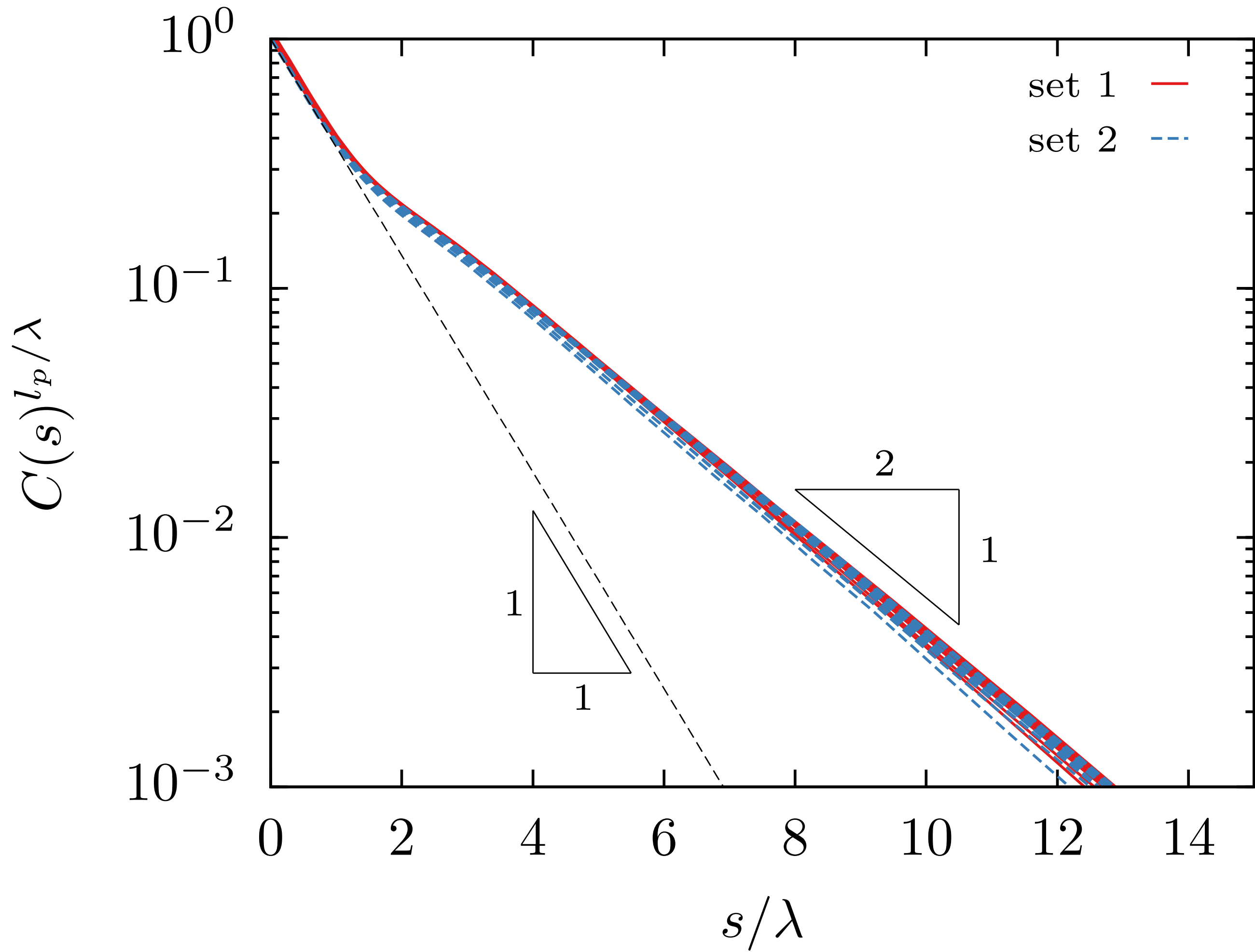
(a)



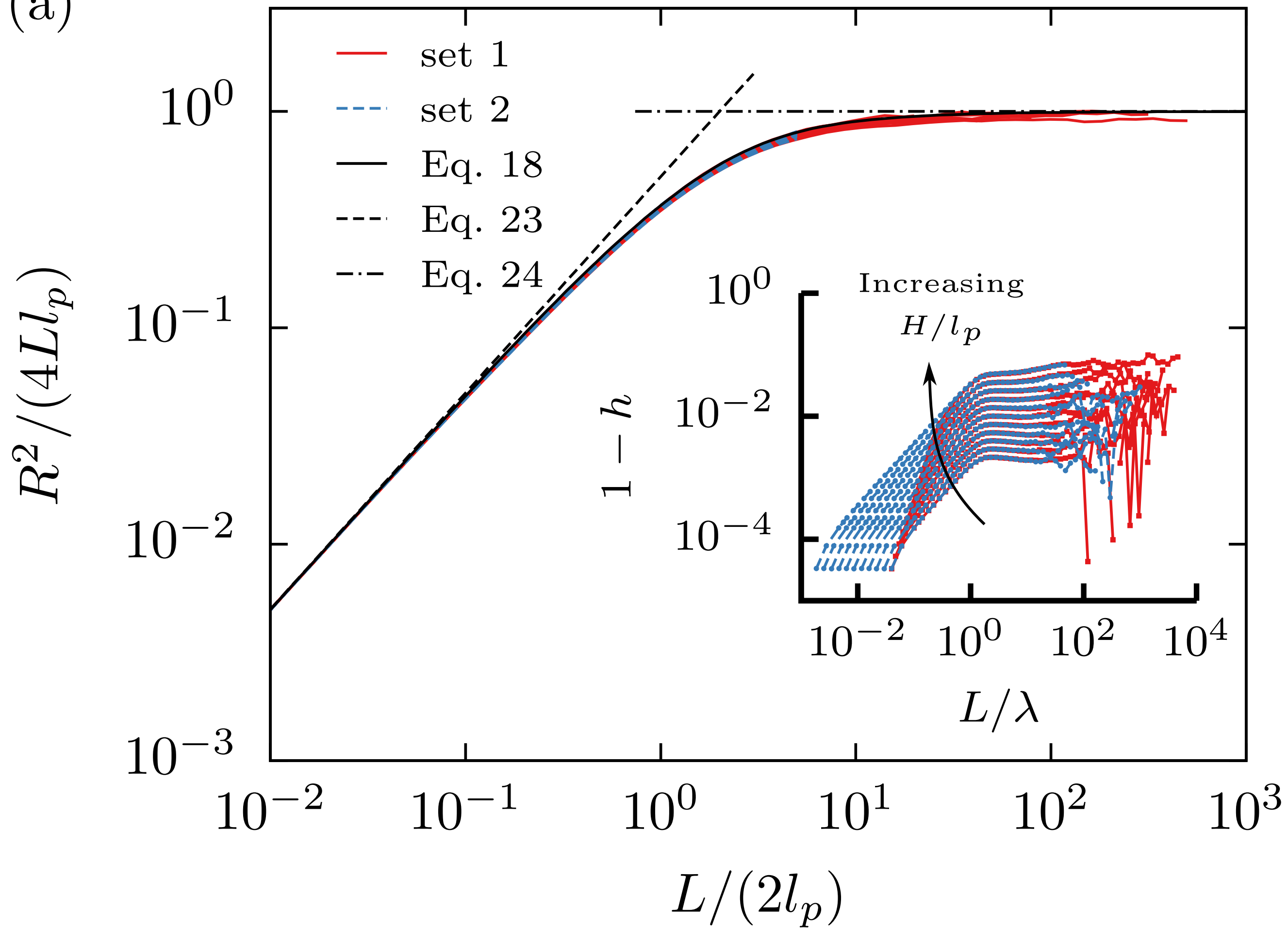
(b)

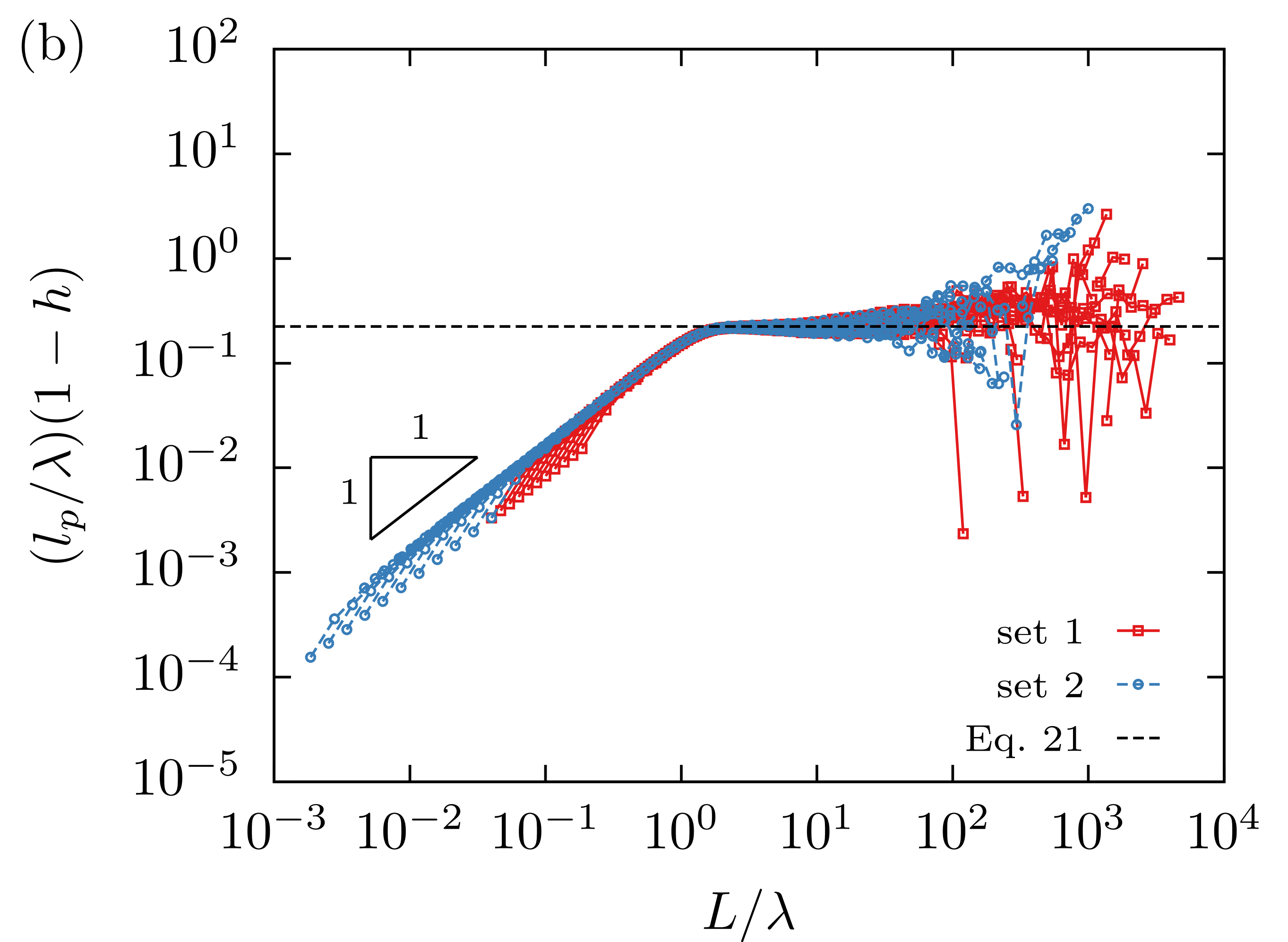
 $\beta \Delta F_c(l_p/L)$ 



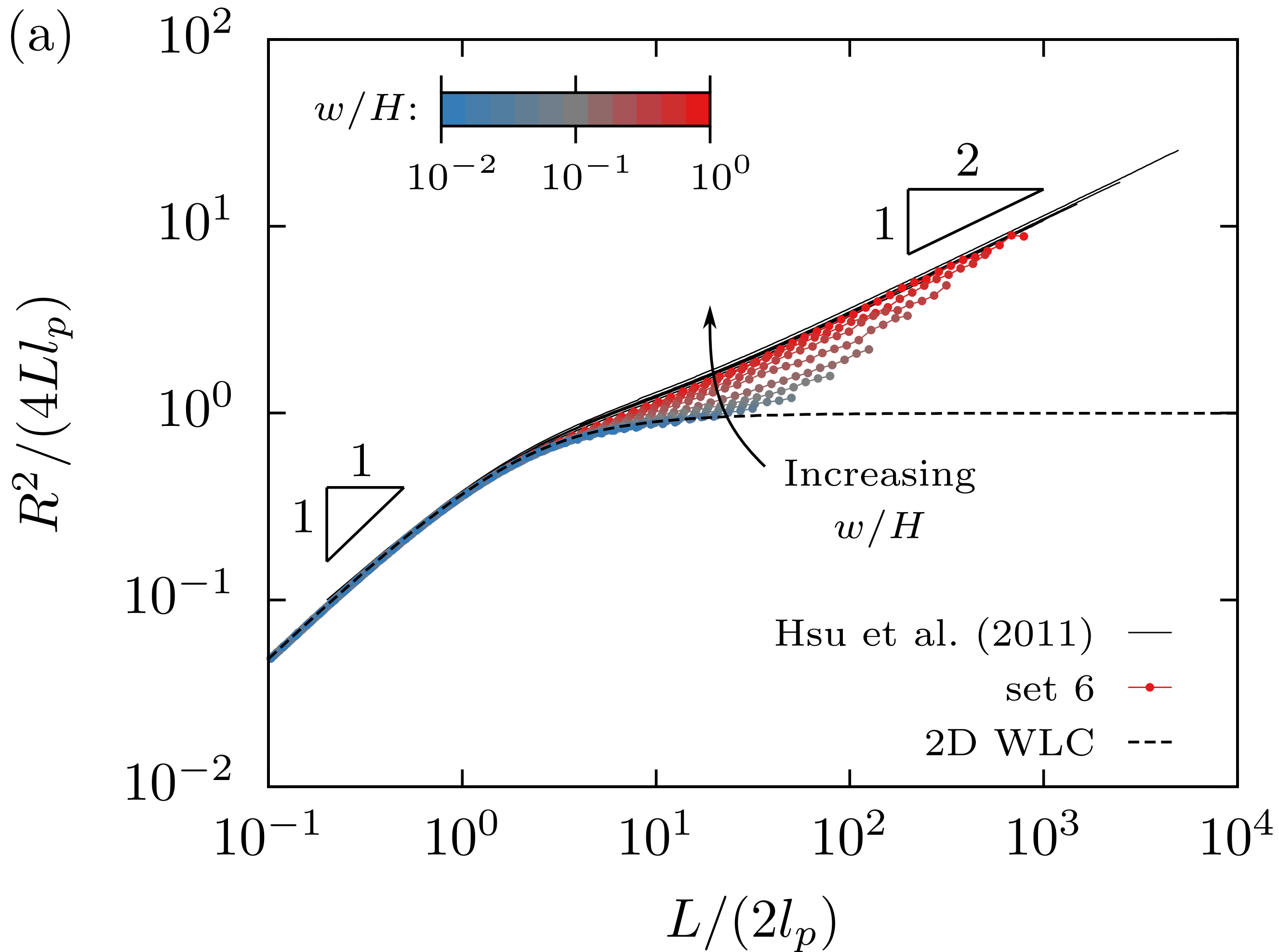


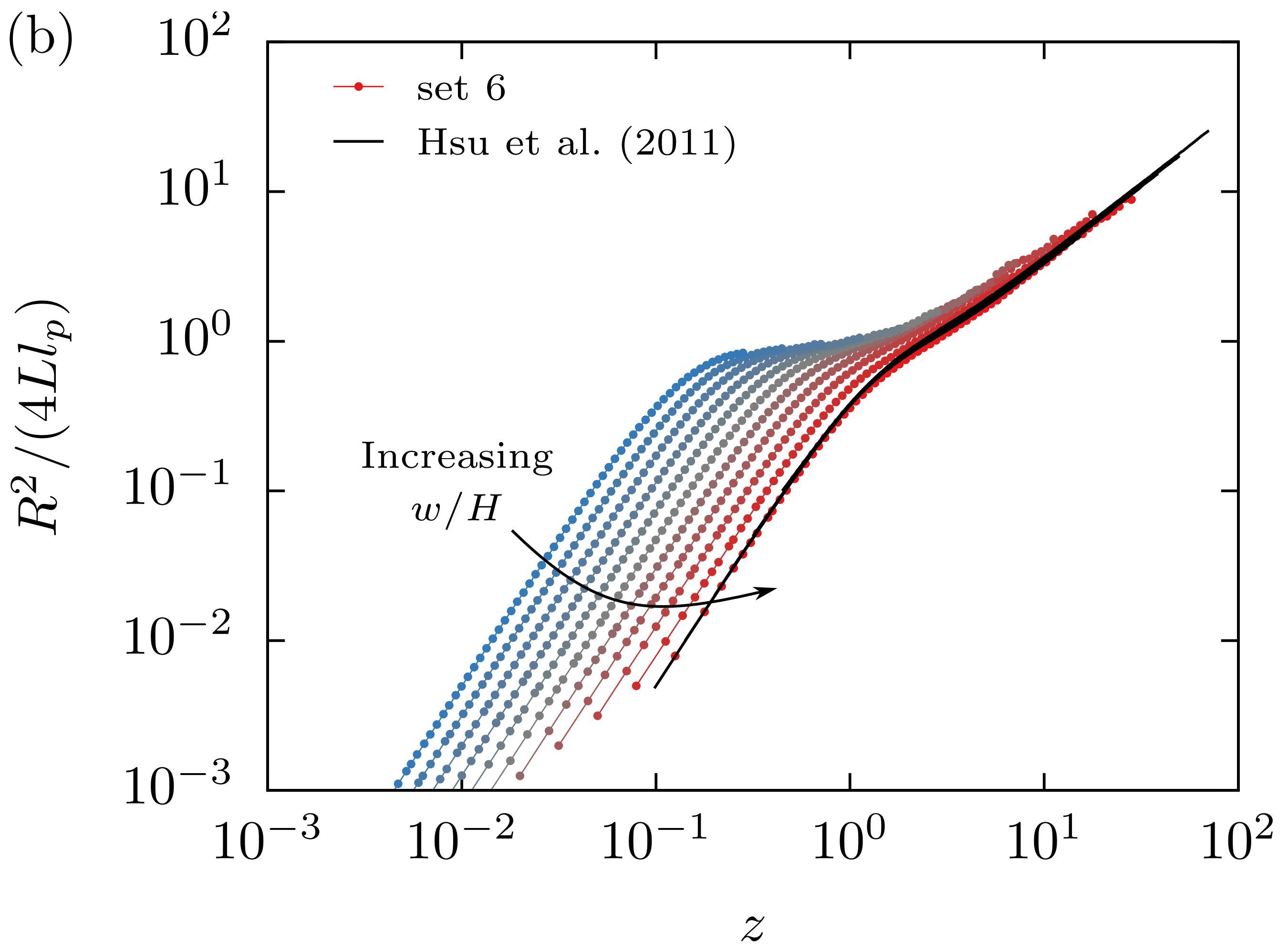
(a)

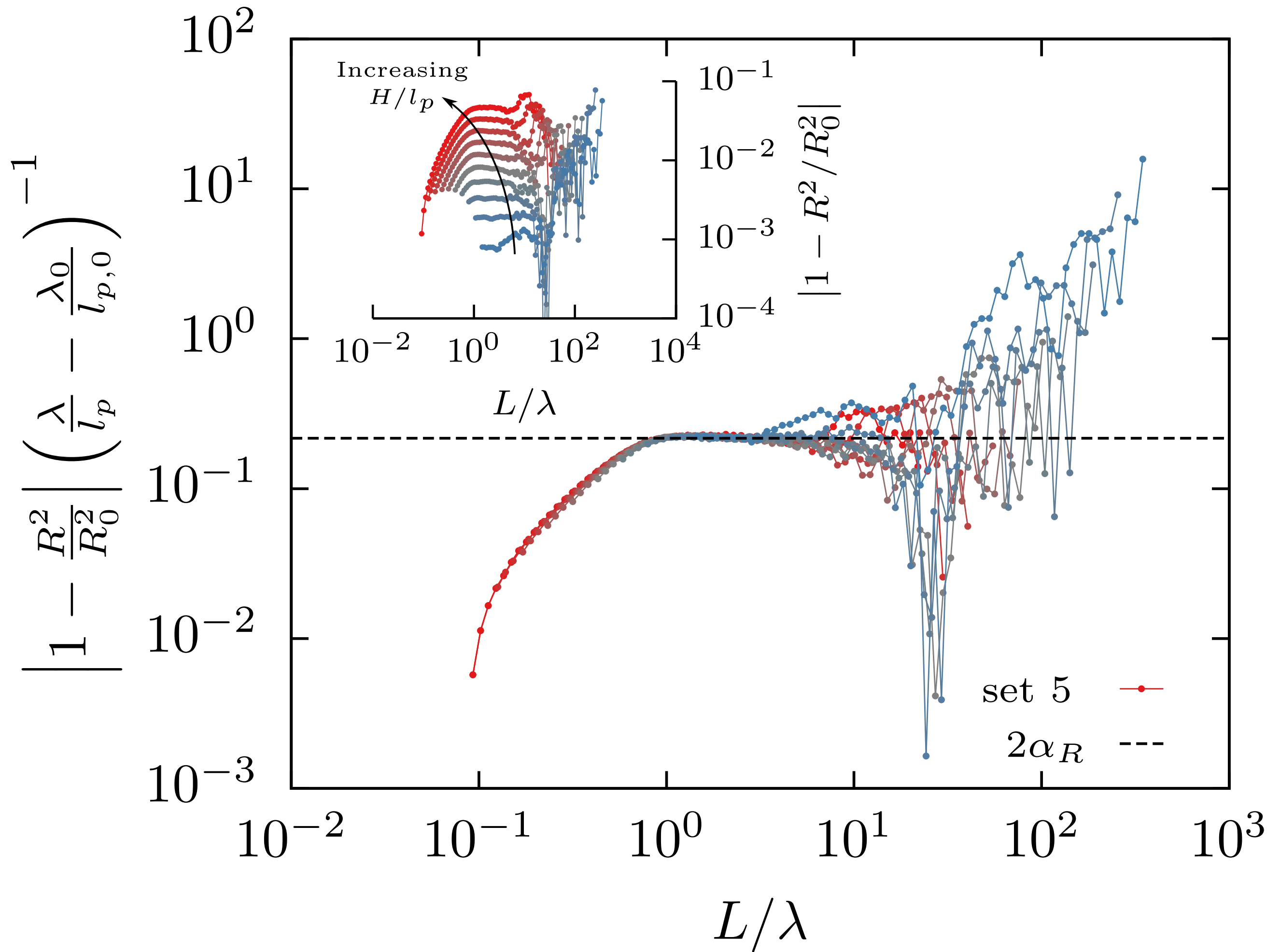


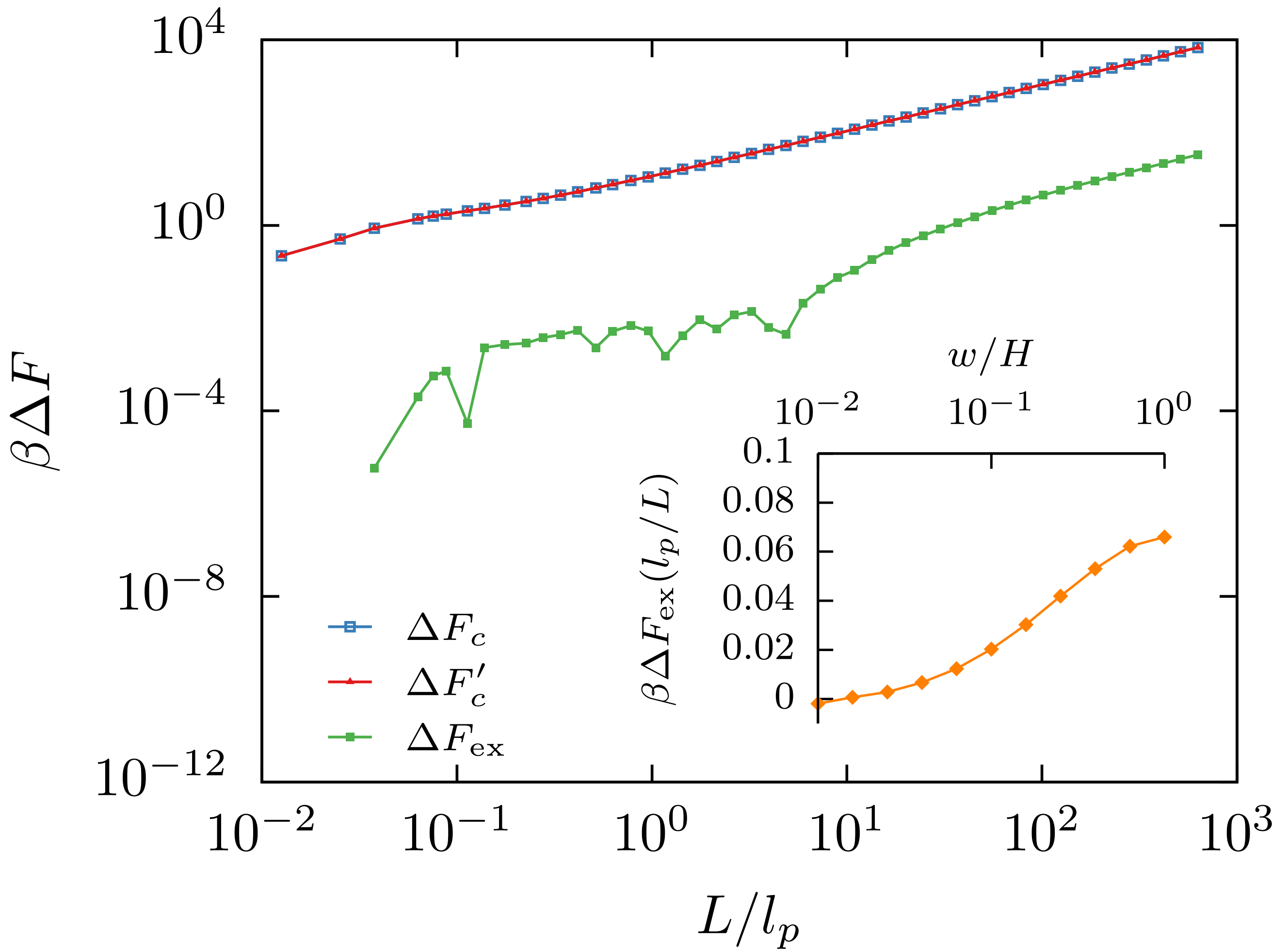


(a)

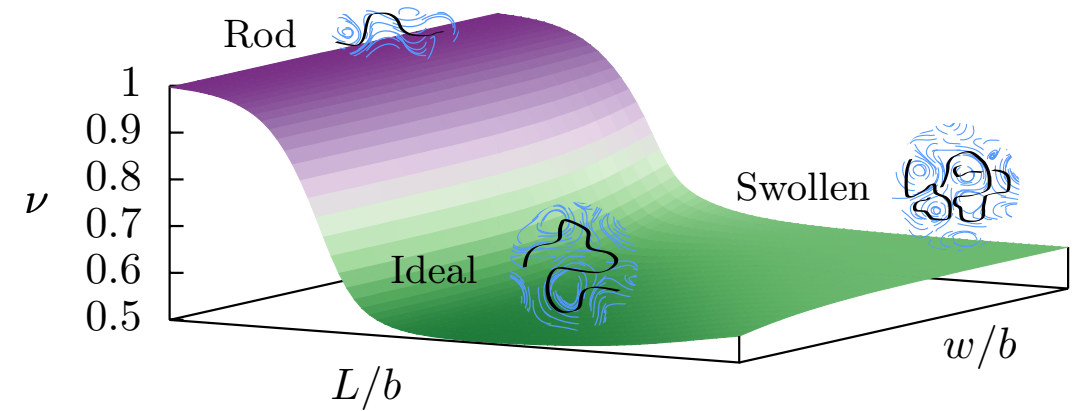






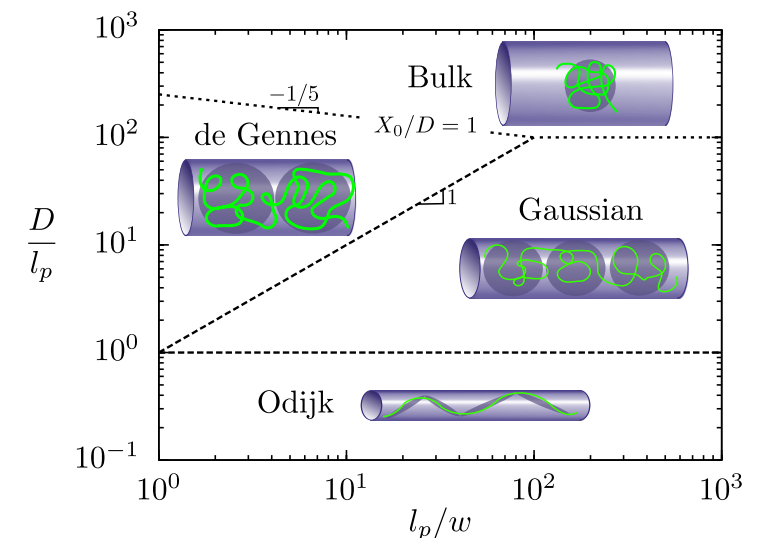


# Polymer Physics of DNA in Solution



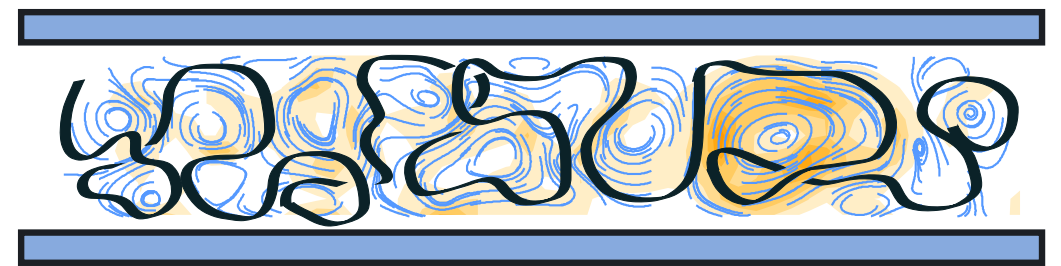
DR Tree, A Muralidhar, PS Doyle and KD Dorfman, *Macromolecules* (2013).

# Regimes of Confined DNA



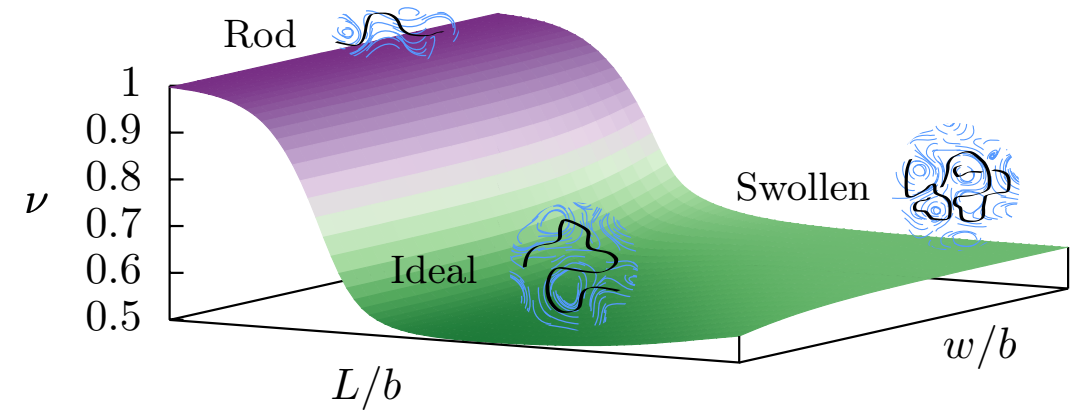
DR Tree, Y Wang and KD Dorfman, *Phys. Rev. Lett.* 110, 208103 (2013).

# Hydrodynamics of Confined DNA



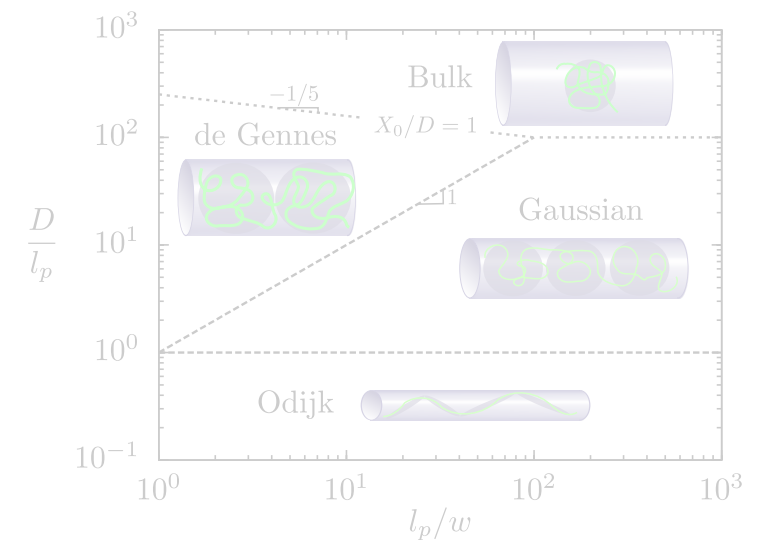
DR Tree, Y Wang and KD Dorfman, *Phys. Rev. Lett.* 108, 228015 (2012).

# Polymer Physics of DNA in Solution



DR Tree, A Muralidhar, PS Doyle and KD Dorfman, *Macromolecules* (2013).

## Regimes of Confined DNA



DR Tree, Y Wang and KD Dorfman, *Phys. Rev. Lett.* 110, 208103 (2013).

## Hydrodynamics of Confined DNA

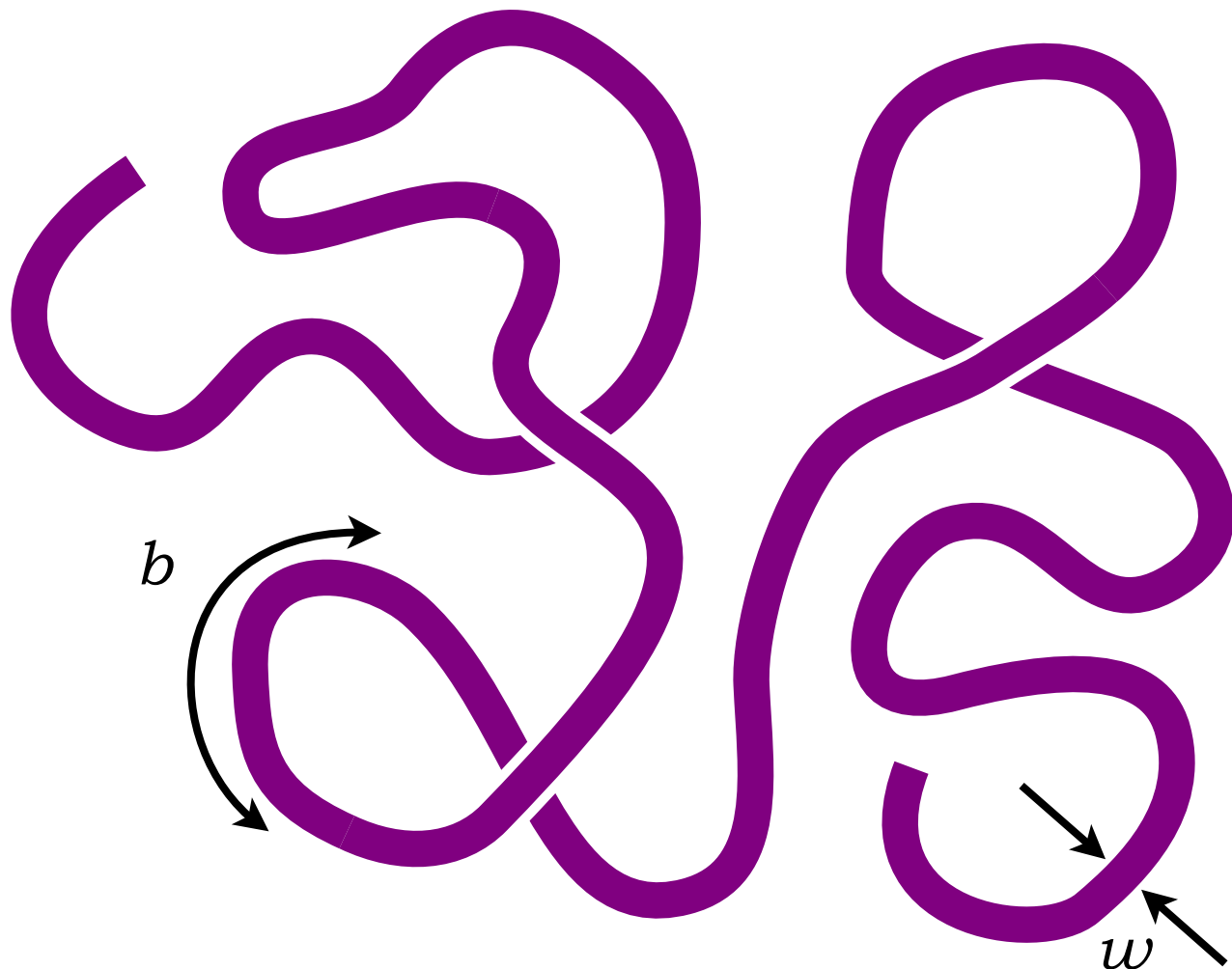


DR Tree, Y Wang and KD Dorfman, *Phys. Rev. Lett.* 108, 228015 (2012).

# DNA is a Wormlike Chain (WLC)

The WLC model assumes DNA to be:

- sequence independent
- locally stiff
- space-filling with short-range interactions

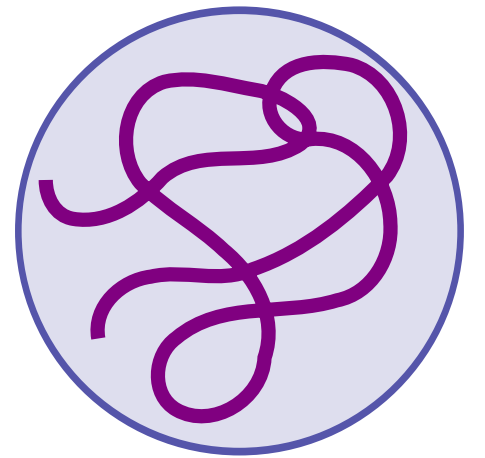


| Physical Quantity       | Symbol        | Value for $\lambda$ -DNA |
|-------------------------|---------------|--------------------------|
| chain length            | $L$           | 21 $\mu\text{m}$         |
| Kuhn length (stiffness) | $b$ or $2l_p$ | 106 nm                   |
| effective width         | $w$           | 4.6 nm                   |
| hydrodynamic radius     | $a$           | 2.9 nm                   |

# Bulk, Infinitely Dilute Solution

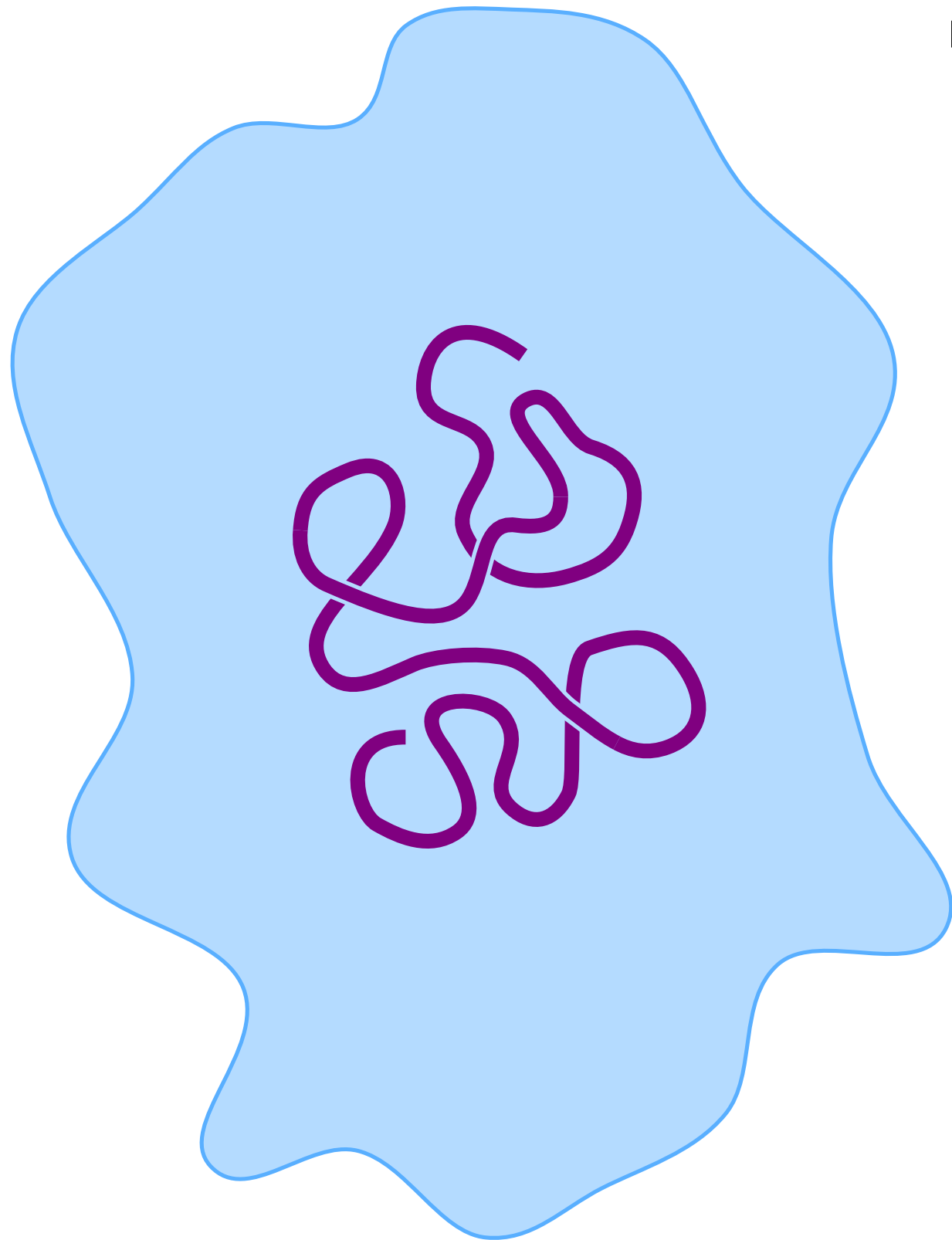
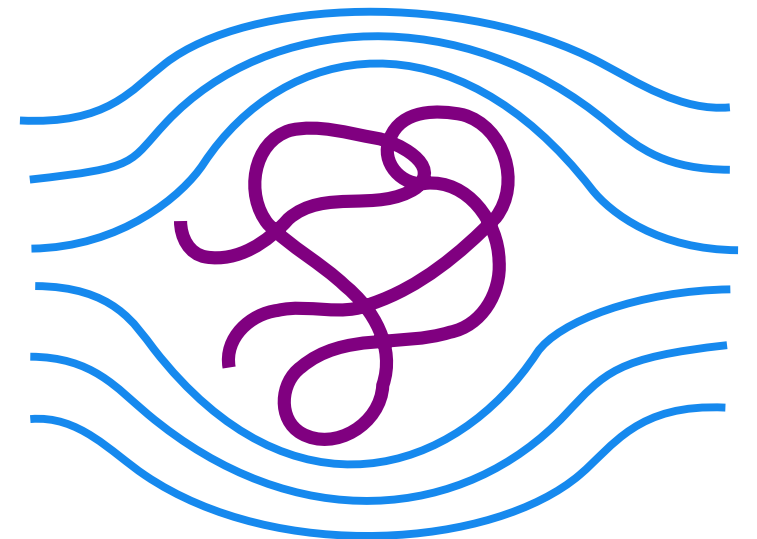
- What is the equilibrium coil size?

$$R_g = f(L, b, w)$$



- What is the coil diffusion coefficient?

$$D = f(L, b, w, a)$$

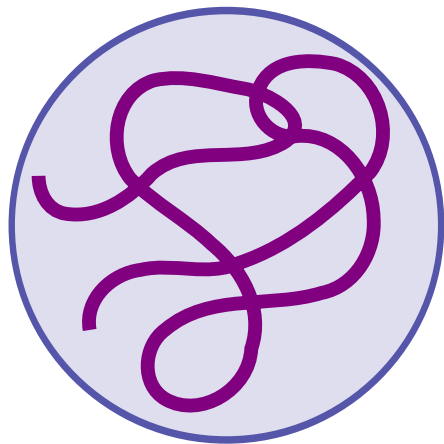


# The Thermodynamic Limit

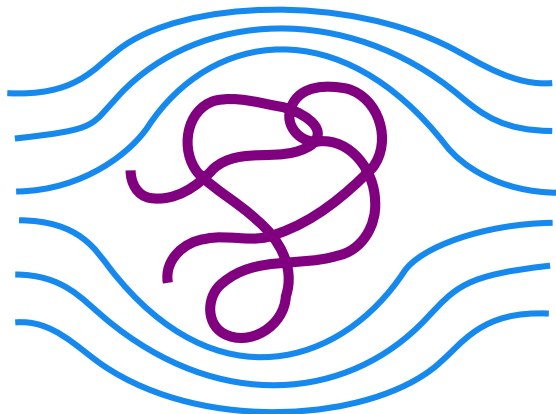
Real (self-avoiding), dilute polymers are expected to show universal behavior in the thermodynamic limit,

$$L \rightarrow \infty$$

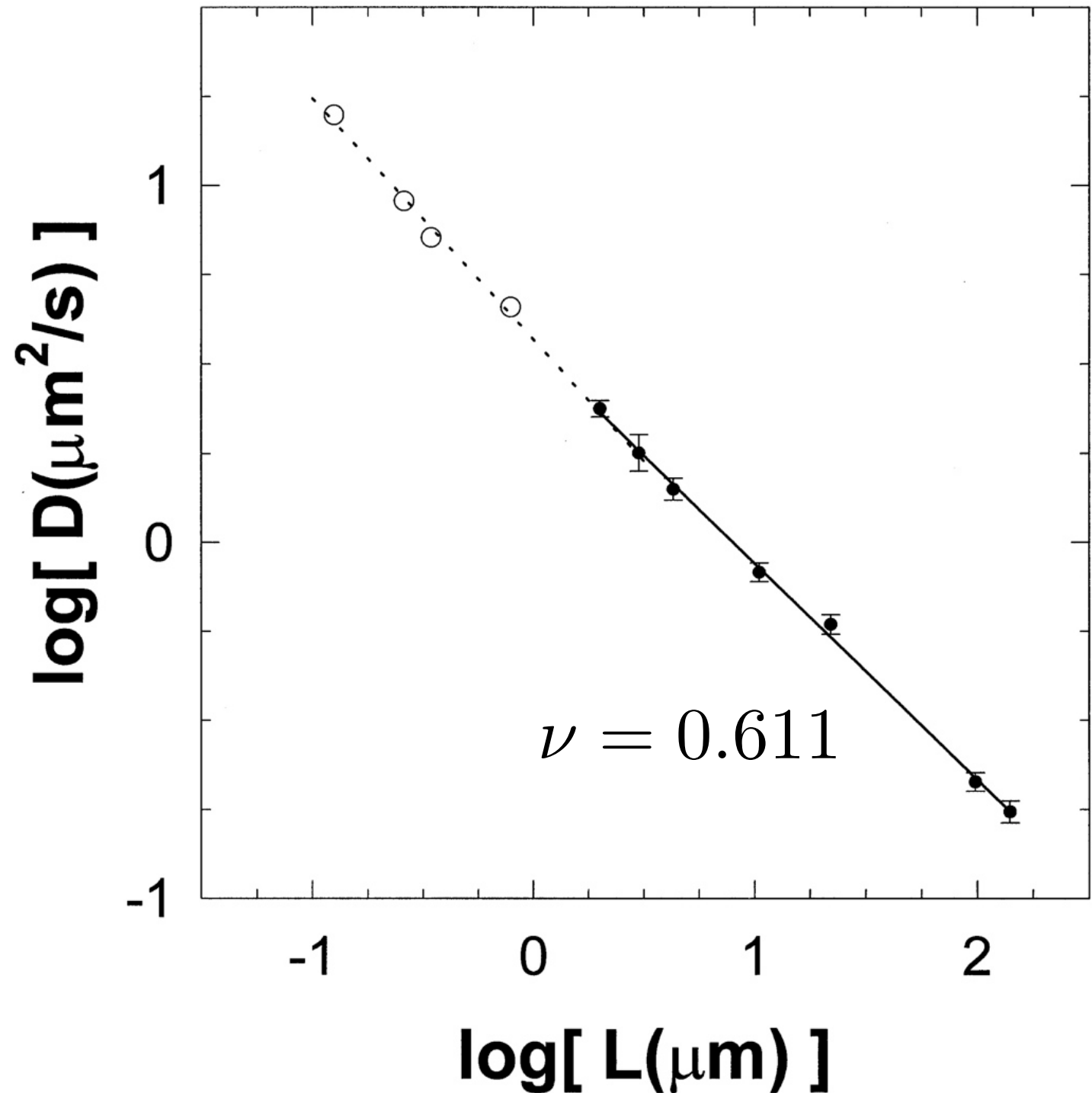
$$R_g \sim L^\nu$$

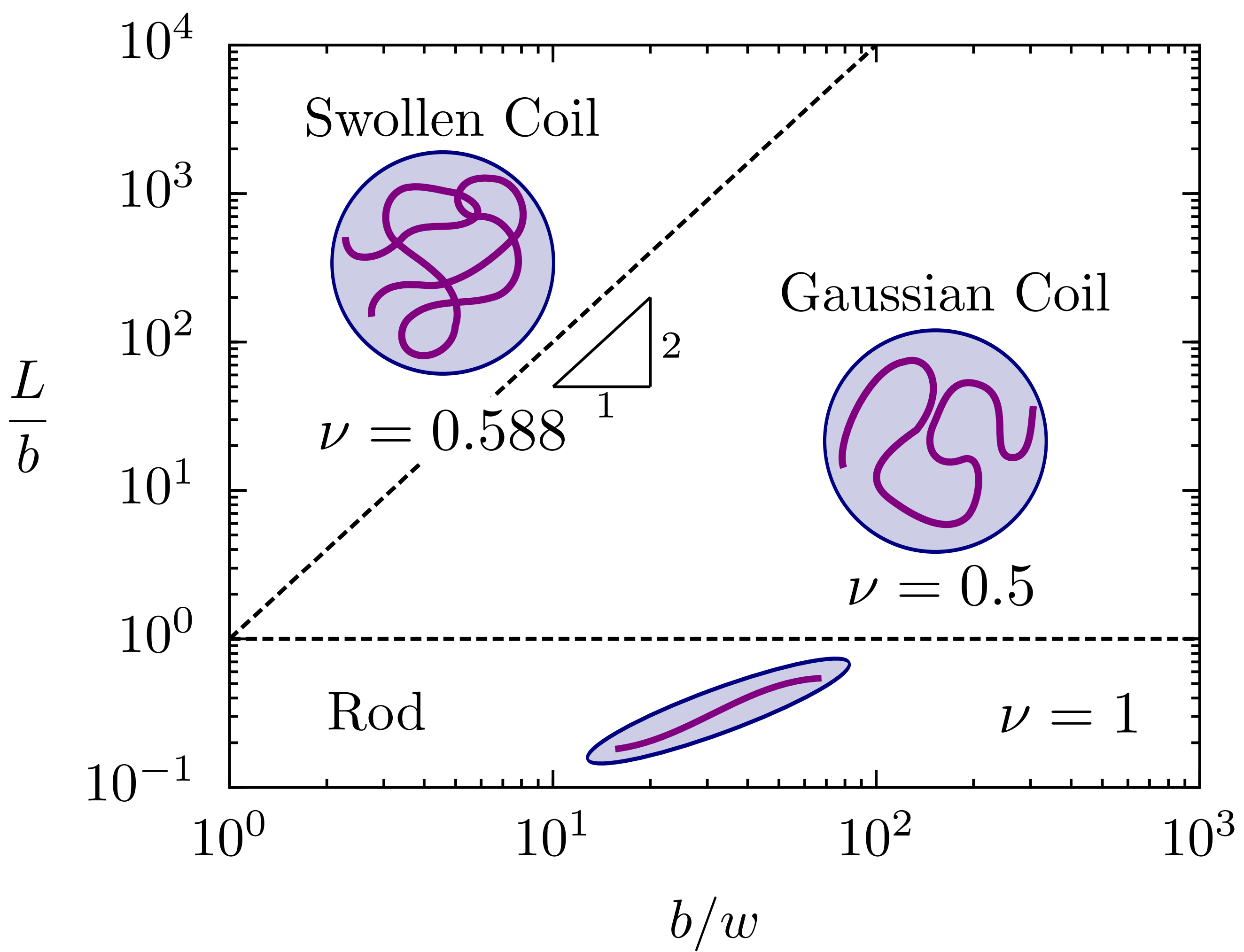


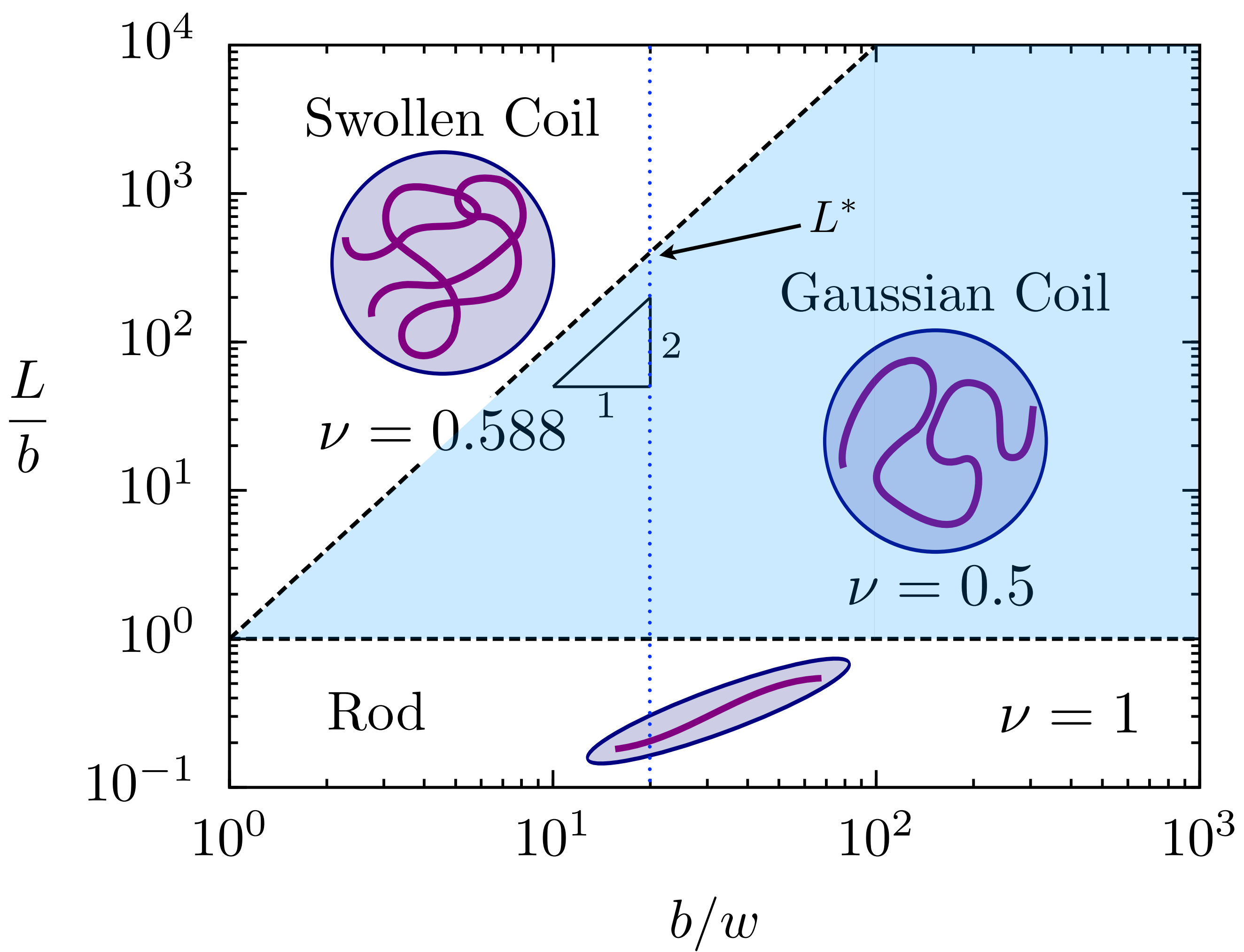
$$D \sim L^{-\nu}$$



$$\nu = 0.588$$

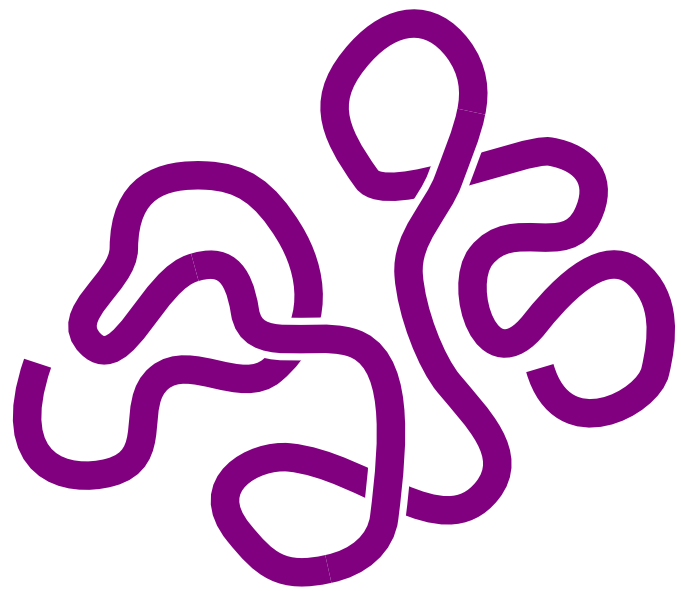




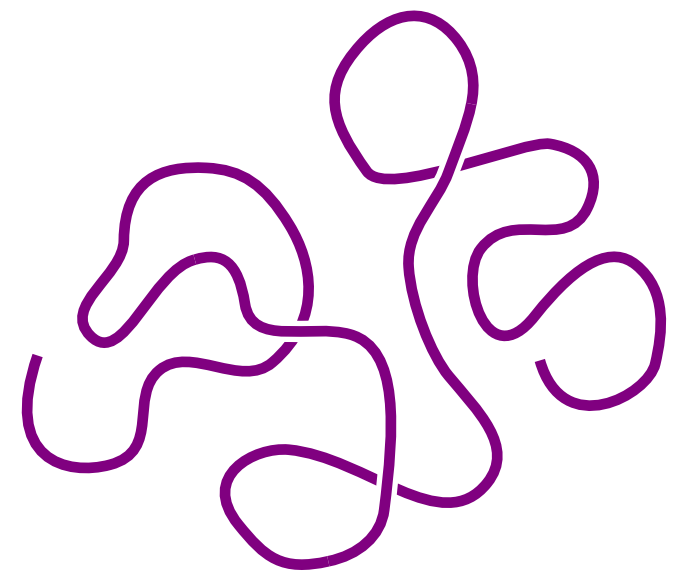


# Monomer Anisotropy

The monomer anisotropy is a dimensionless ratio of the strength of the excluded volume to the stiffness.



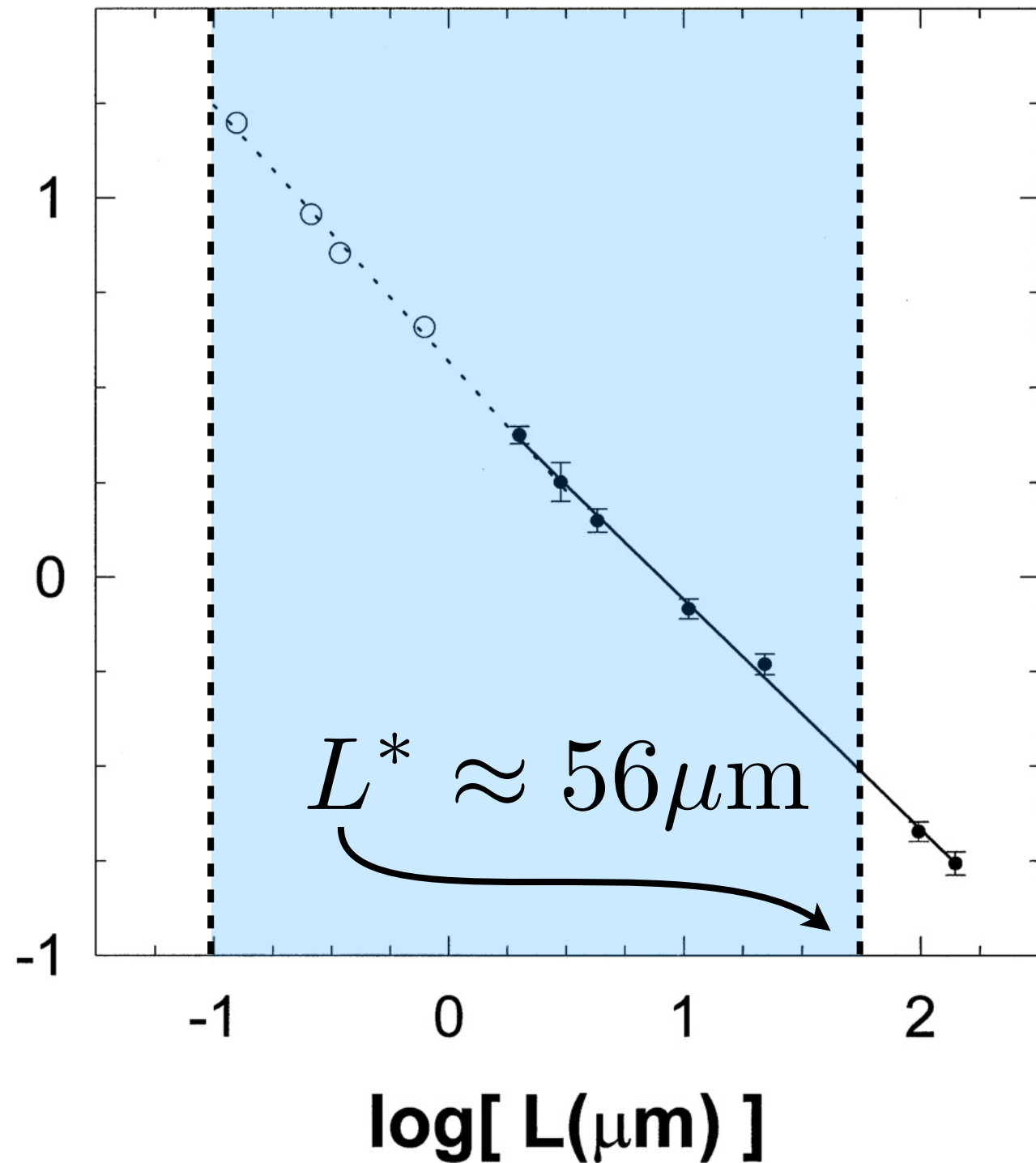
$$\frac{w}{b} \sim 1$$



$$\frac{w}{b} \ll 1$$

$$\frac{L^*}{b} \sim \left( \frac{w}{b} \right)^{-2}$$

$\log[ D(\mu\text{m}^2/\text{s}) ]$



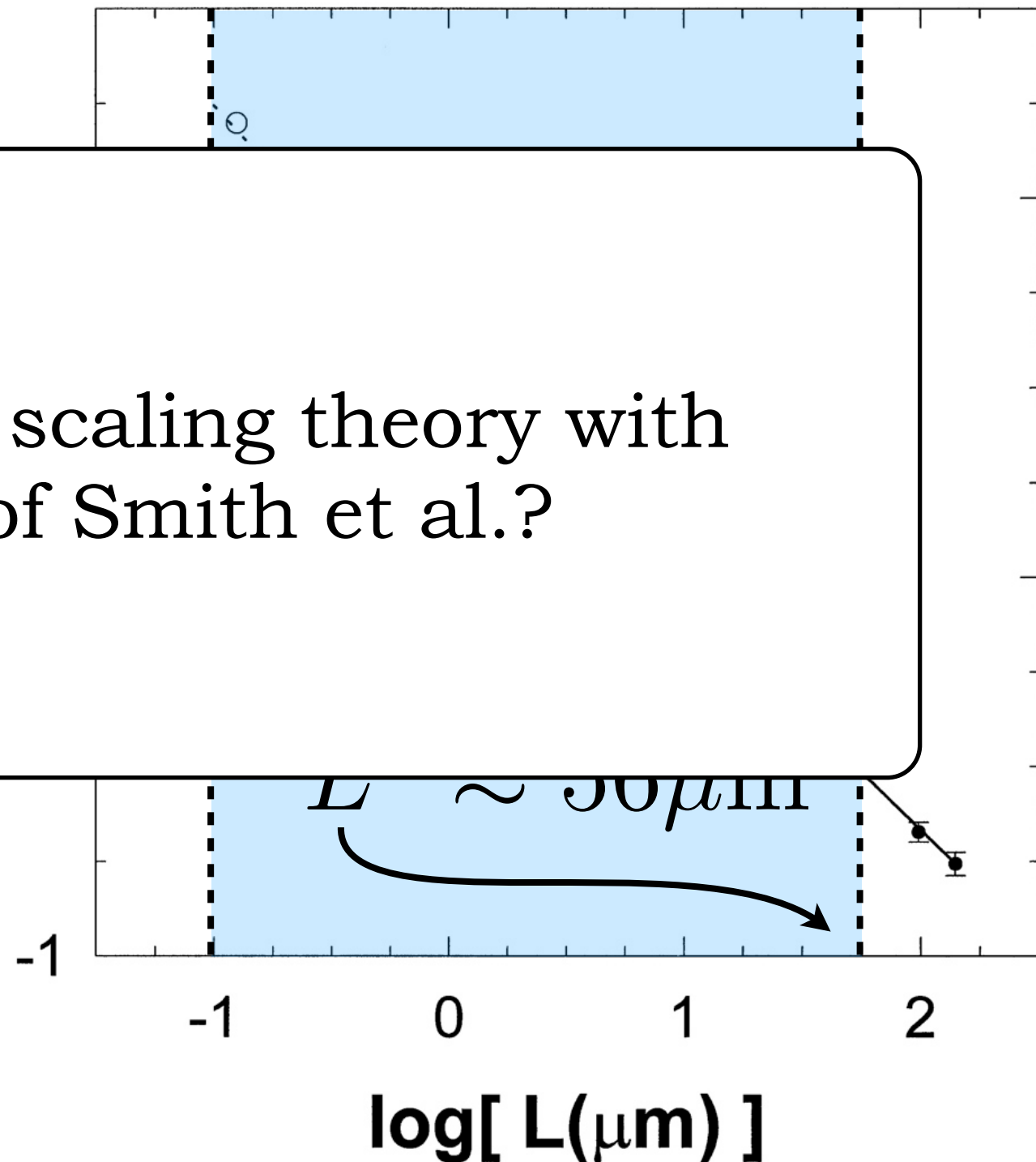
# Monomer Anisotropy

The monomer anisotropy is a dimensionless ratio of the strength of the excluded volume to the stiffness.

$$\frac{L^*}{b} \sim \left( \frac{w}{b} \right)^{-2}$$

Can we reconcile scaling theory with the experiments of Smith et al.?

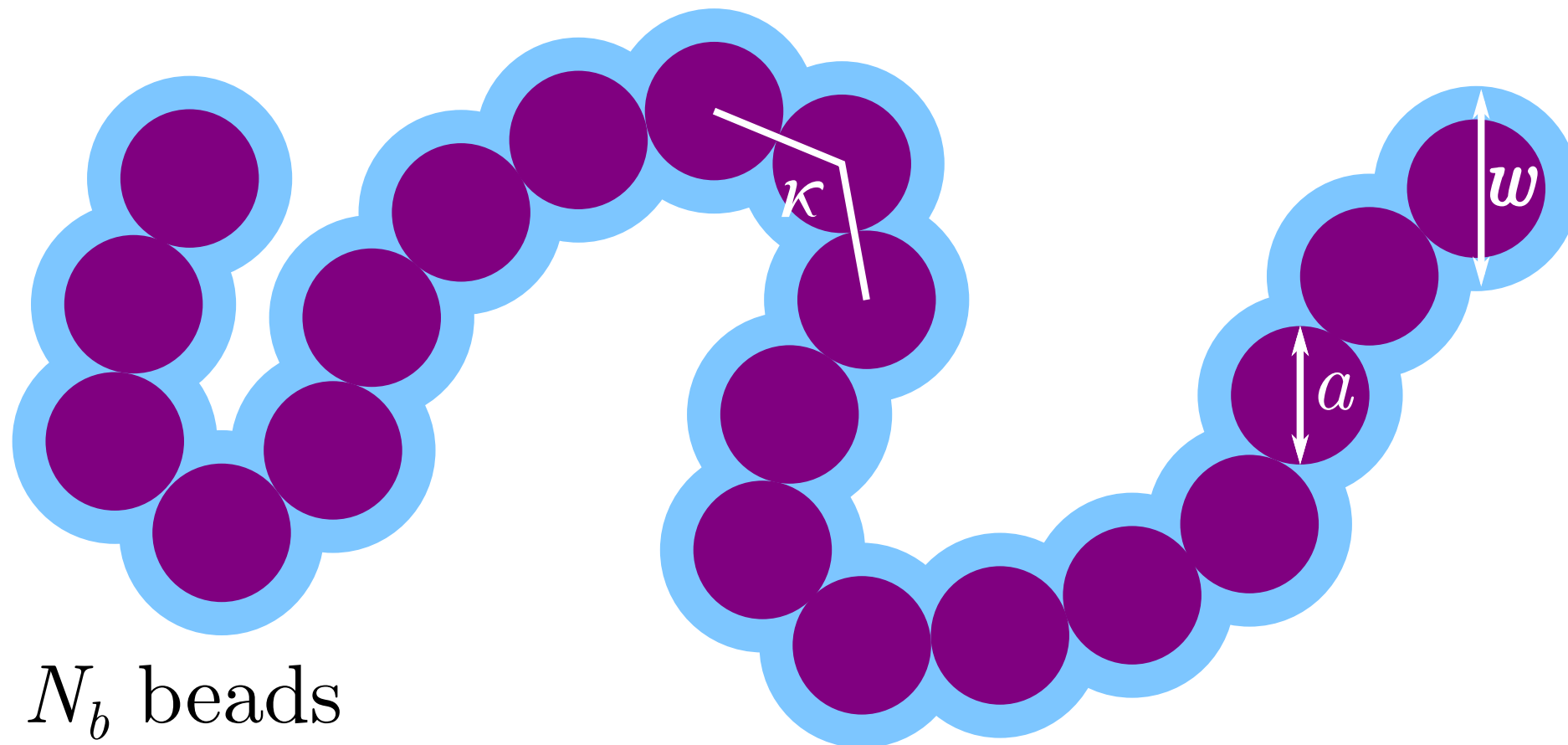
$$\frac{w}{b} \ll 1$$



# A Numerical Approach

## Discrete wormlike chain (DWLC)

- $N_b$  touching hydrodynamic beads of size,  $a$
- Hard-core beads of size,  $w$
- Bending constant,  $\kappa \approx l_p/a$



# A Numerical Approach

- Long chains are needed for experimentally relevant length scales ( $\lambda$ -DNA requires about 5500 beads).
- Metropolis algorithm: chain relaxation time scales like  $L^2$

## Pruned-enriched Rosenbluth Method (PERM)

- MC chain-growth method
- Off-lattice
- Chain lengths up to  $10^5$  beads

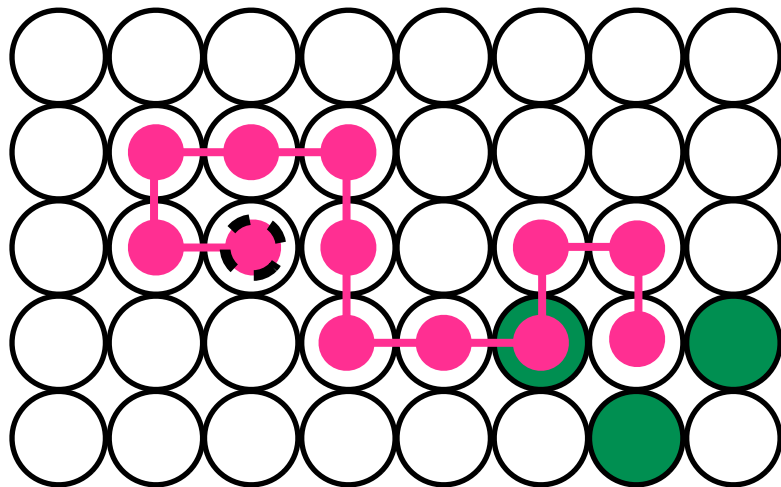
Grassberger. Phys. Rev. E. (1997).

Prellberg and Krawczyk. Phys. Rev. Lett. (2004).

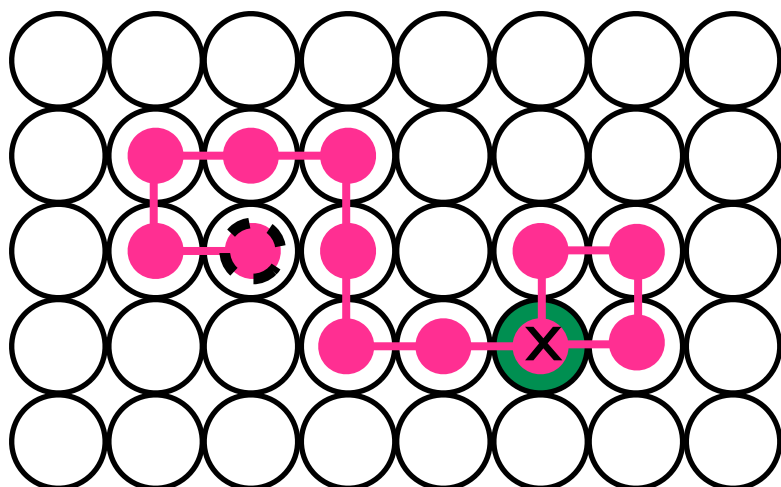
# Basics of PERM

- ▶ Rosenbluth Method suffers from exponential attrition rate

# Growth

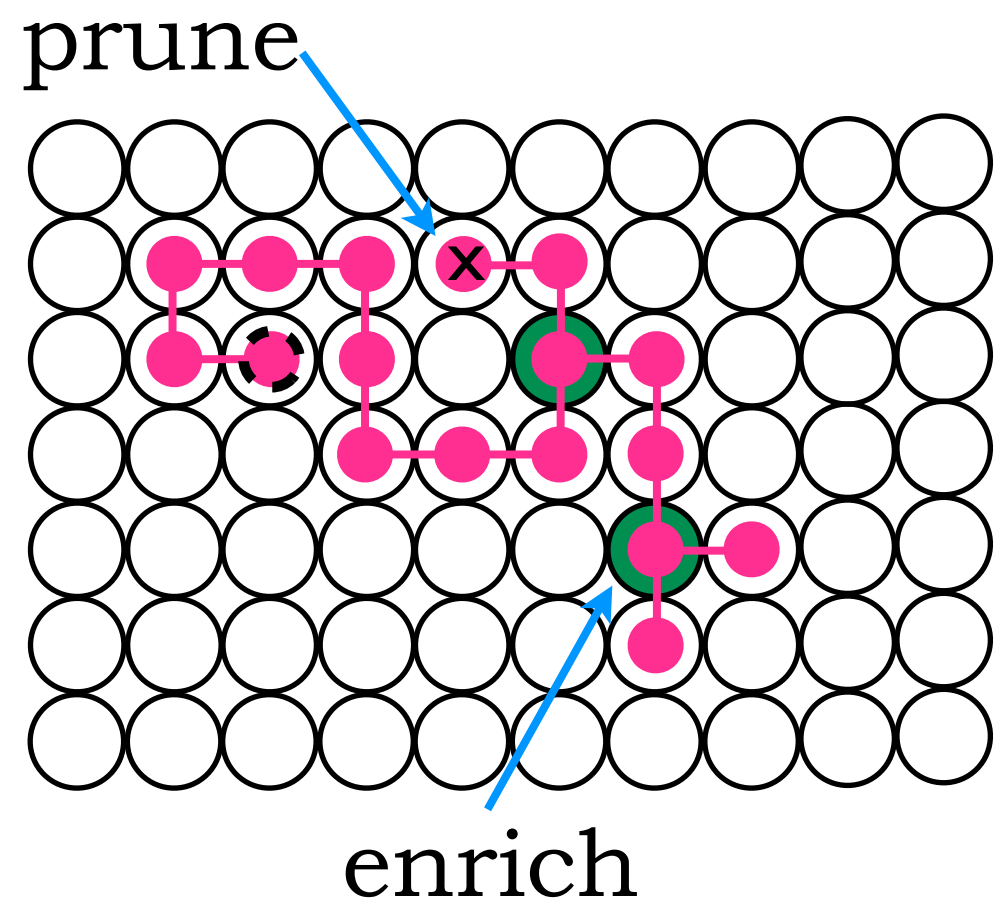


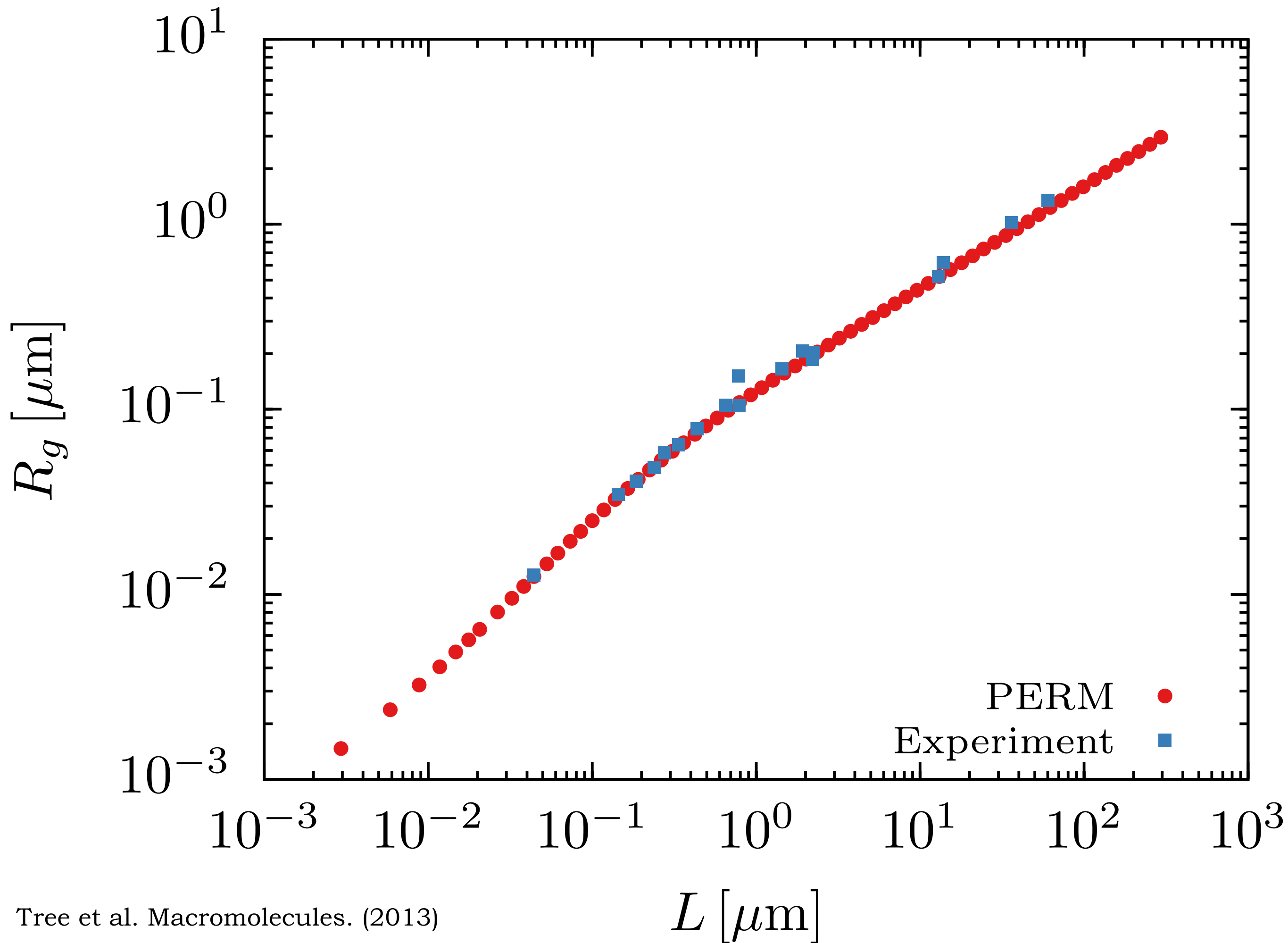
# Attrition

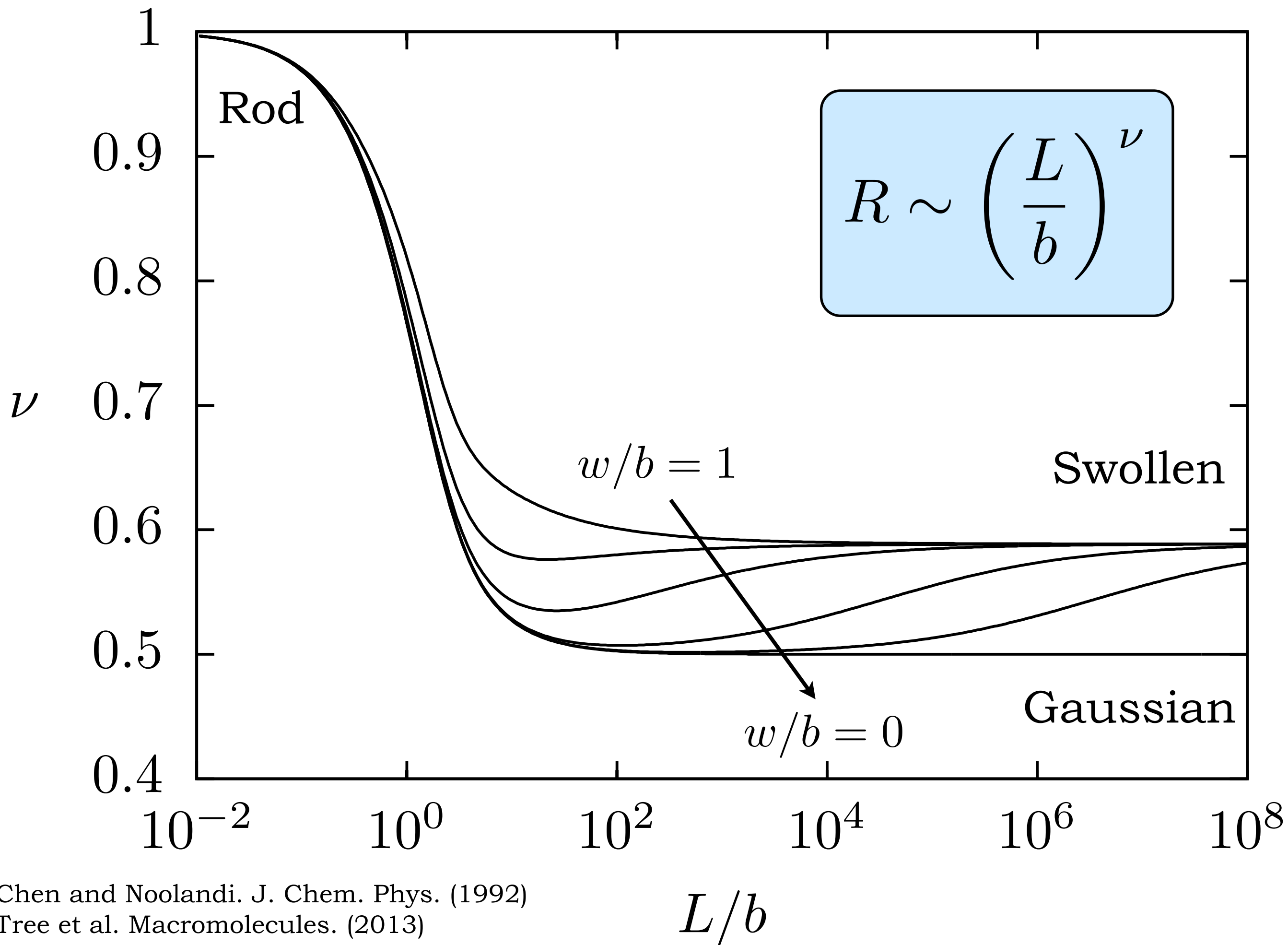


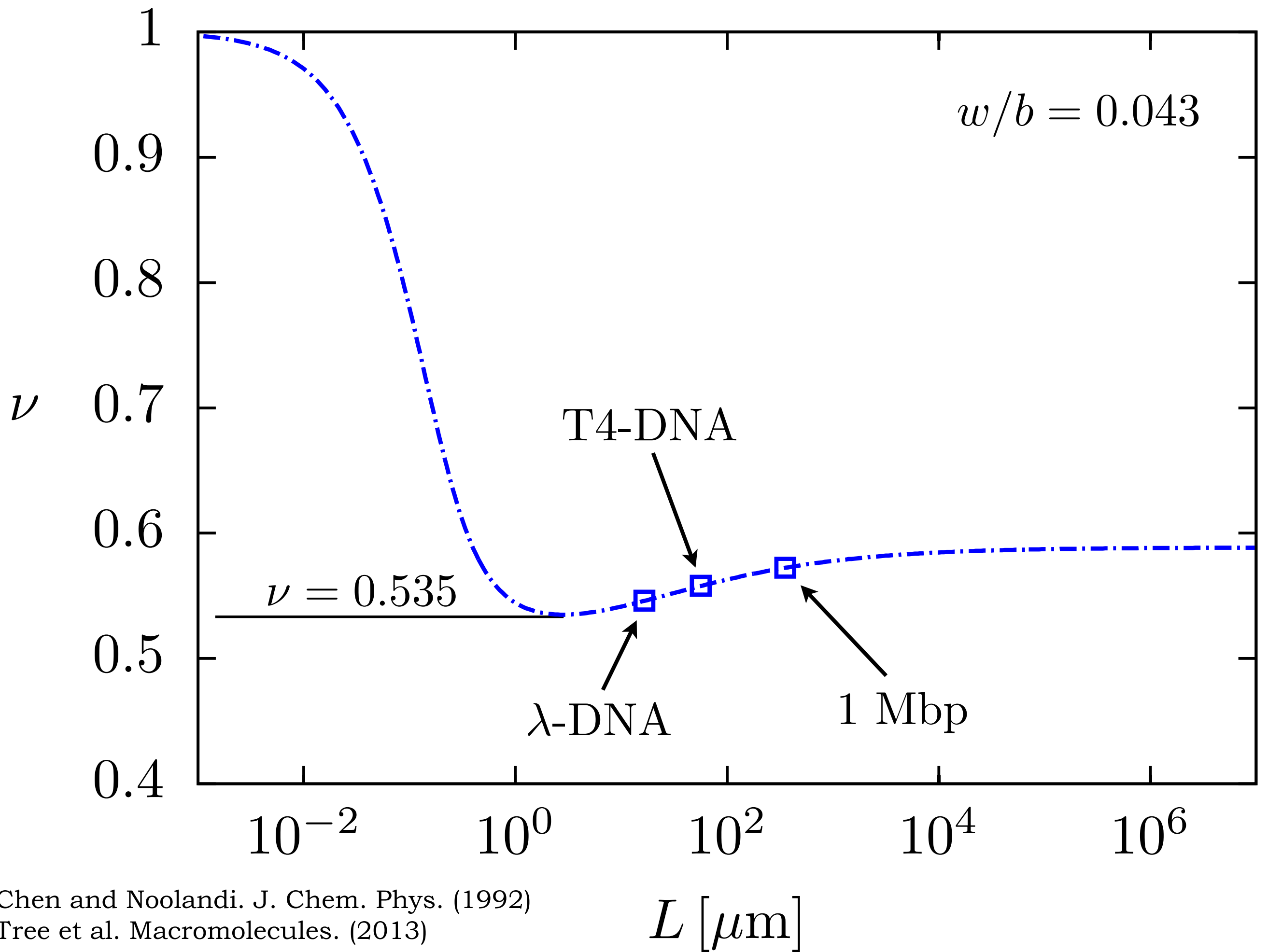
- ▶ PERM grows *tours* of chains and handles attrition statistically by *pruning* and *enrichment*.

# Tour









Chen and Noolandi. J. Chem. Phys. (1992)  
Tree et al. Macromolecules. (2013)

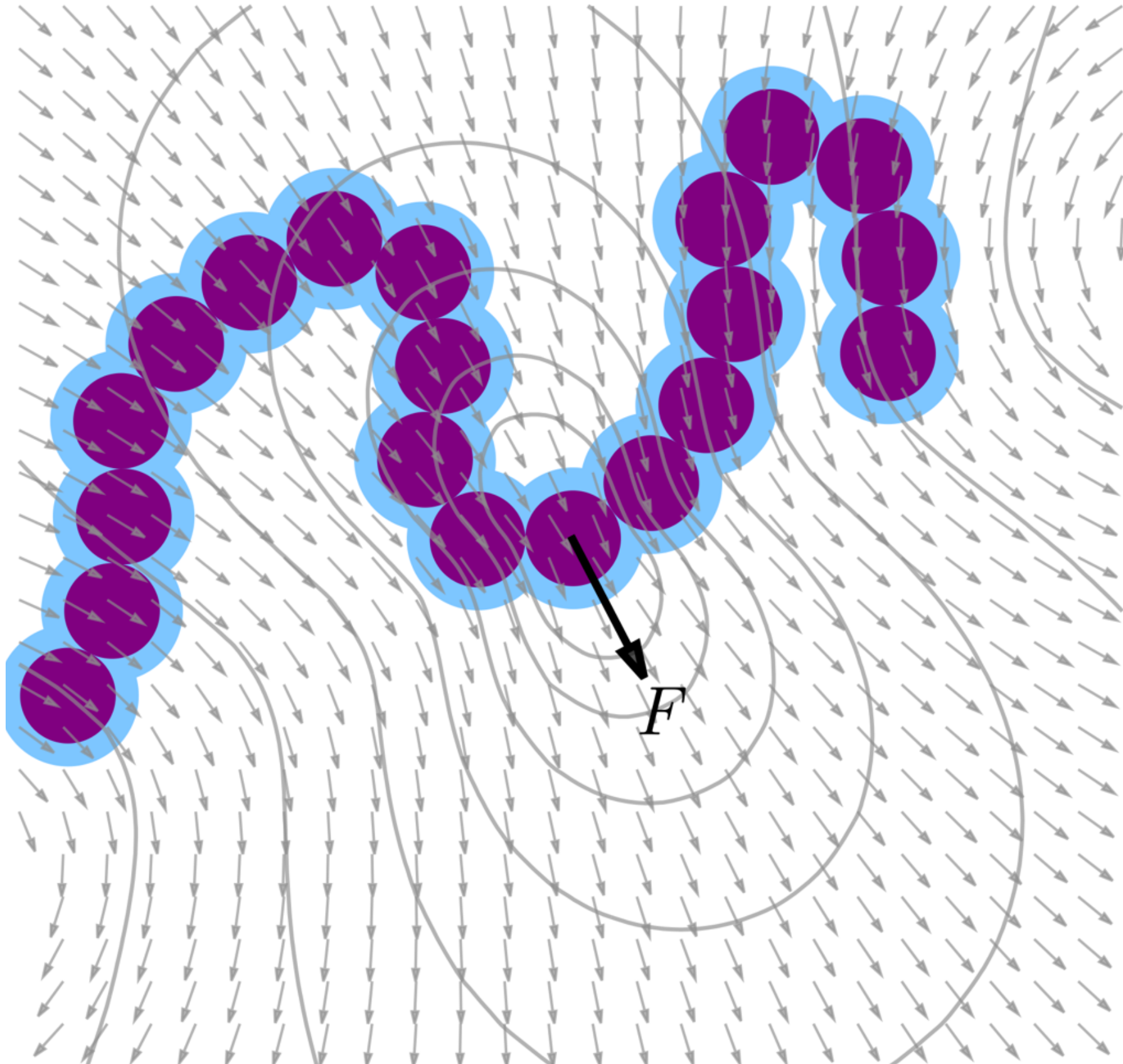
# Results

Radius of gyration calculations:

- Confirm basic results of scaling theory (3 regimes, dependence on anisotropy)
- Scaling transitions are broad and continuous.
- Crossover length is *longer* than anticipated (1 Mbp)  
-- opposite of what we expected from experiments.

What do dynamics tell us?

# Hydrodynamic interactions

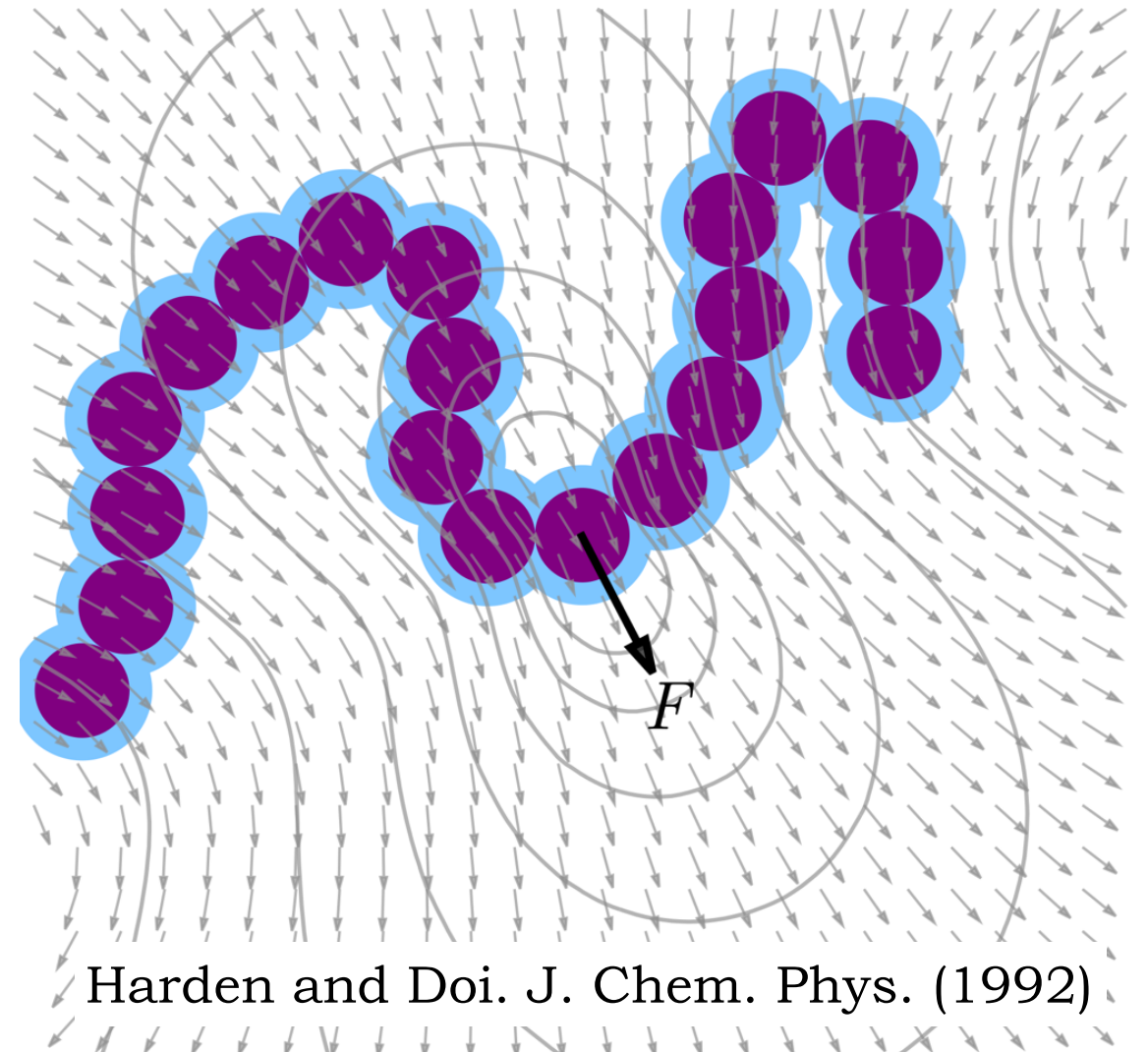


# Rigid-Body Approximation to HI

Diffusivity is approximated as the thermal averaged hydrodynamic mobility.

$$D = \frac{k_B T}{3} \langle \text{Tr}(\boldsymbol{\Omega}) \rangle$$

- Assumes near-equilibrium configurations which are perturbed by infinitesimal forces.
- Neglects correlations due to Brownian motion (mean field approximation).



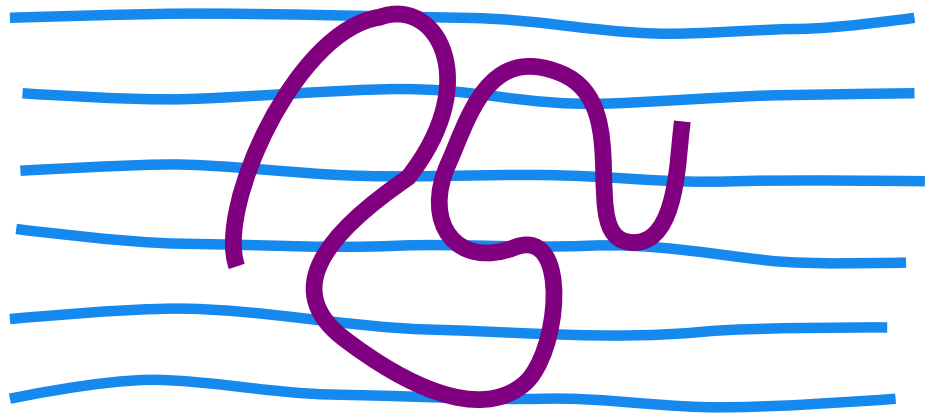
$$\boldsymbol{\Omega} = \frac{1}{N^2} \sum_{i,j} \left[ \left( \frac{\delta_{ij}}{3\pi\eta a} + \frac{1 - \delta_{ij}}{8\pi\eta r_{ij}} \right) \mathbf{I} + \frac{\mathbf{r}_{ij}\mathbf{r}_{ij}}{r_{ij}^2} \right]$$

# Hydrodynamics of WLCs



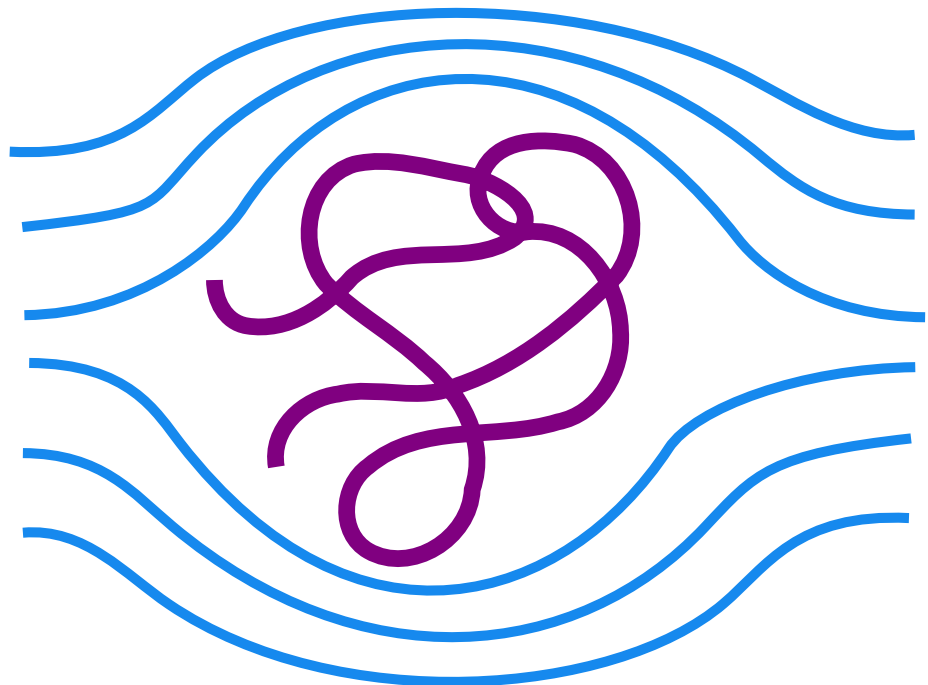
Rod

$$D \sim \frac{k_B T}{\eta L} \ln \left( \frac{L}{2a} \right)$$



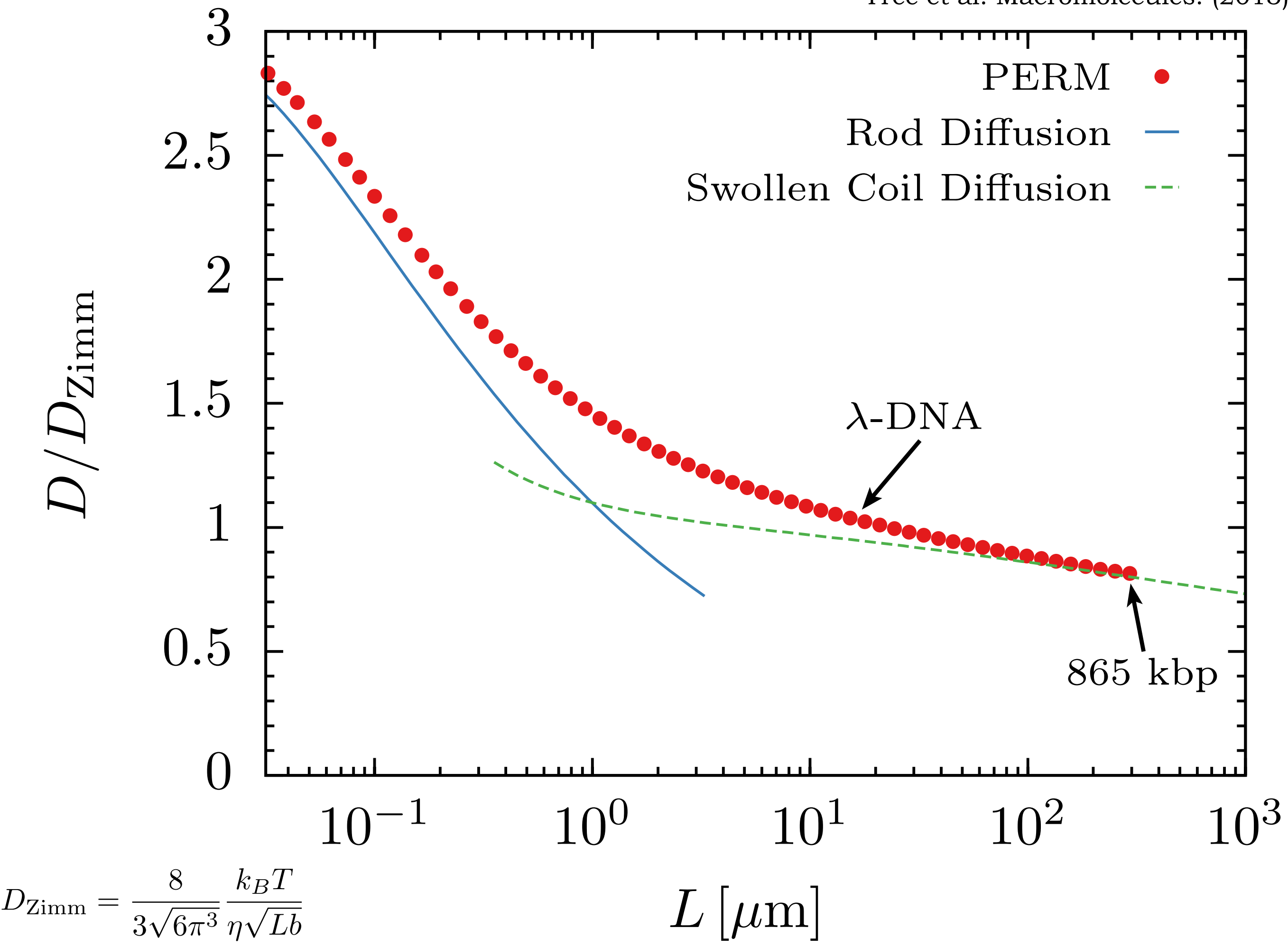
Draining Coil  
(weak HI)

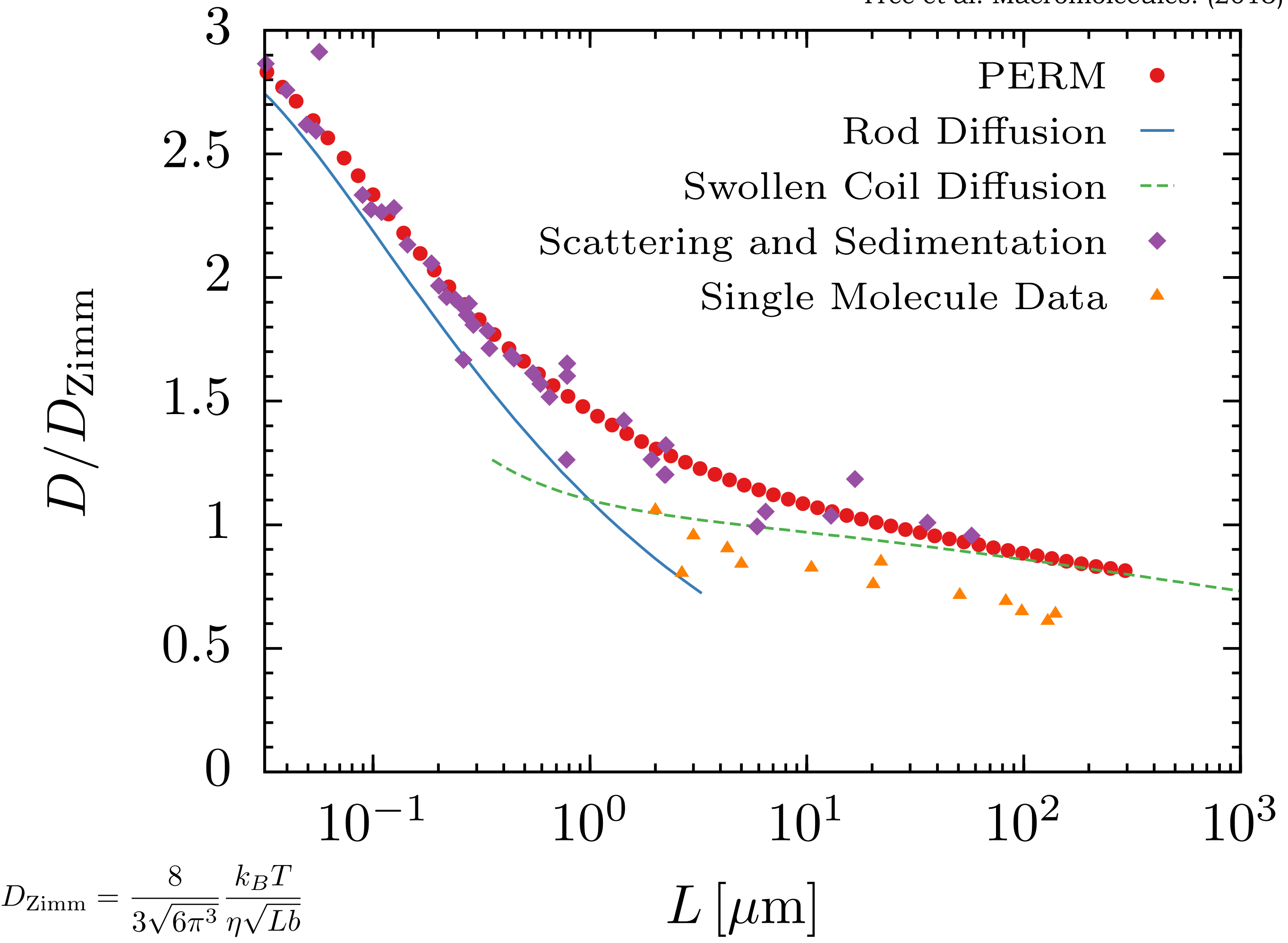
$$D \sim \frac{k_B T}{\eta L}$$

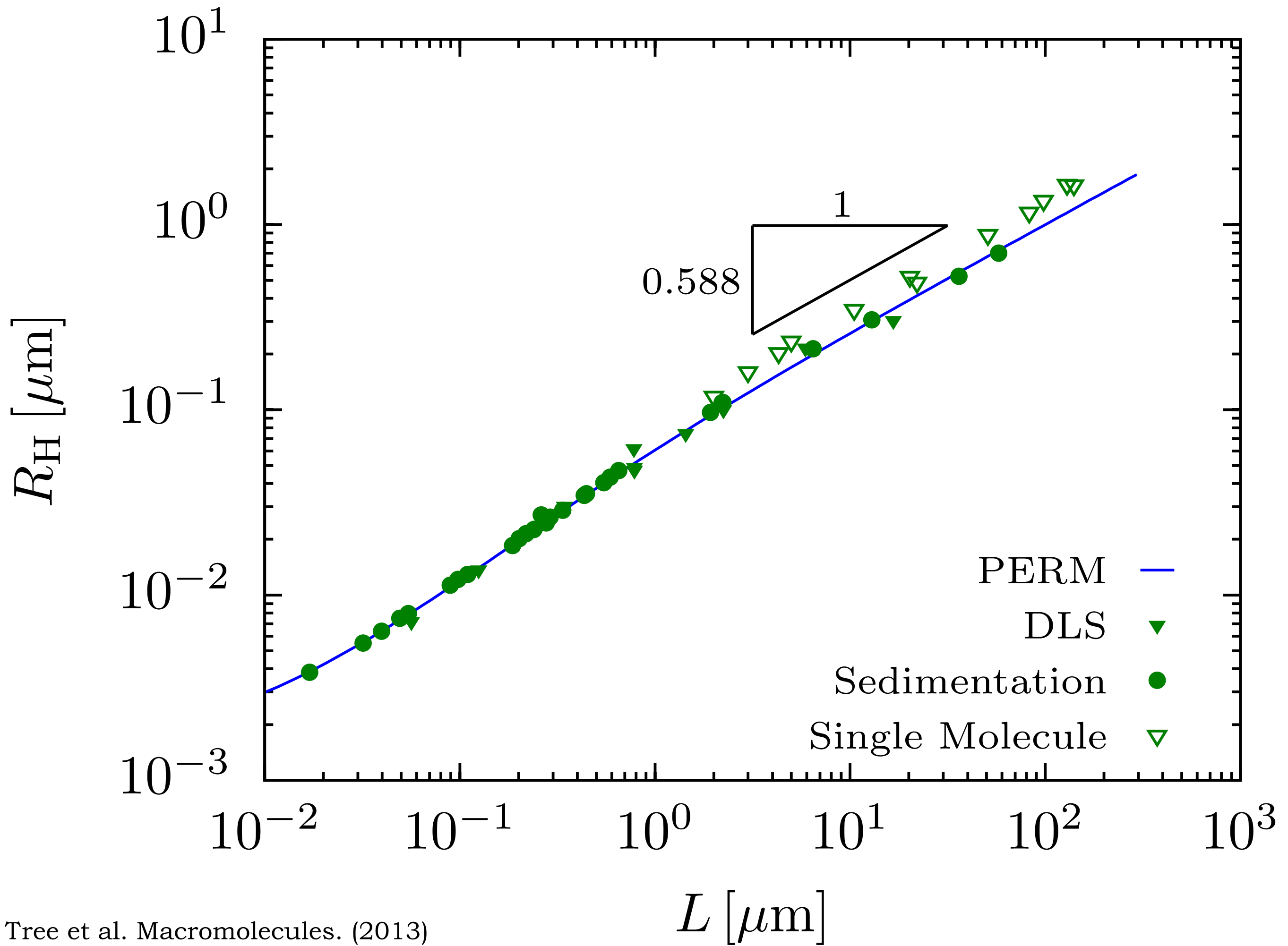


Non-Draining Coil  
(strong HI)

$$D \sim \frac{k_B T}{\eta R_g}$$





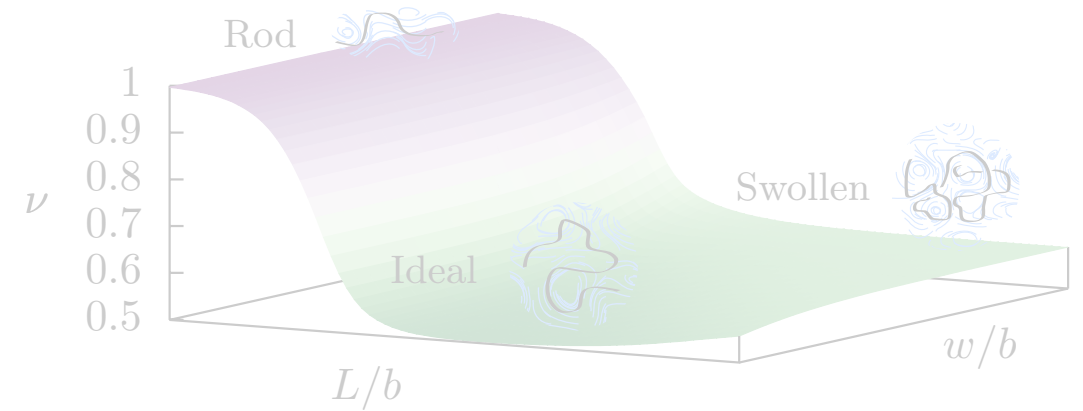


# Conclusions

- Can model DNA as a wormlike chain.
- Basic scaling theory results are confirmed using Monte Carlo simulation.
- Crossover to universal behavior happens at very long contour lengths -- greater than 1 Mbp.
- The measured scaling exponent is not sensitive enough to indicate subtle changes in polymer draining behavior.

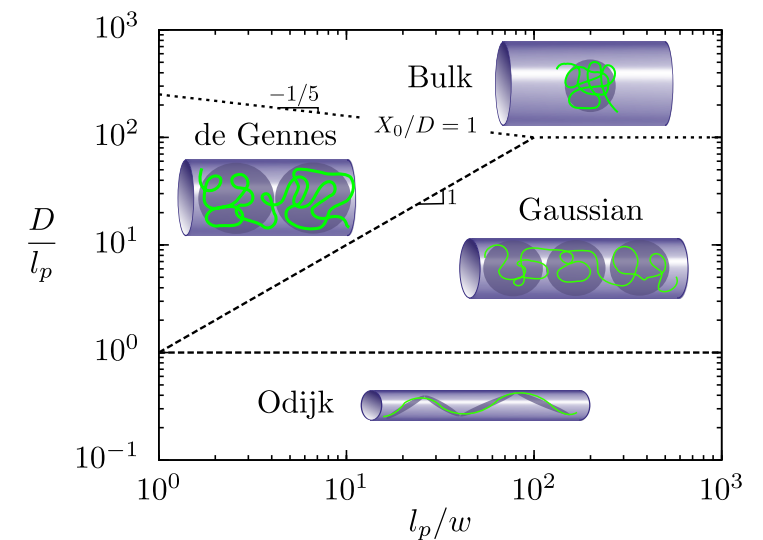
DR Tree, A Muralidhar, PS Doyle and KD Dorfman,  
Macromolecules. 46 (20) 8369-8382 (2013).

# Polymer Physics of DNA in Solution



DR Tree, A Muralidhar, PS Doyle and KD Dorfman, *Macromolecules* (2013).

# Regimes of Confined DNA



DR Tree, Y Wang and KD Dorfman, *Phys. Rev. Lett.* 110, 208103 (2013).

# Hydrodynamics of Confined DNA

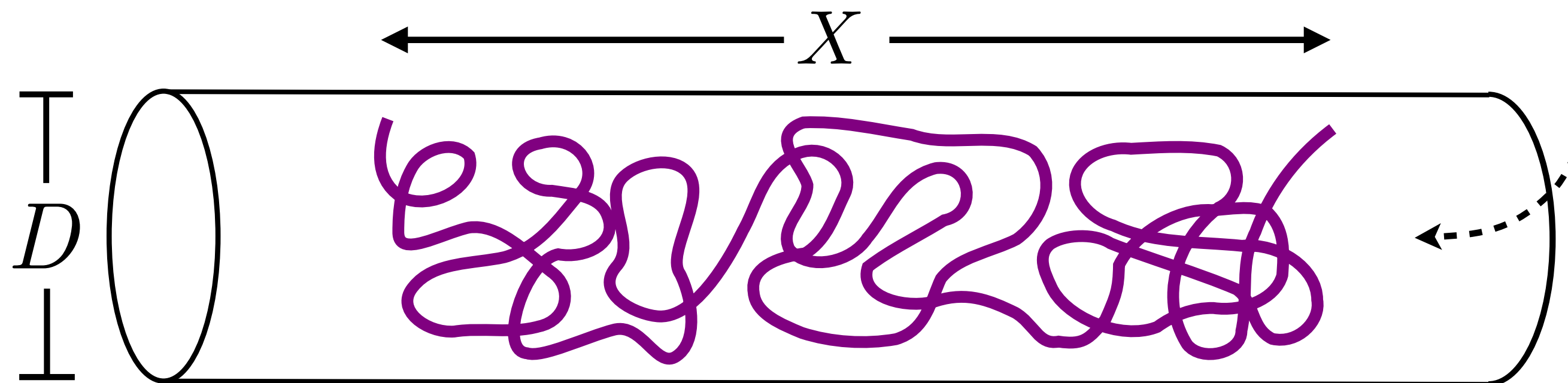
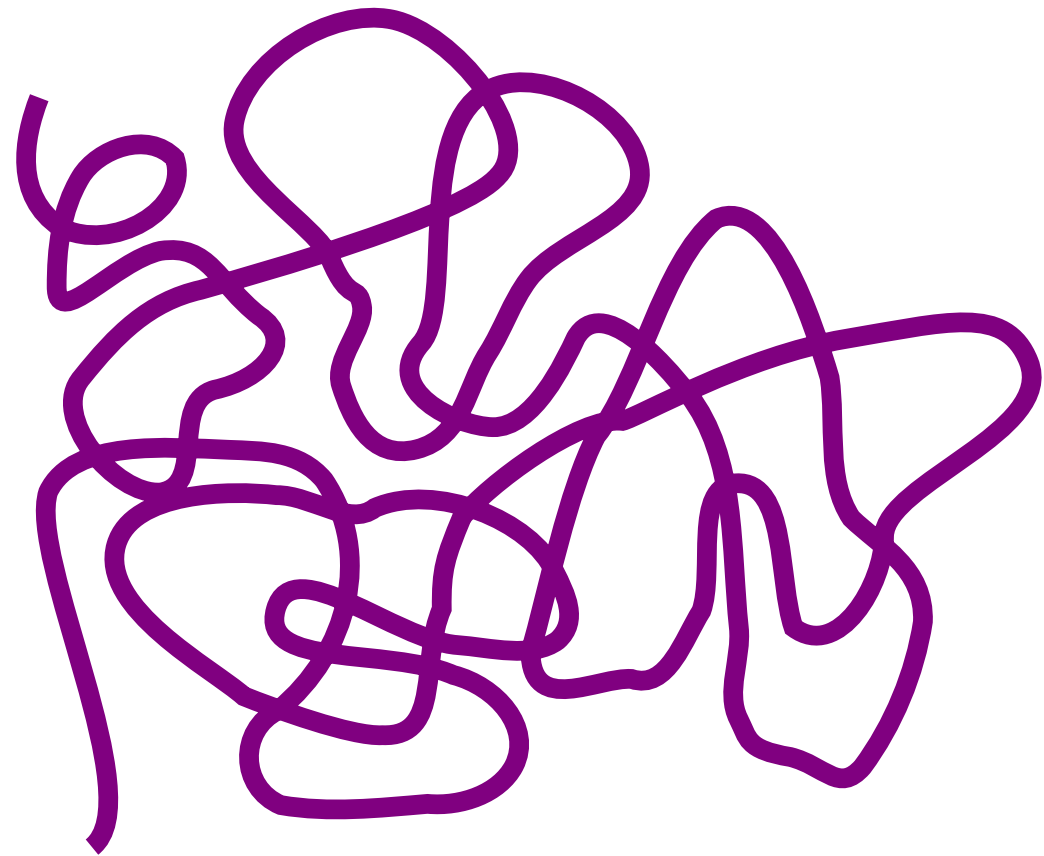


DR Tree, Y Wang and KD Dorfman, *Phys. Rev. Lett.* 108, 228015 (2012).

# Confined Semiflexible Polymer

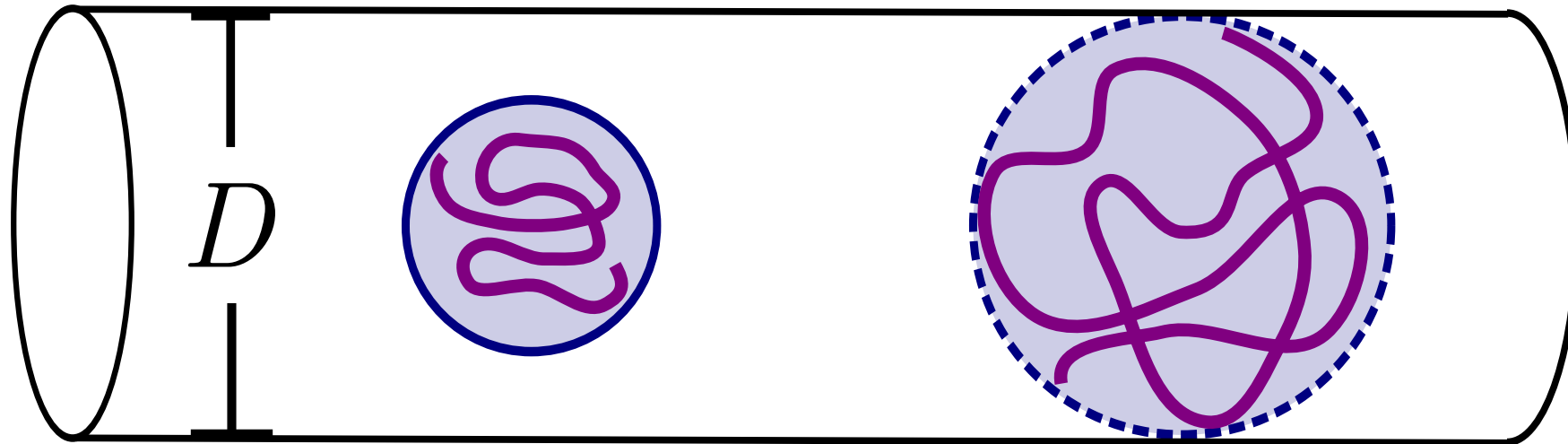
Place a WLC in a tube with diameter  $D$ .

- Let the tube be long compared to the size of the chain.
- What is the span,  $X$ ?
- What is the confinement free energy,  $\Delta F_c$ ?



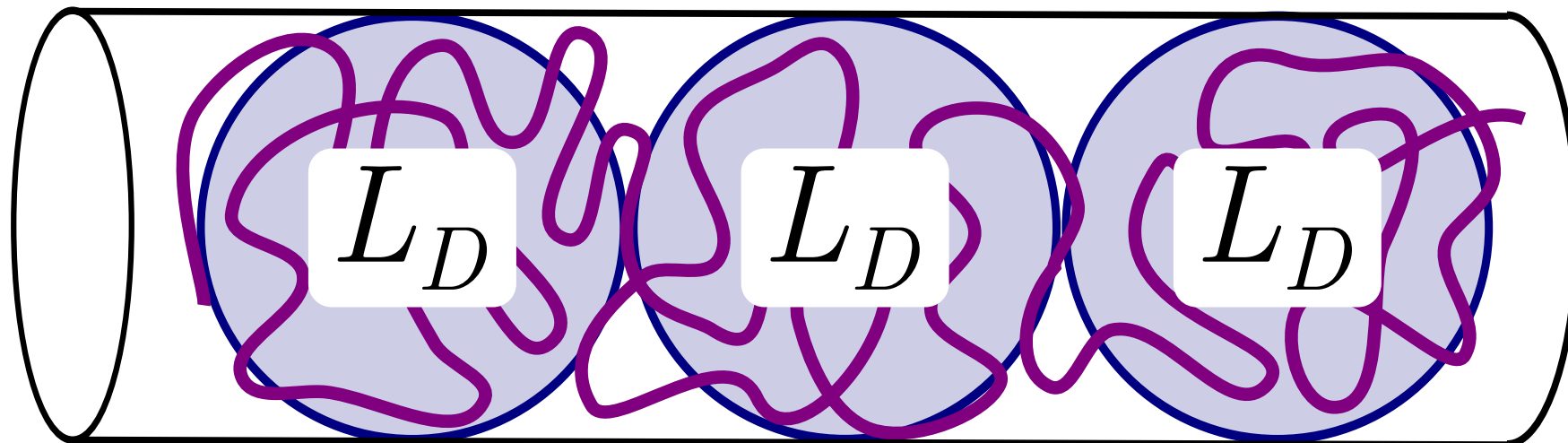
# Confining a Chain

$$L < L_D \quad L = L_D$$



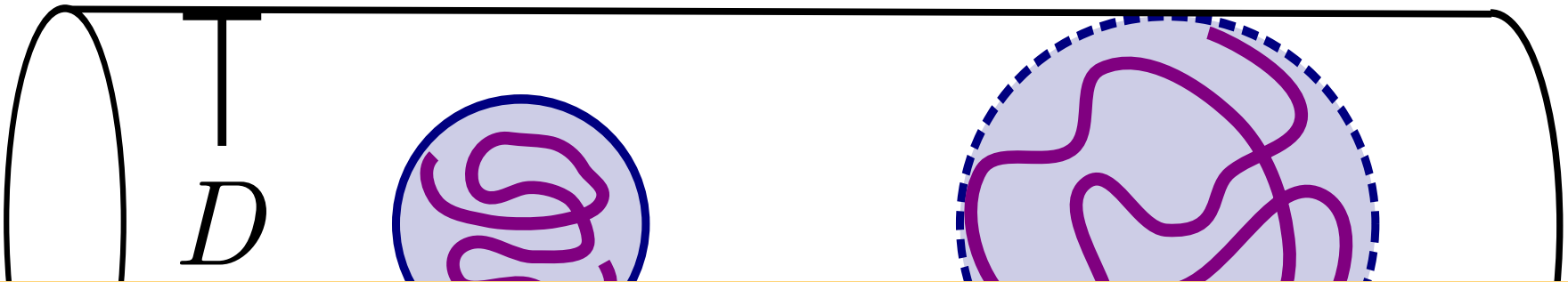
- Properties for long chains are extensive in subunits made of length  $L_D$ .

$$L > L_D$$



# Confining a Chain

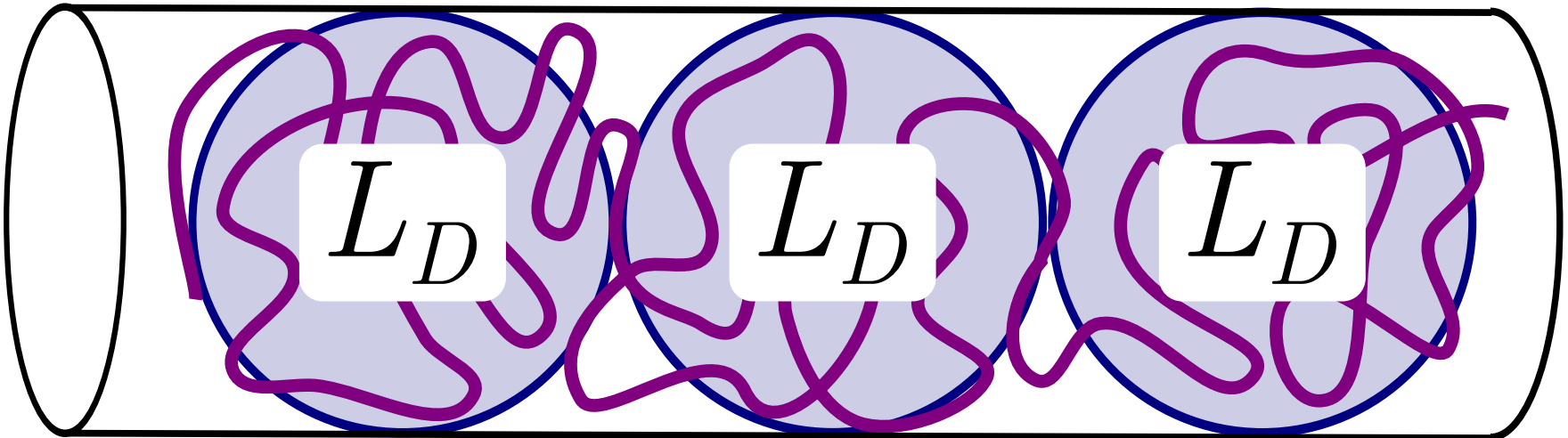
$$L < L_D \qquad L = L_D$$

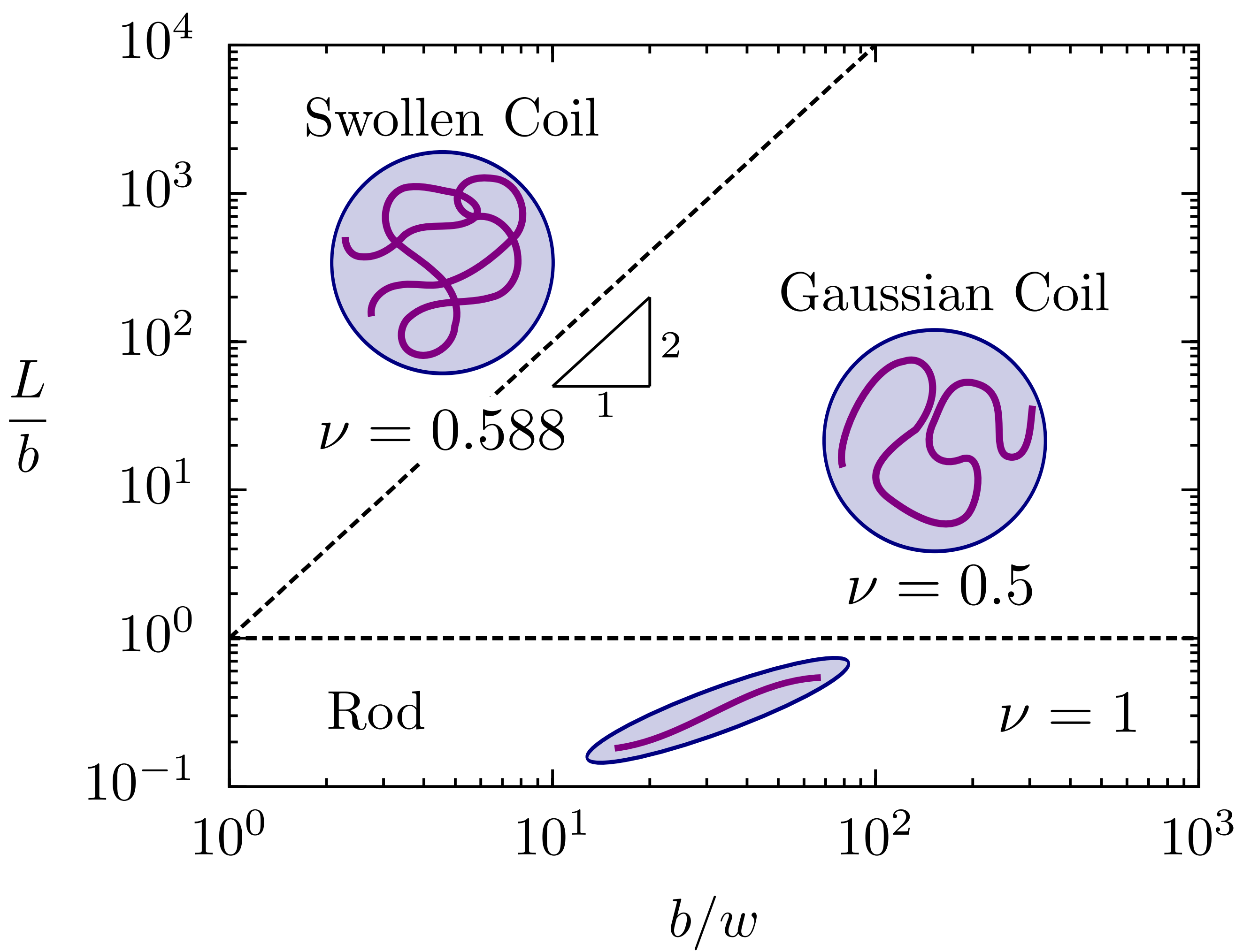


- Prop of len

$$\Delta F_c = f\left(\frac{L}{b}, \frac{D}{b}, \frac{w}{b}\right) \rightarrow \Delta F_c = L f\left(\frac{D}{b}, \frac{w}{b}\right)$$

made



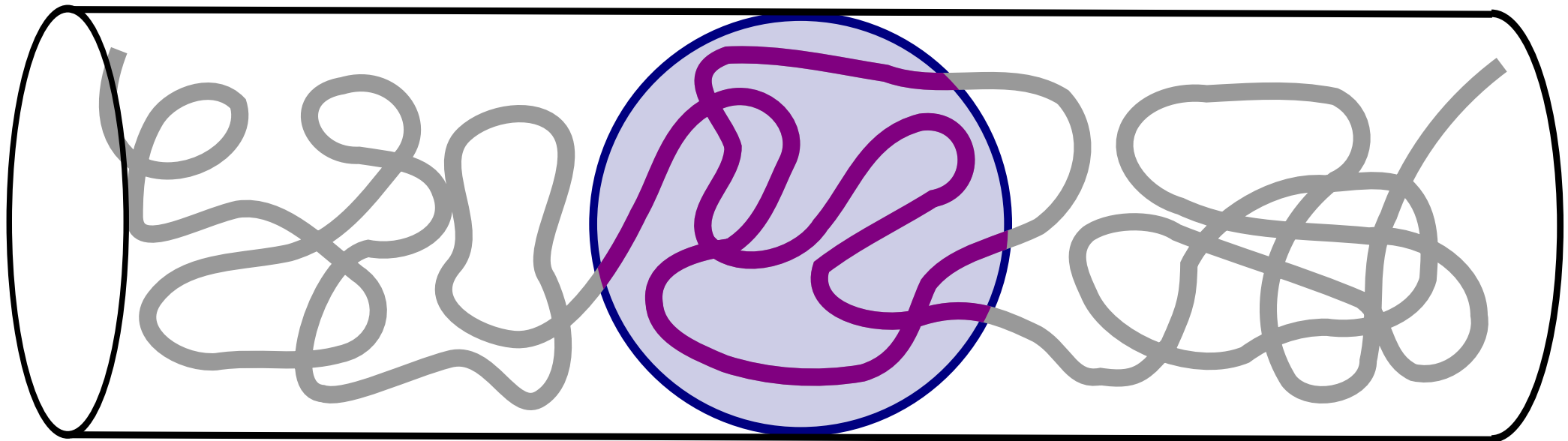


# Confined Chain -- de Gennes

- The chain is composed of a linear string of blobs.

$$l_p \ll D \ll R_g$$

$$D \sim L_{\text{blob}}^{3/5} \qquad F \sim k_B T \frac{L}{L_{\text{blob}}}$$

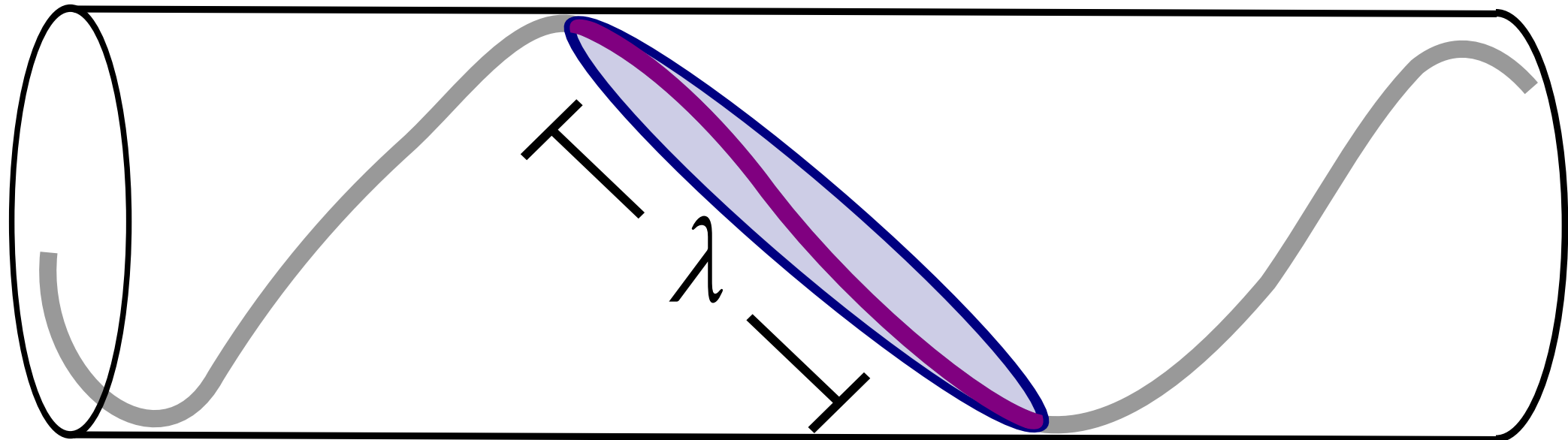


$$F \sim k_B T L D^{-5/3}$$

# Confined Chain -- Odijk

$$D \ll l_p$$

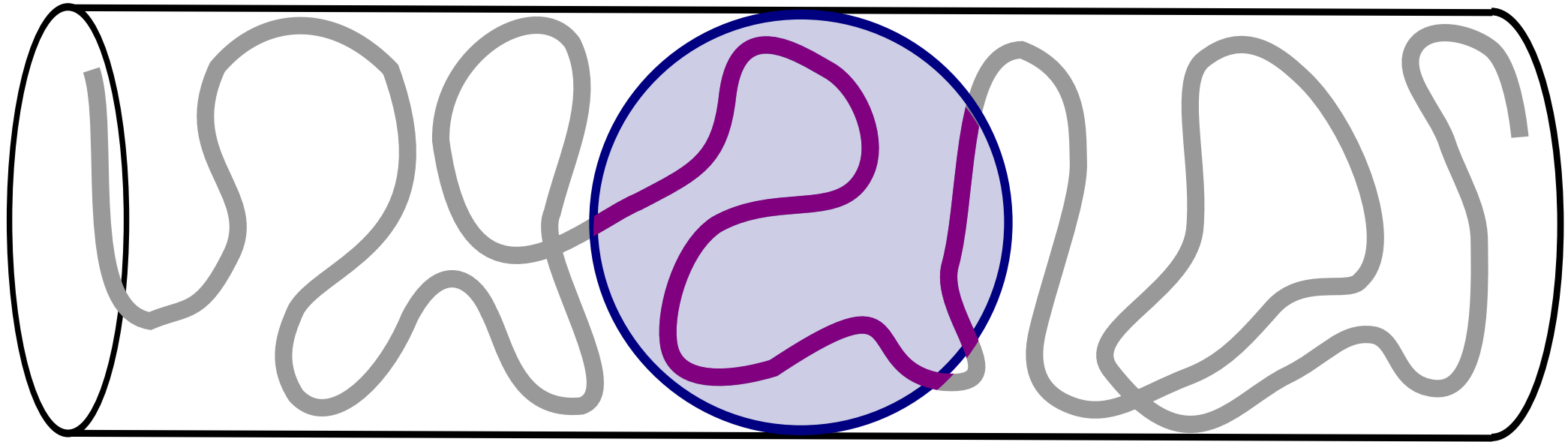
$$\lambda \sim D^{2/3} l_p^{1/3} \quad F \sim k_B T \frac{L}{\lambda}$$



$$F \sim k_B T L D^{-2/3}$$

# What about a Gaussian-like regime?

$$D \sim L_{\text{blob}}^{1/2}$$



$$F \sim k_B T L D^{-2}$$

# Summary of Confined Chain Theory

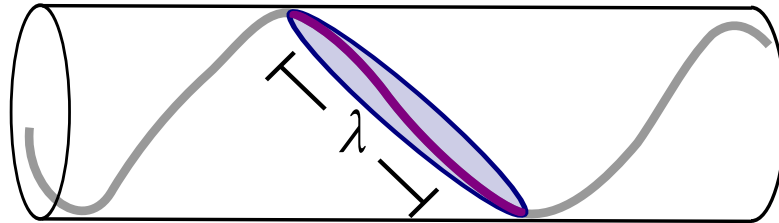
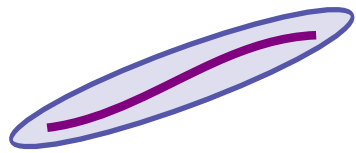
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Unconfined

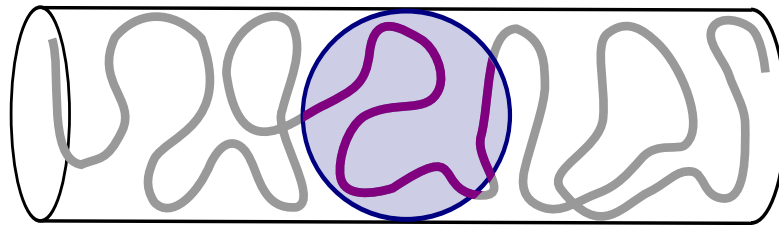
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Confined

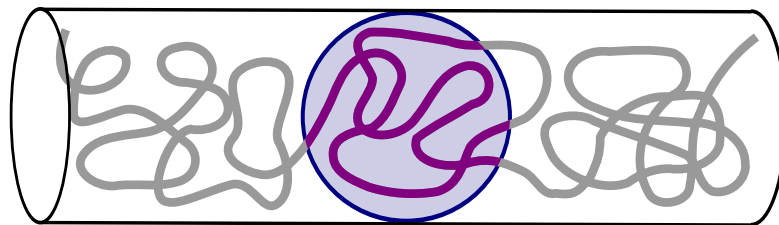
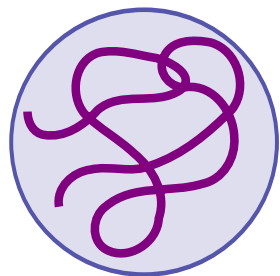
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$$F \sim D^{-2/3}$$



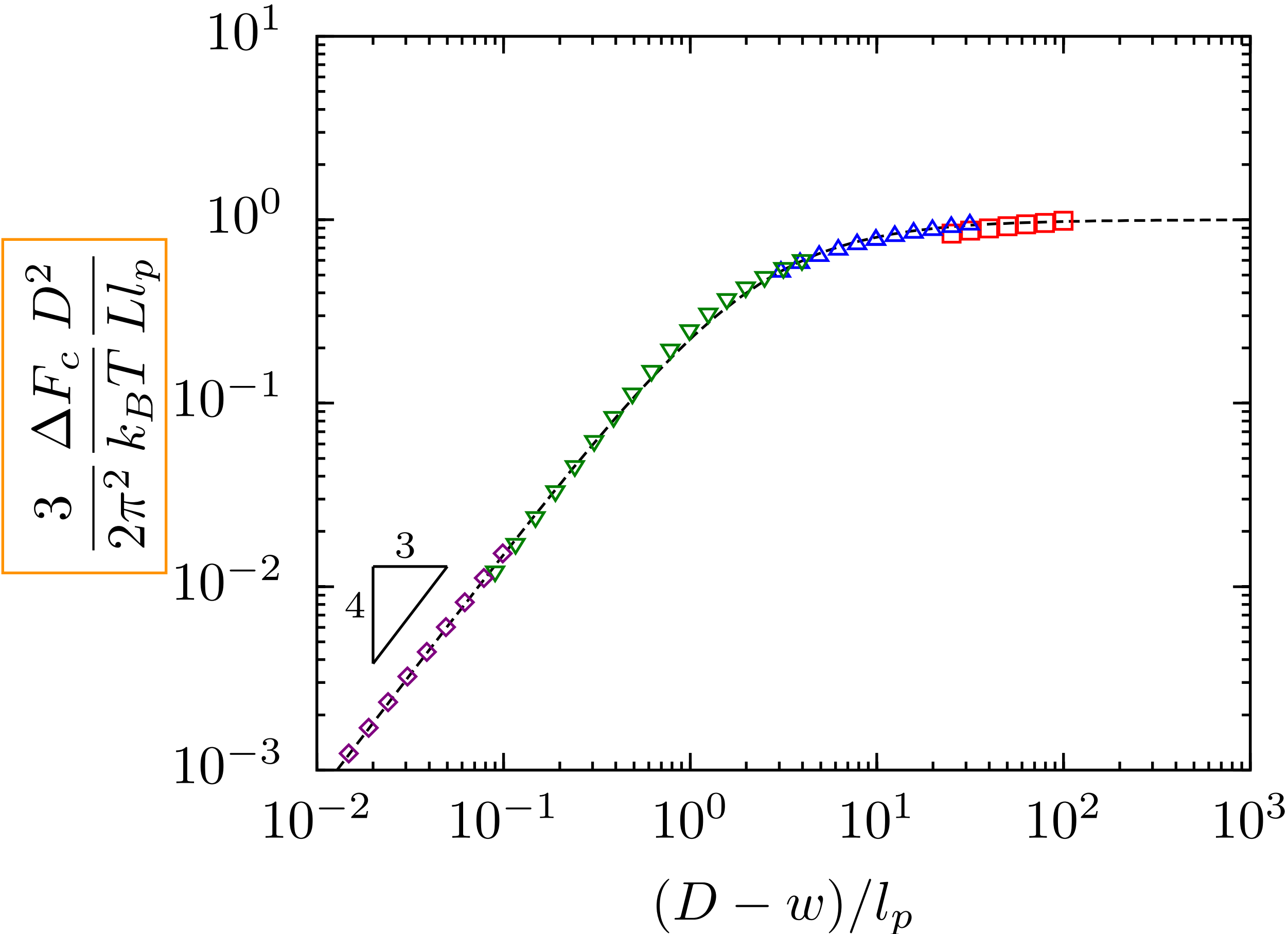
$$F \sim D^{-2}$$



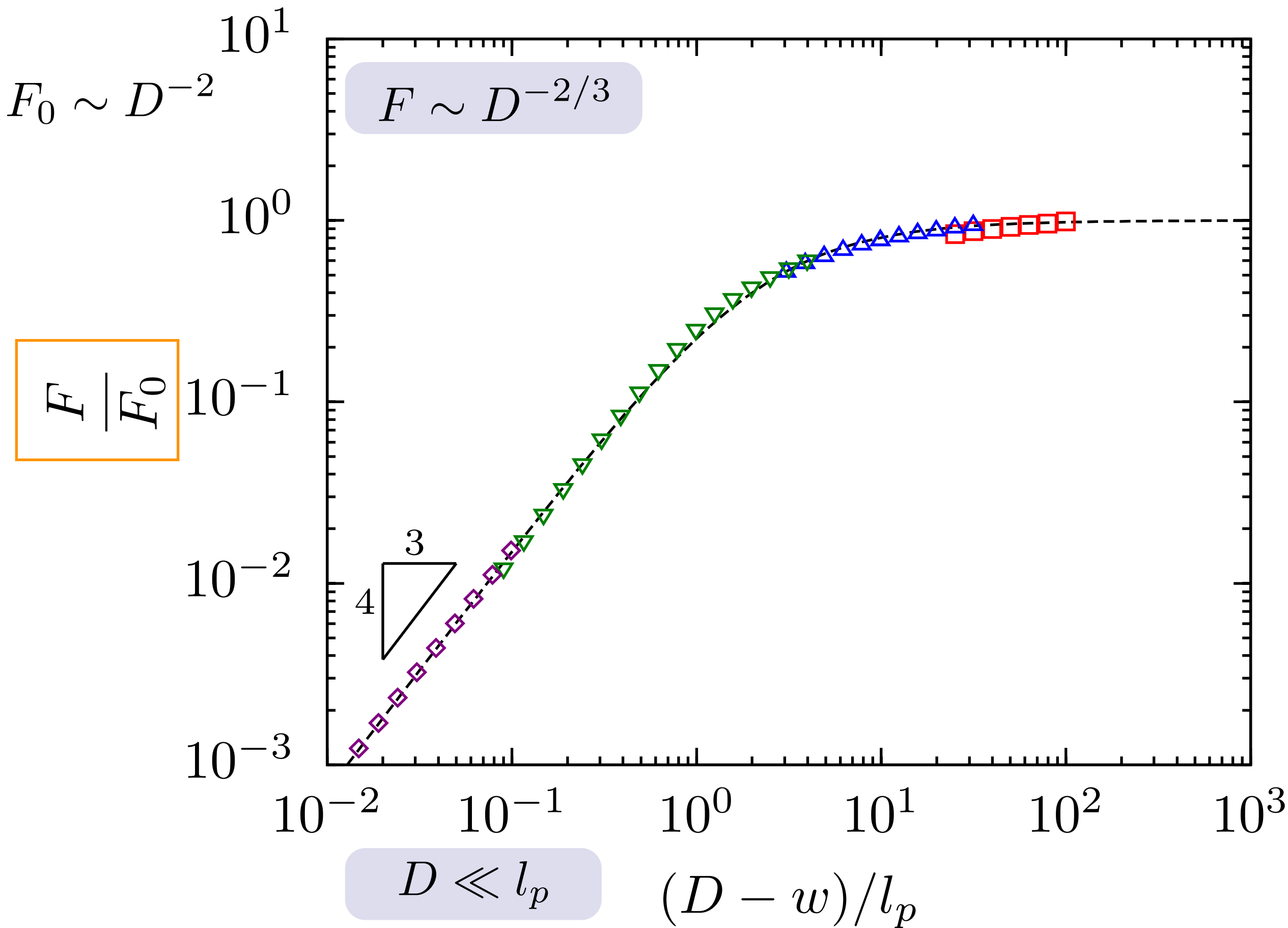
$$F \sim D^{-5/3}$$

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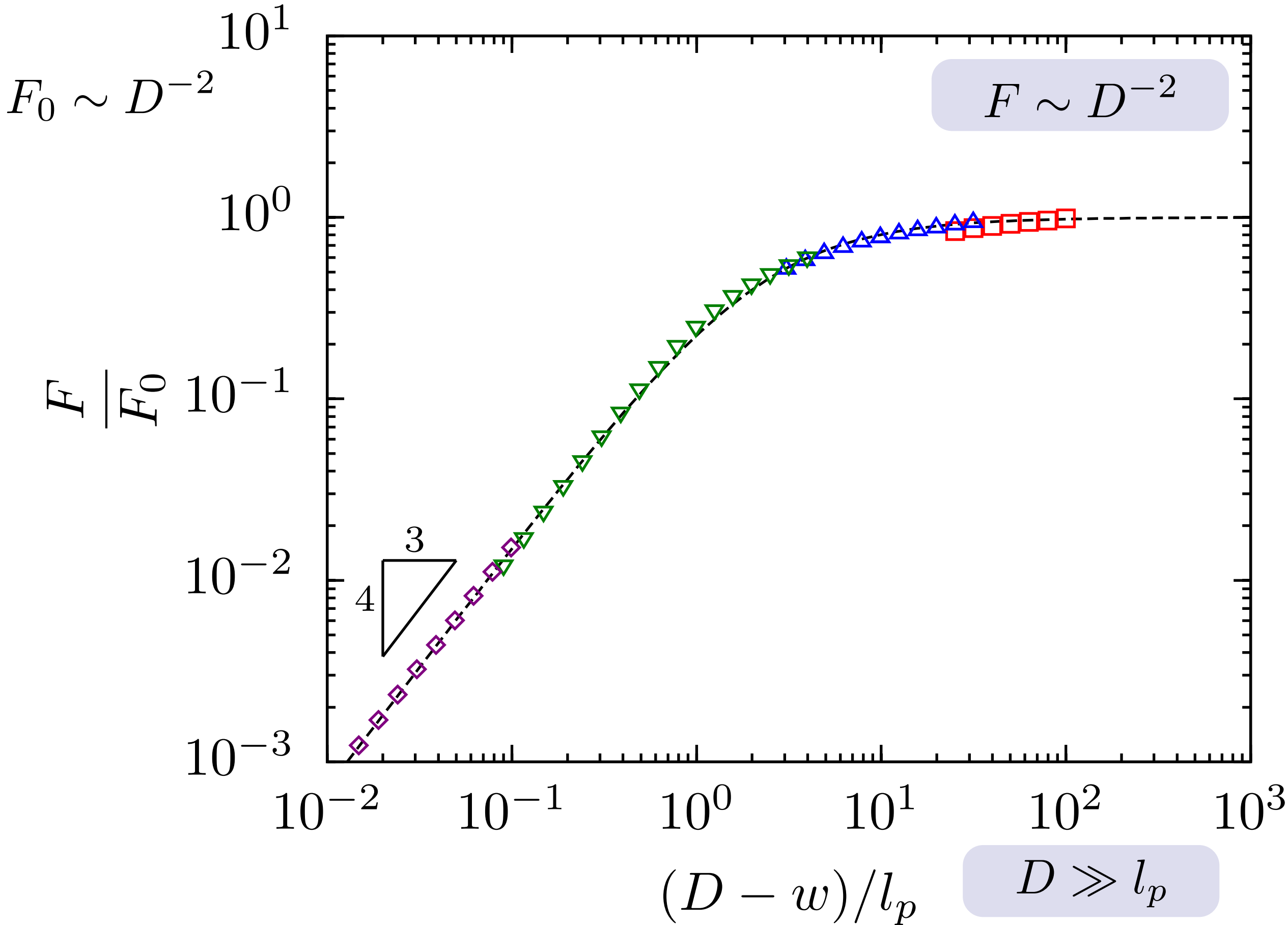
# Confined Chain without Excluded Volume



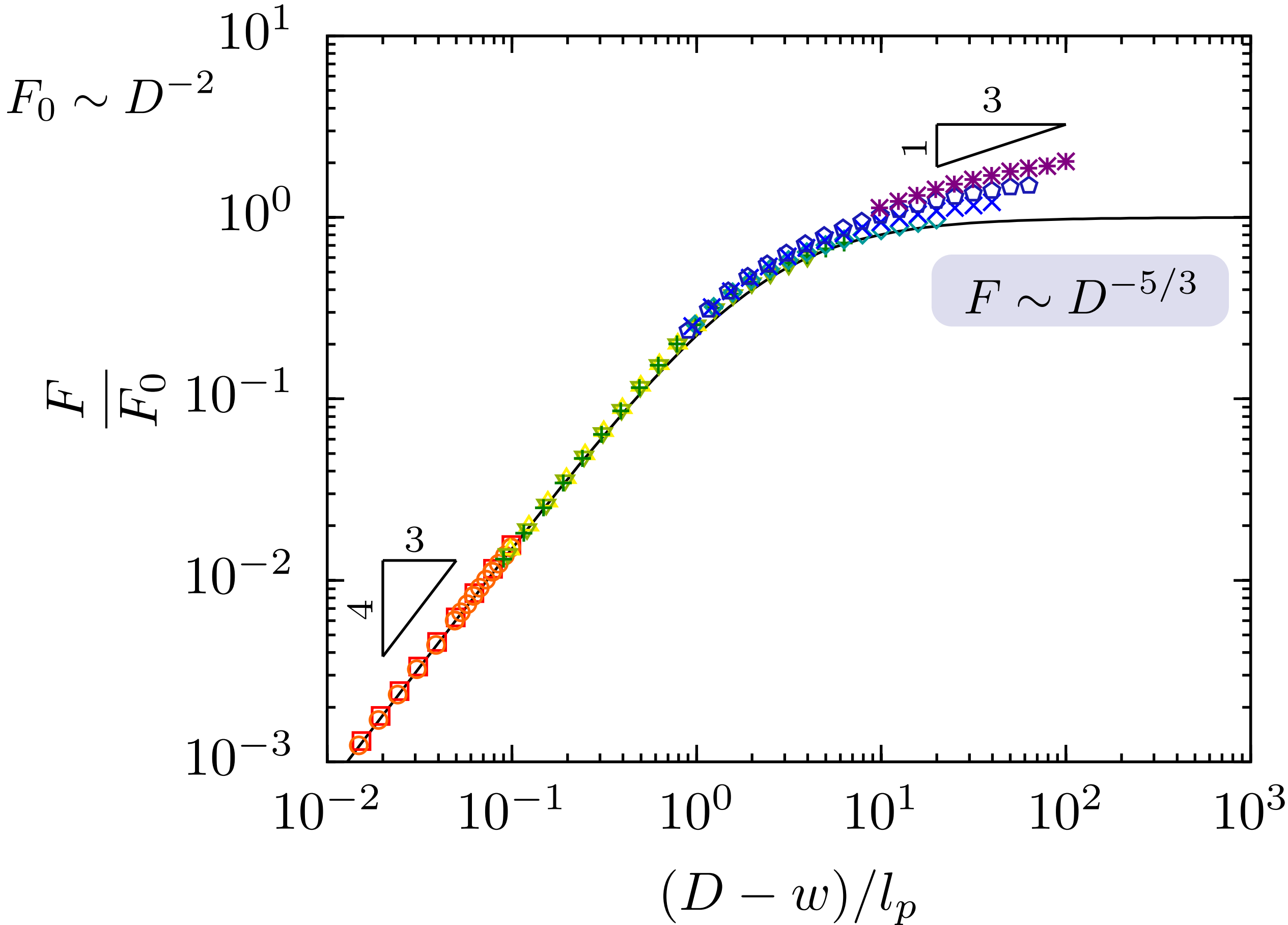
# Confined Chain without Excluded Volume



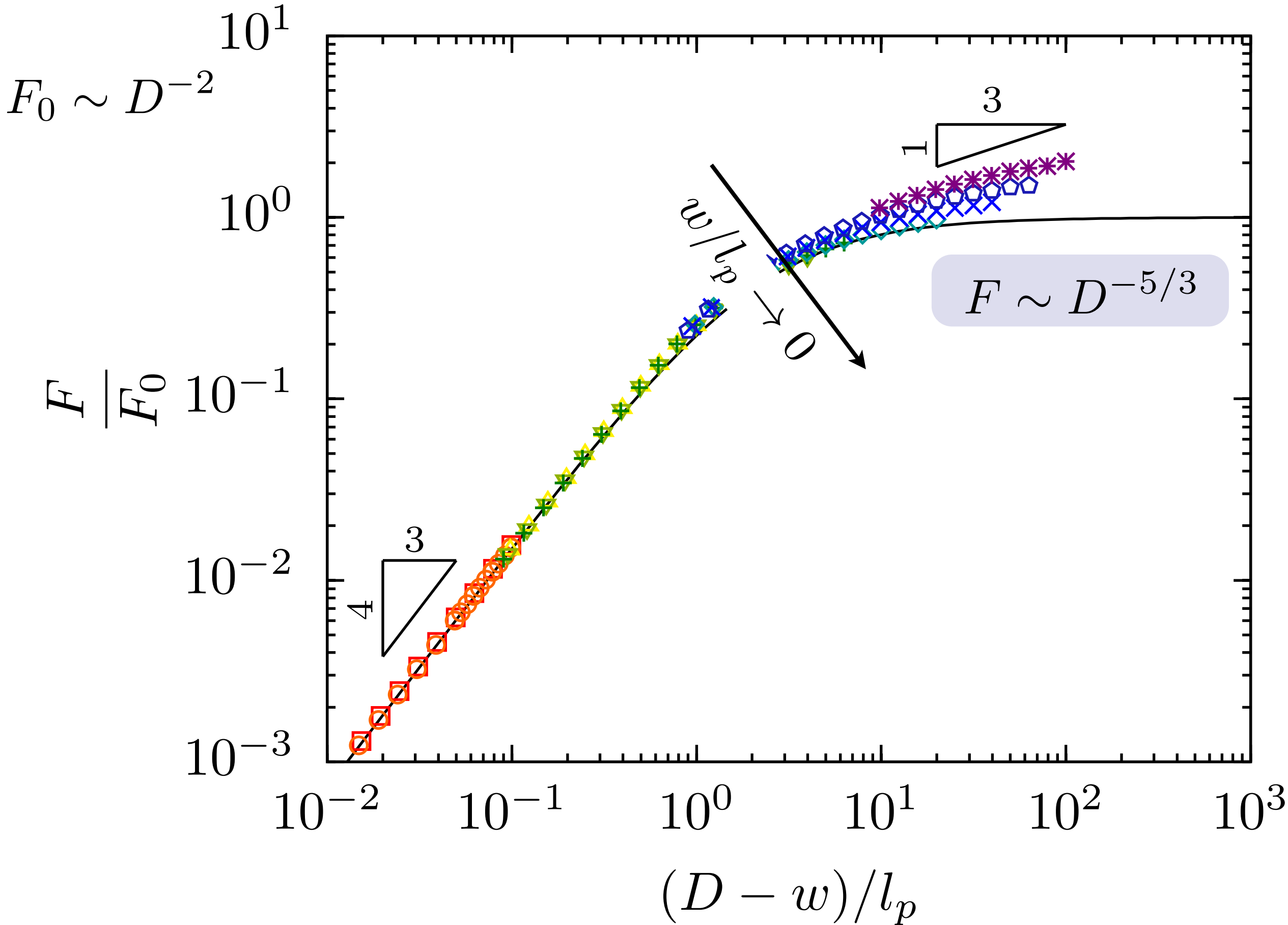
# Confined Chain without Excluded Volume



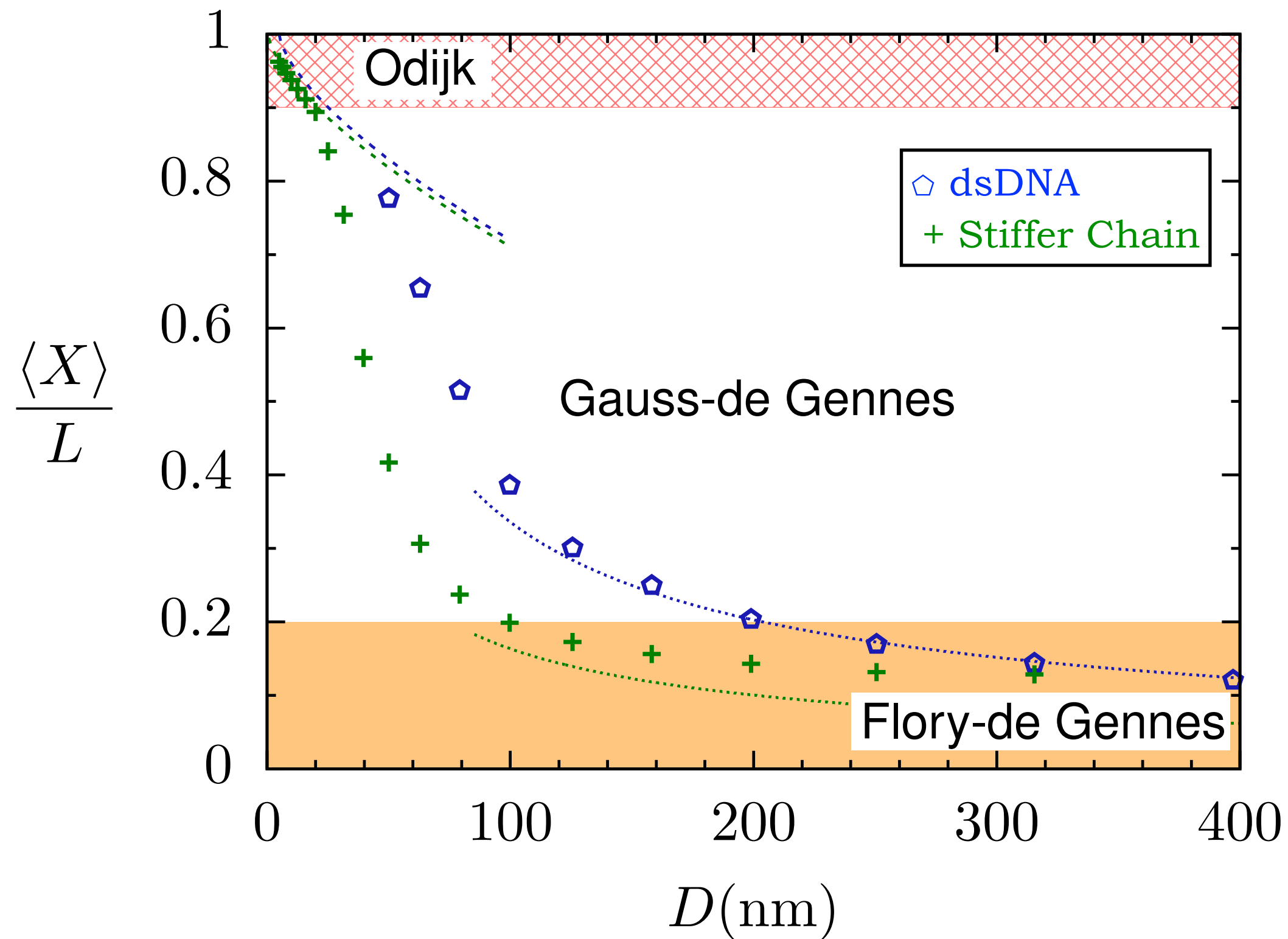
# Confined Chain with Excluded Volume



# Confined Chain with Excluded Volume



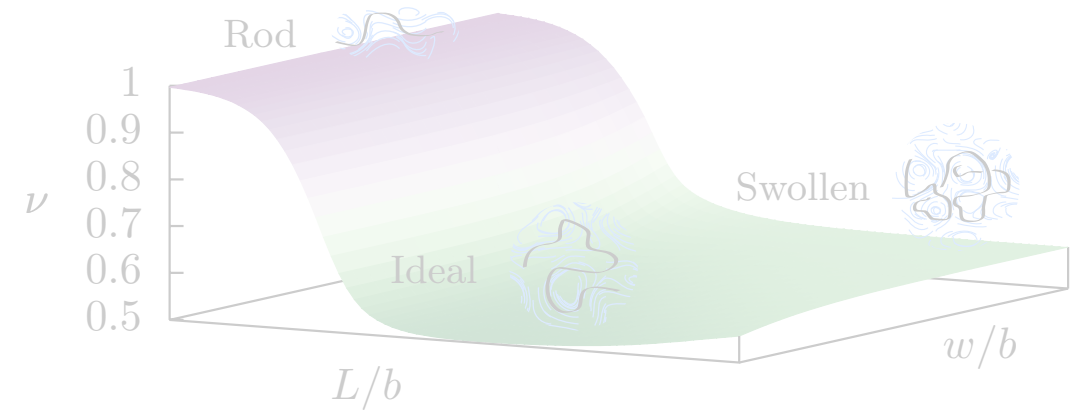
# Where are the regimes?



# Conclusion

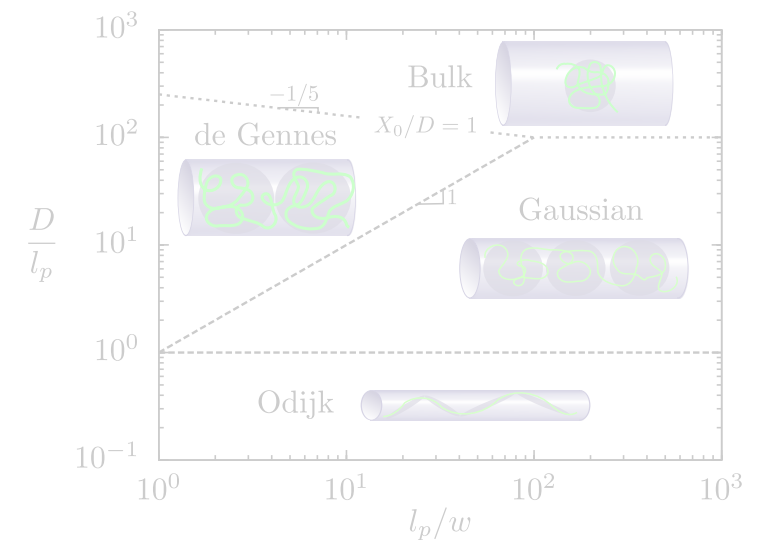
- There is an analogy between the behavior of confined chains and free solution chains
- There is a Gaussian-like regime for large monomer anisotropies.
- This regime is the most practically relevant regime for confined DNA.

# Polymer Physics of DNA in Solution



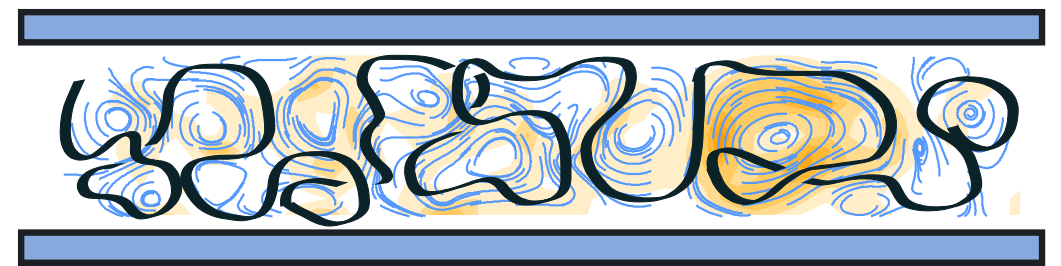
DR Tree, A Muralidhar, PS Doyle and KD Dorfman, *Macromolecules* (2013).

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DR Tree, Y Wang and KD Dorfman, *Phys. Rev. Lett.* 110, 208103 (2013).

# Hydrodynamics of Confined DNA



DR Tree, Y Wang and KD Dorfman, *Phys. Rev. Lett.* 108, 228015 (2012).

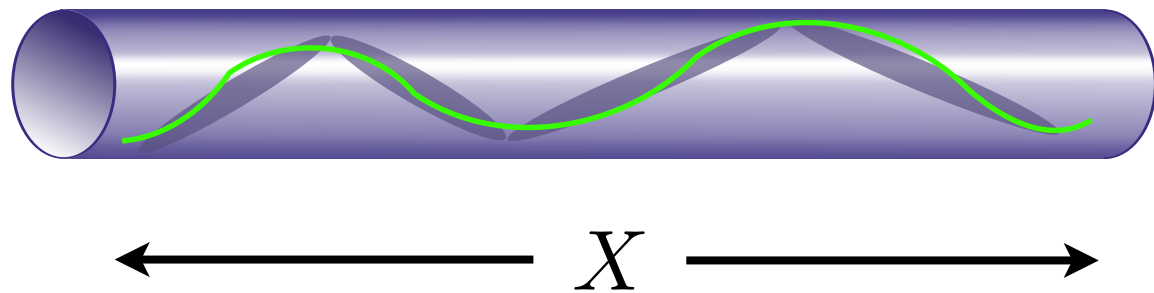
# Classic Analytical Results

Definition of the mobility in the Kirkwood approximation

$$\mu \equiv \langle \Omega_{xx} \rangle = N^{-1} \int g(\mathbf{r}) \Omega_{xx}(\mathbf{r}) d^3 \mathbf{r}$$

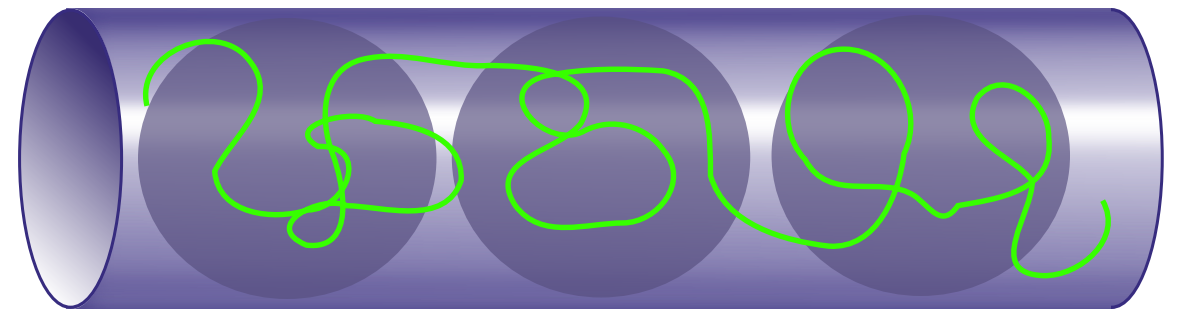
$$\mu = \frac{\mathcal{D}}{k_B T}$$

Odijk (rods):  $D \ll l_p$



$$\mu \sim \frac{1}{\eta L} \ln \left( \frac{D}{2a} \right)$$

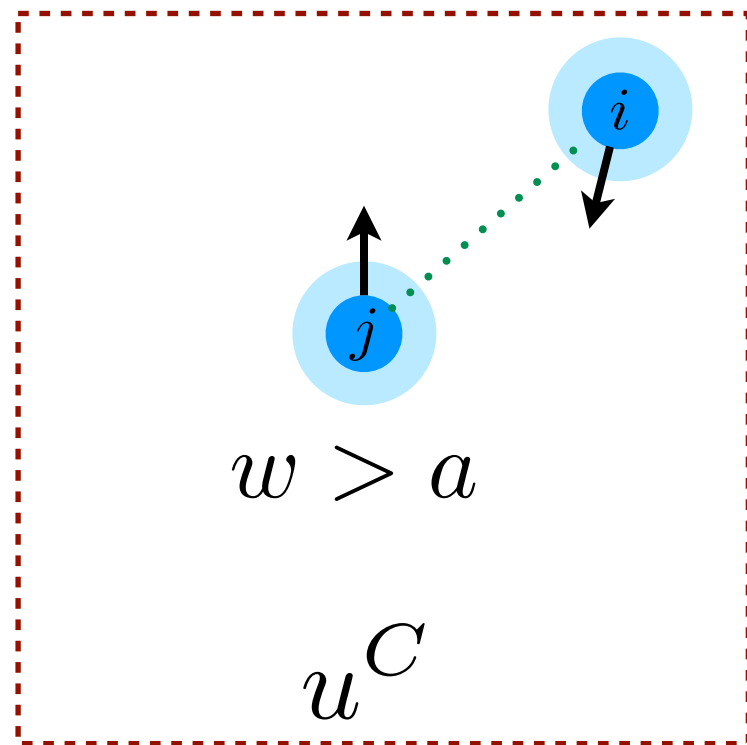
de Gennes (coils):  $D \gg l_p$



$$\mu \sim \frac{1}{\eta \langle X \rangle} \sim \left( \frac{1}{\eta L} \right) \frac{1}{\langle X/L \rangle}$$

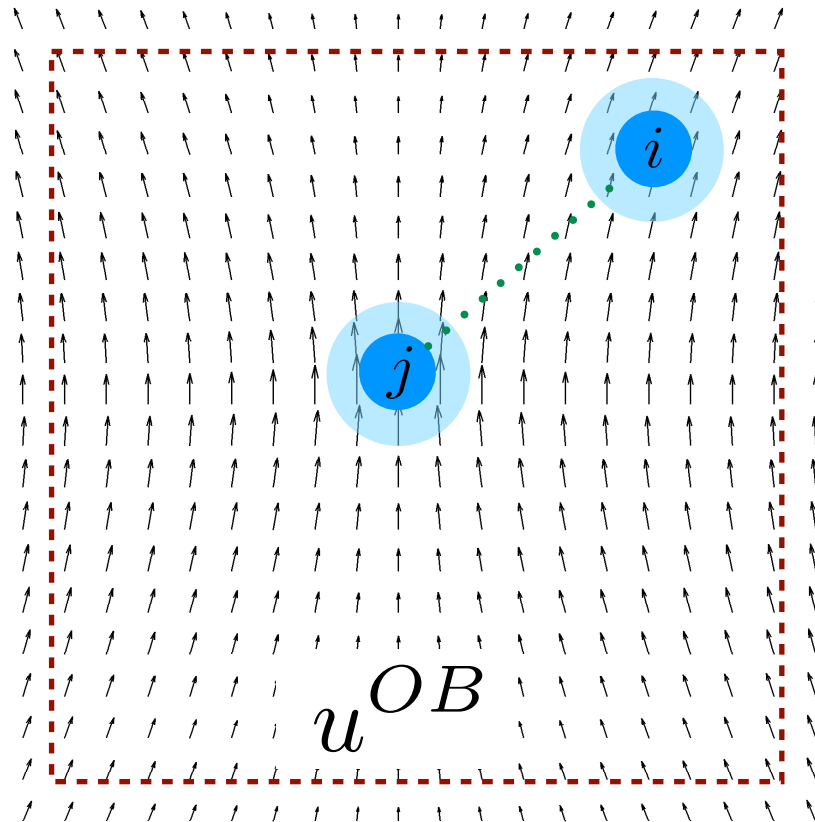
# Confined Chain Hydrodynamics

Mobility of the  
confined chain



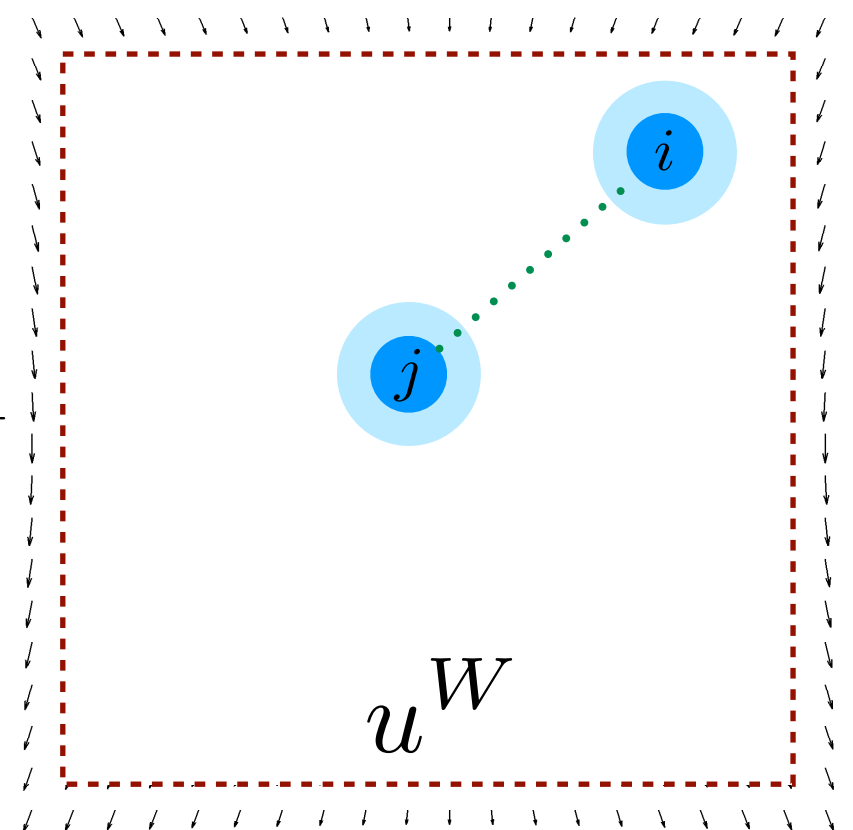
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Free solution  
hydrodynamics



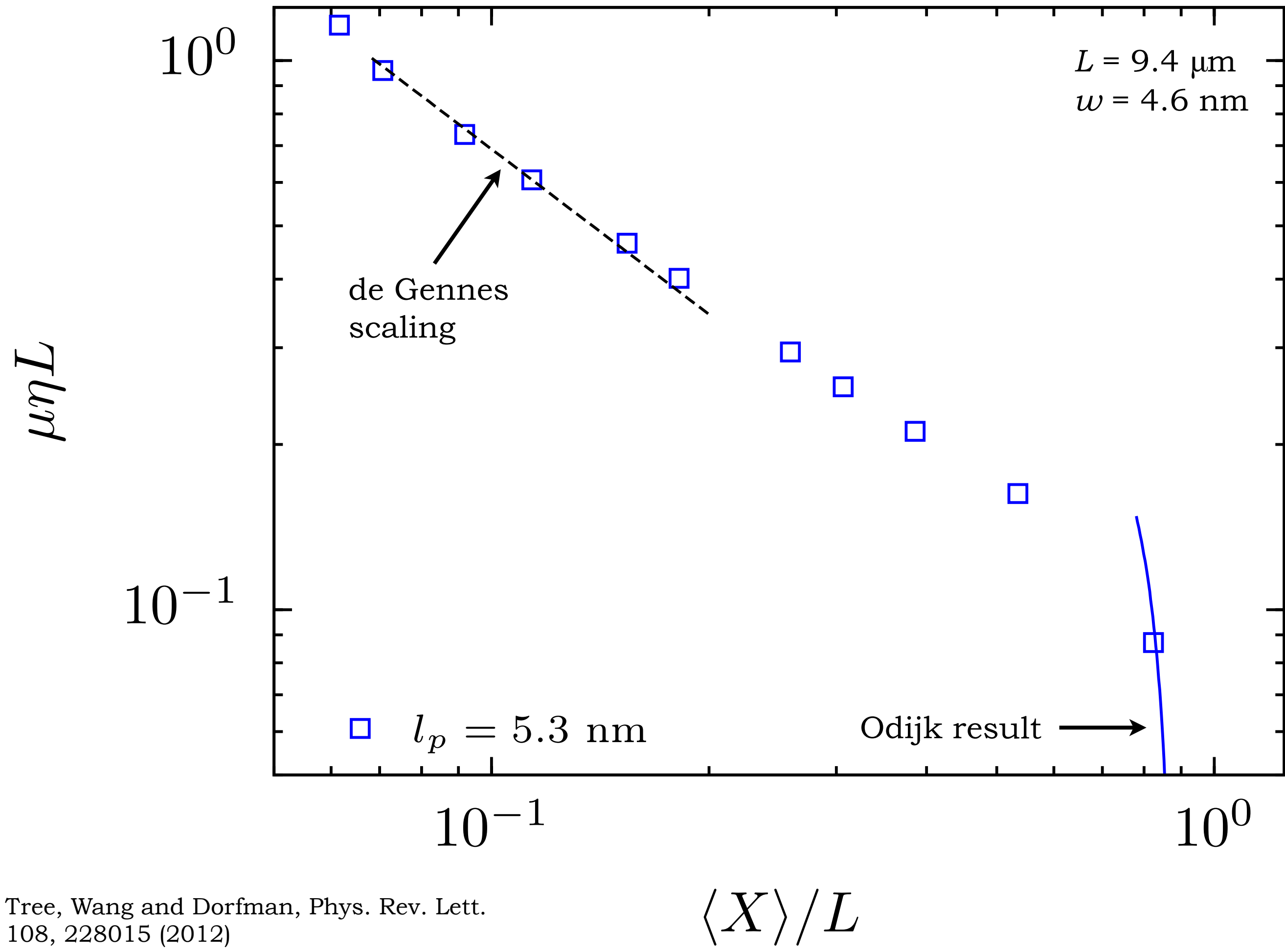
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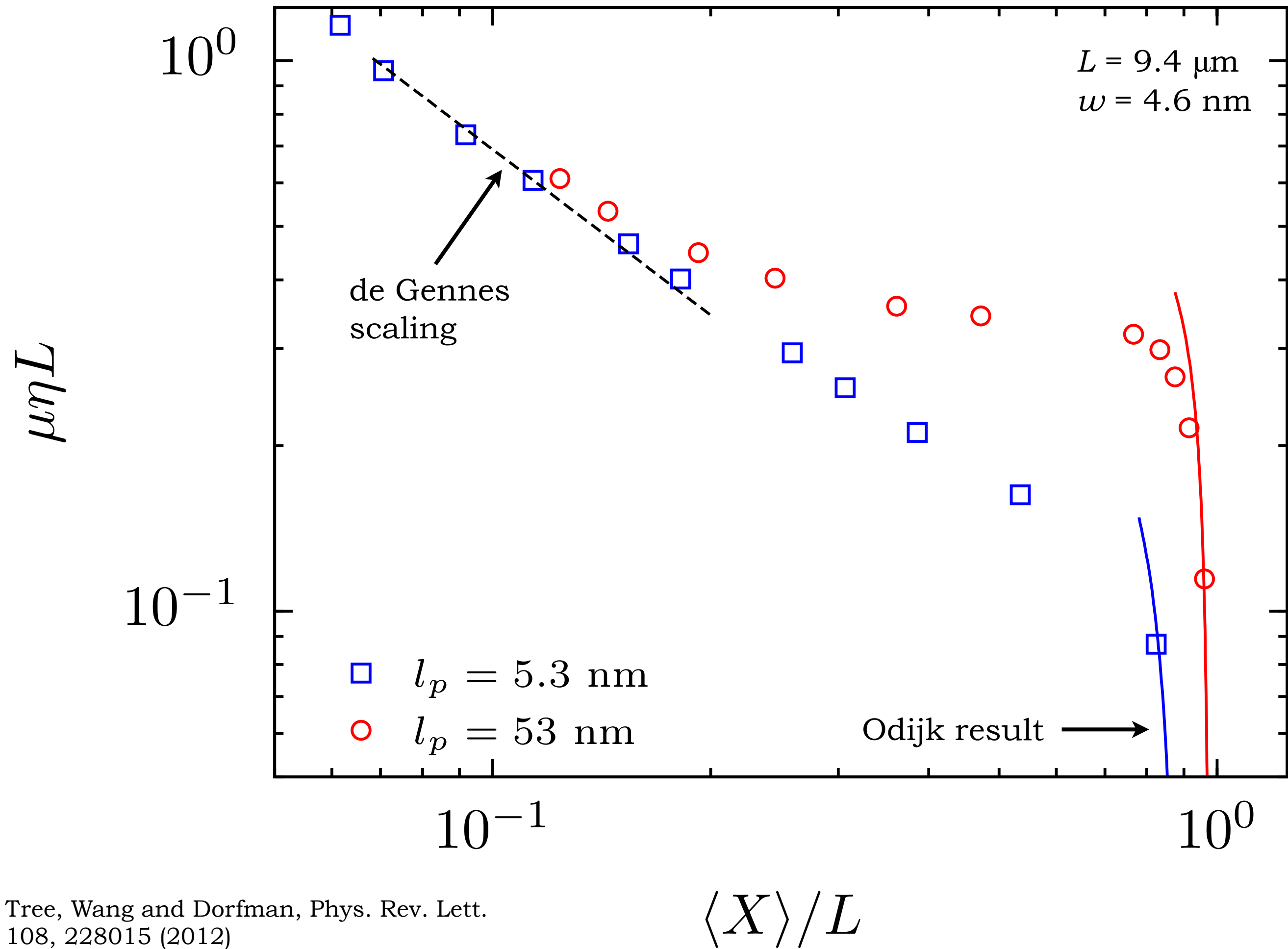
Wall  
hydrodynamics

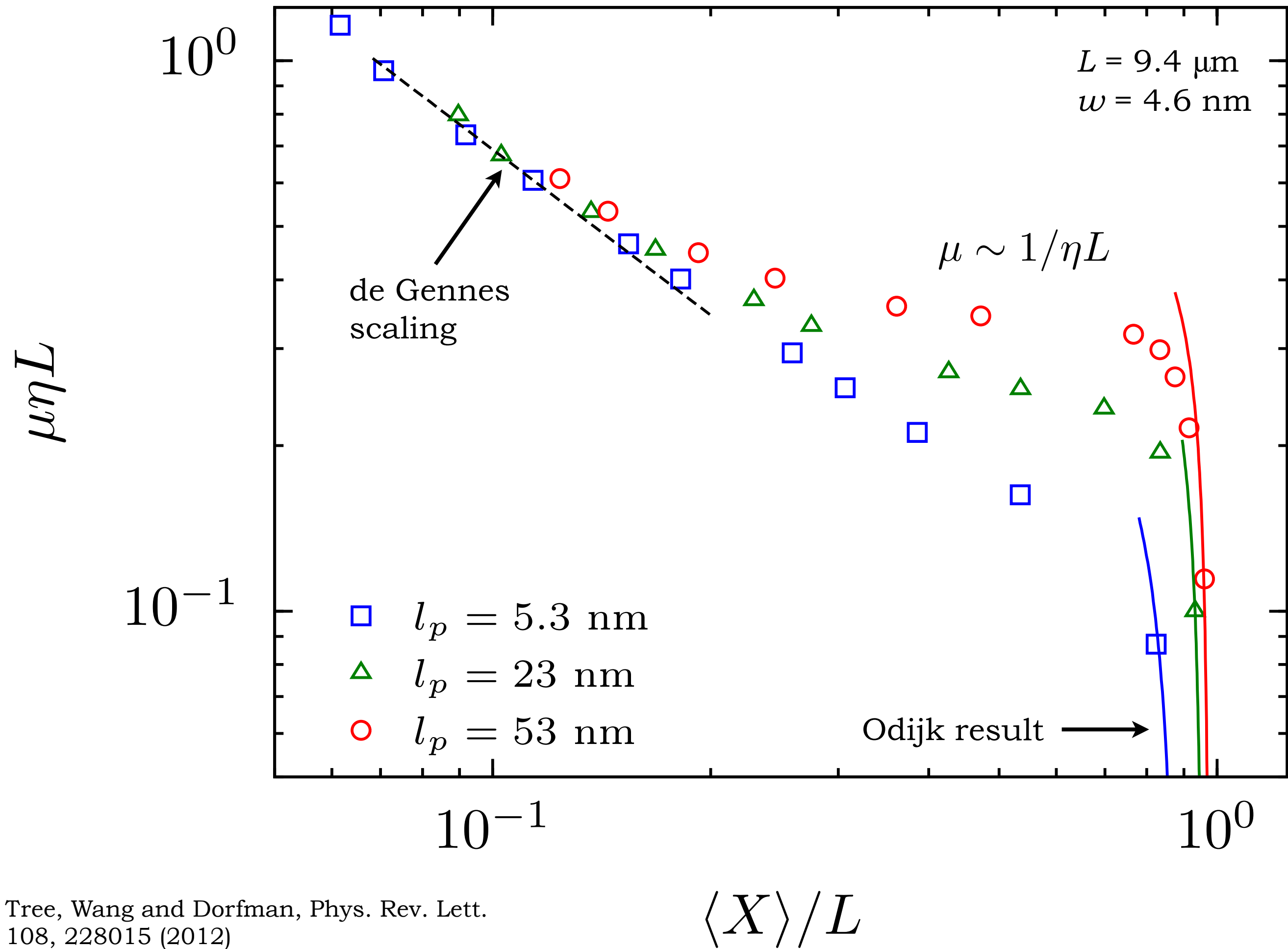


The three dimensional mobility tensor of the confined chain is the sum of effects

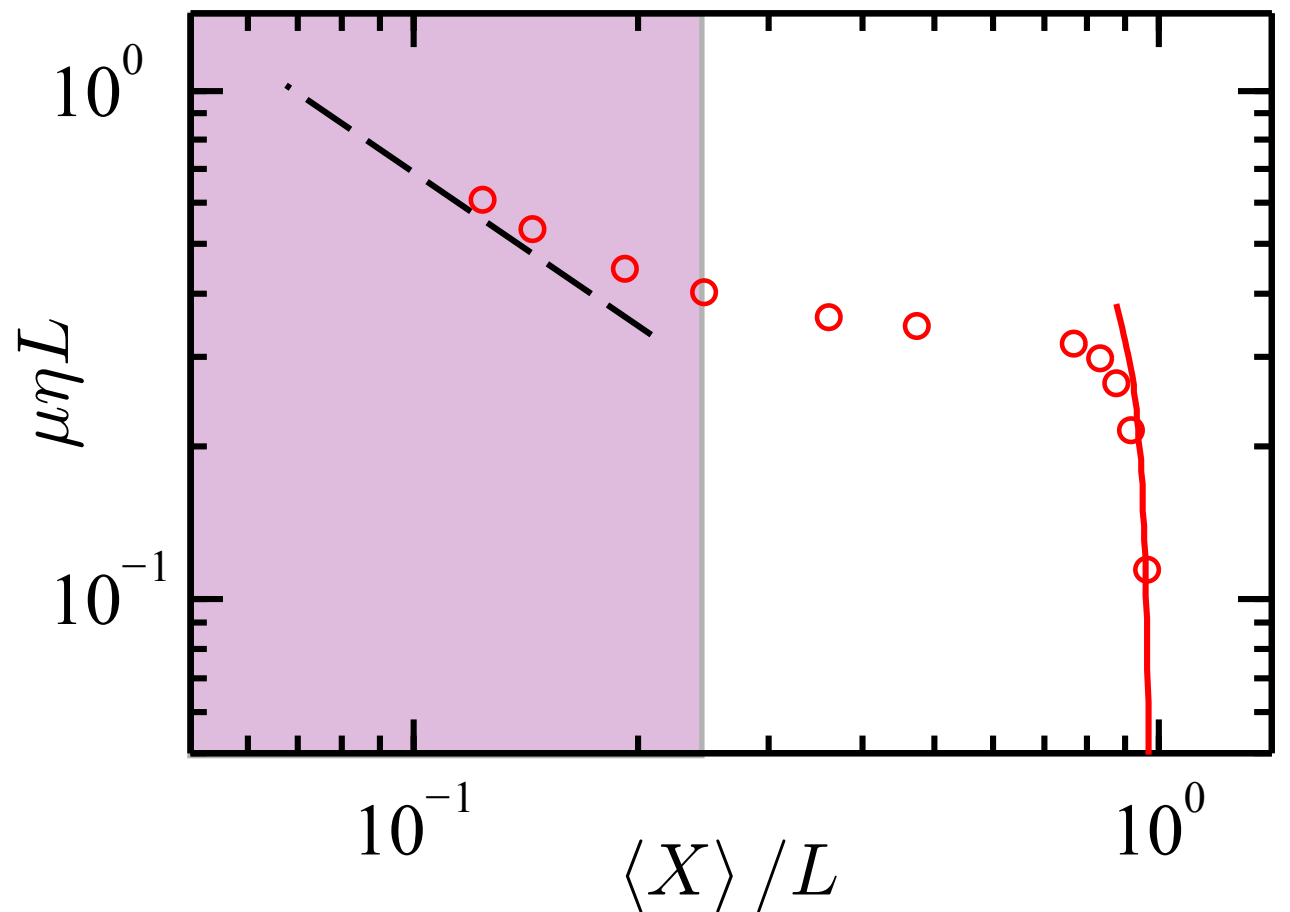
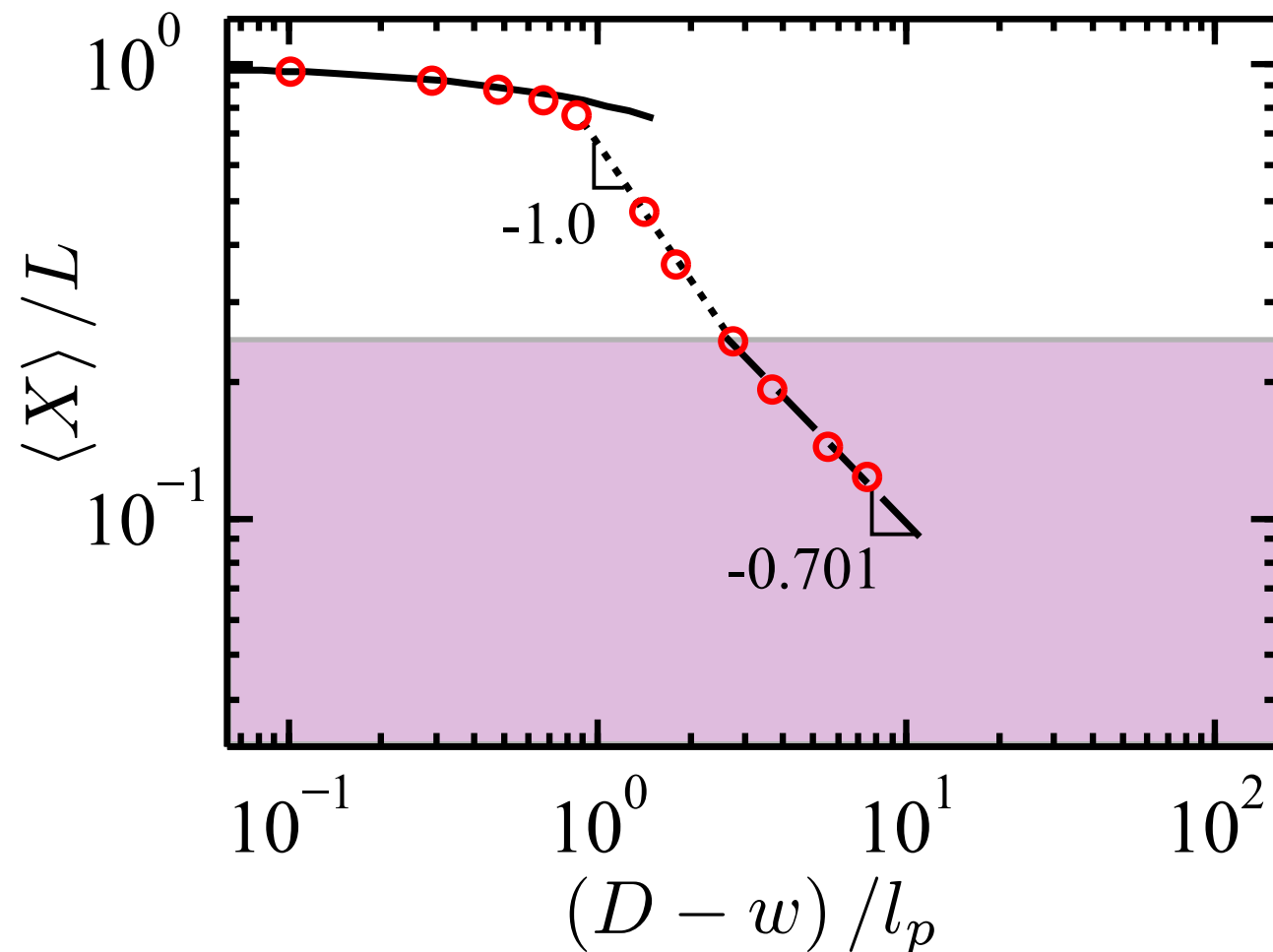
$$\boldsymbol{\Omega} = \frac{1}{N_b^2} \sum_{i,j}^{N_b} \left[ \frac{\delta_{ij}}{6\pi\eta a} \mathbf{I} + (1 - \delta_{ij}) \boldsymbol{\Omega}^{OB}(\mathbf{r}_{ij}) + \boldsymbol{\Omega}^W(\mathbf{r}_i, \mathbf{r}_j) \right]$$



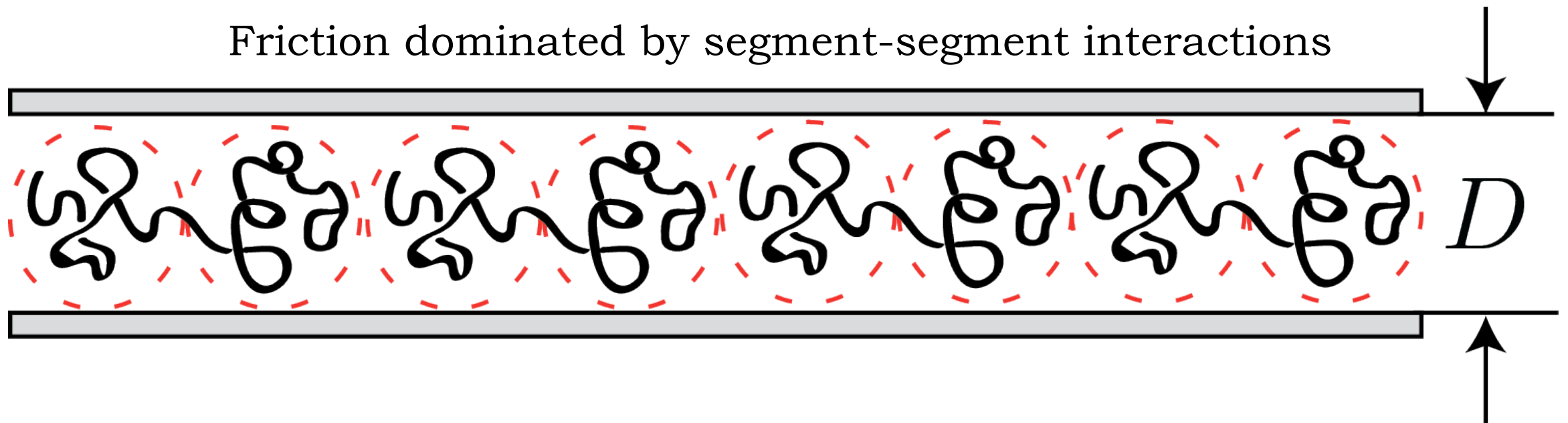




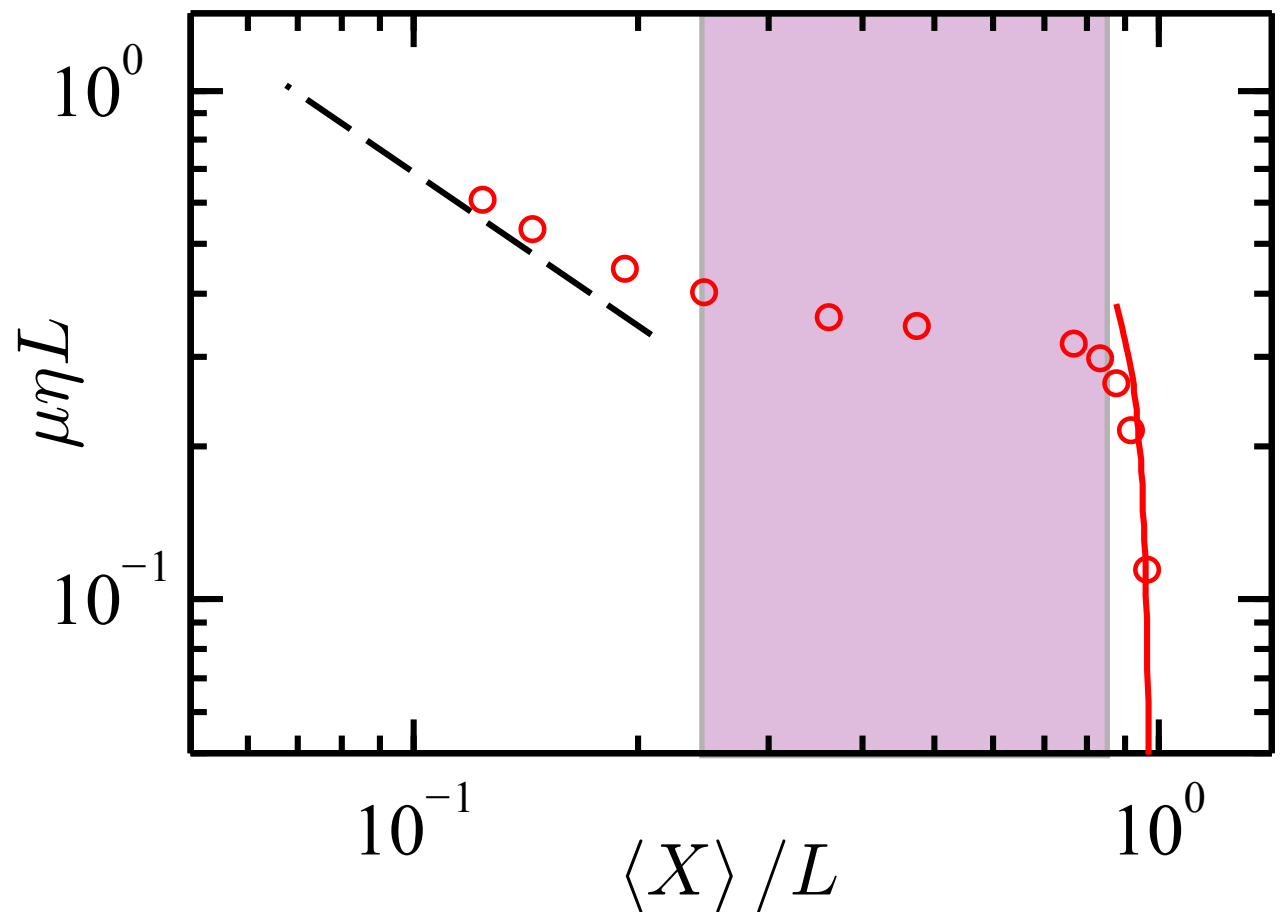
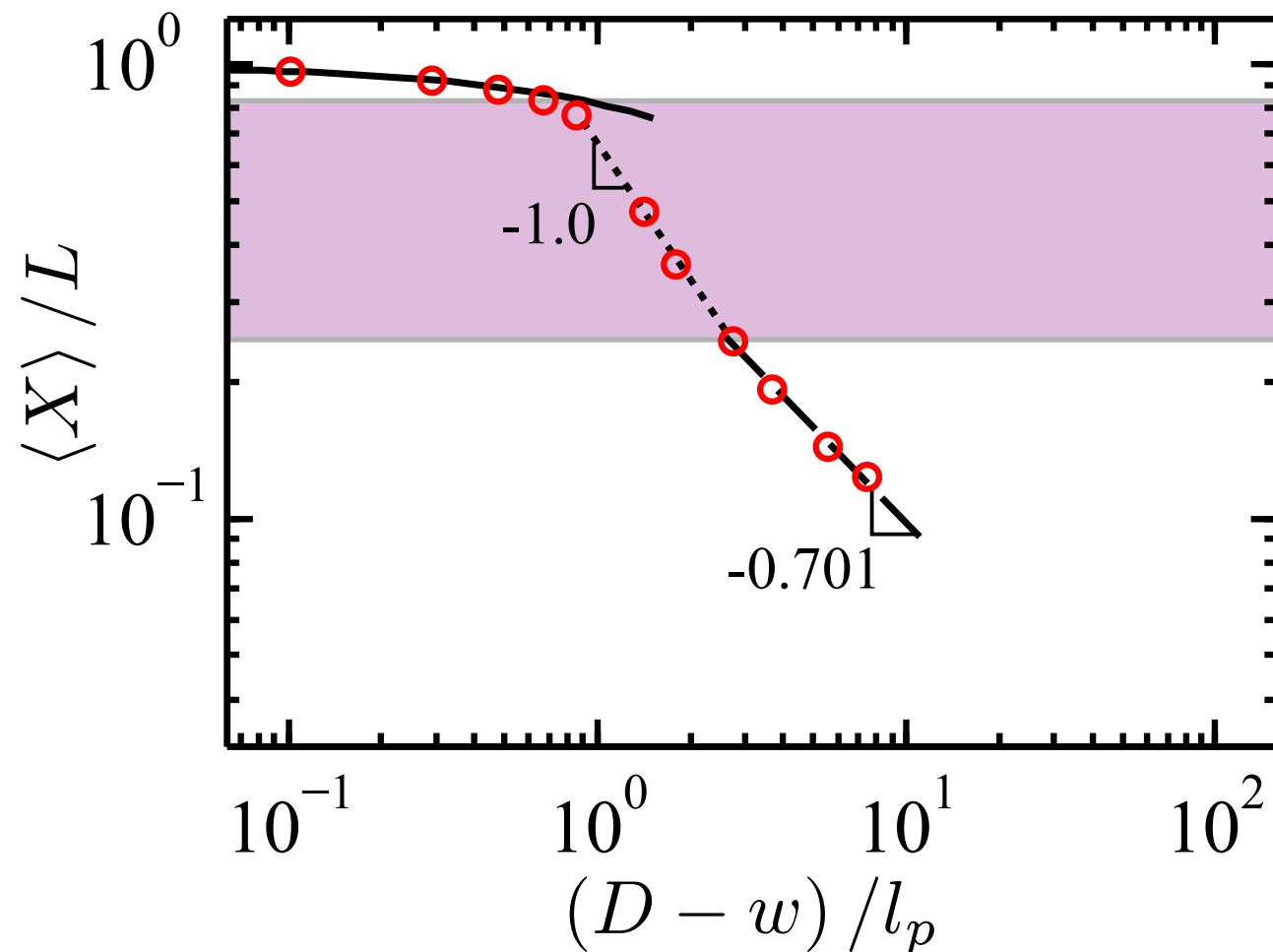
# Connecting Mobility to Conformation



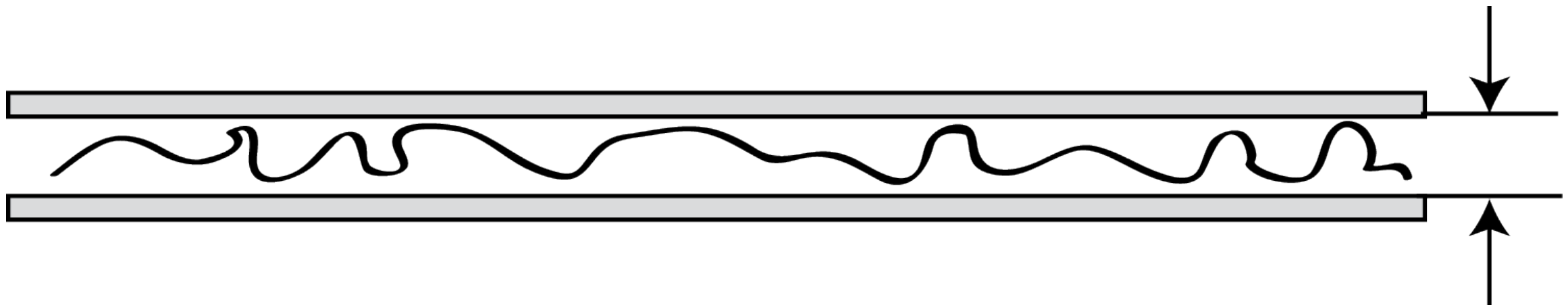
Friction dominated by segment-segment interactions



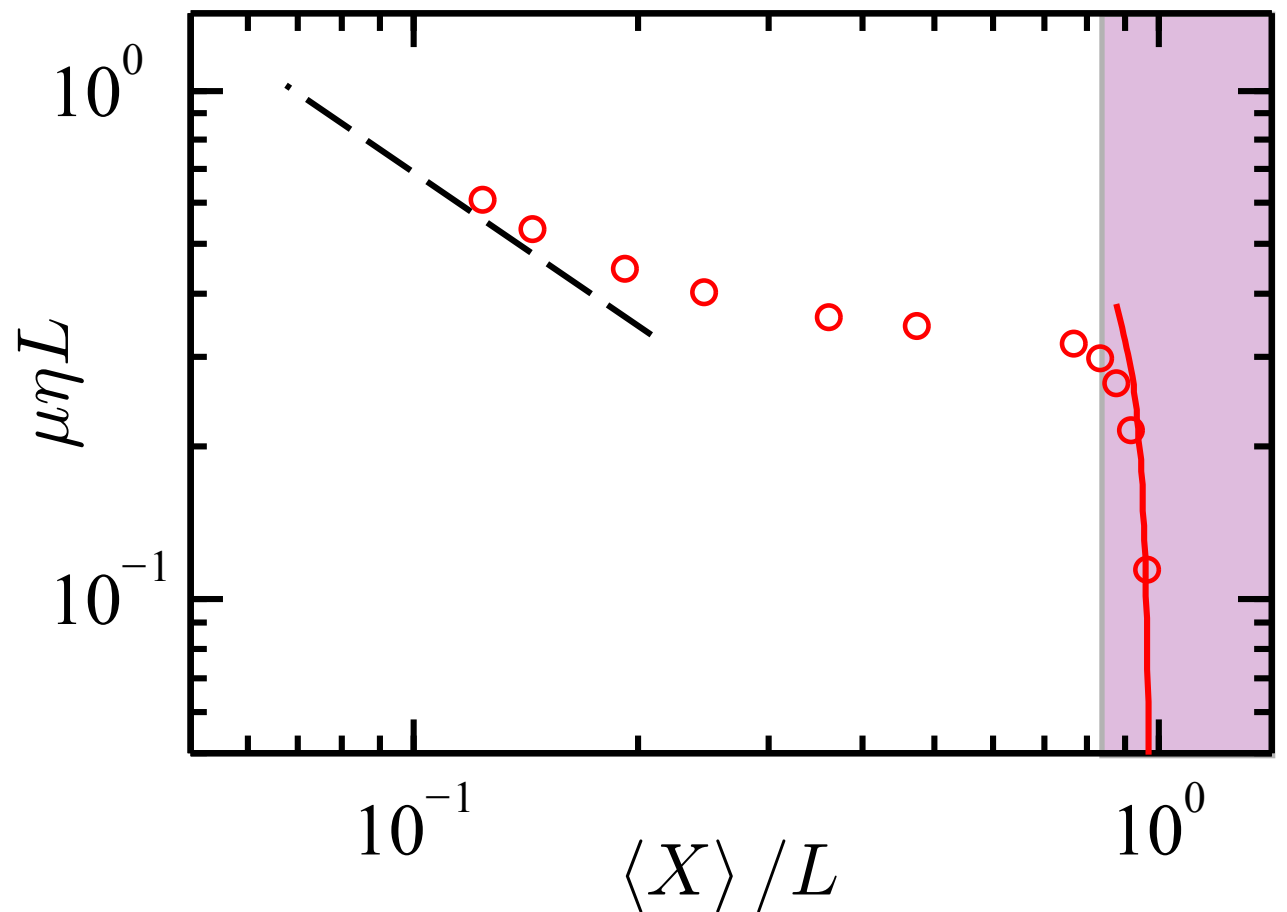
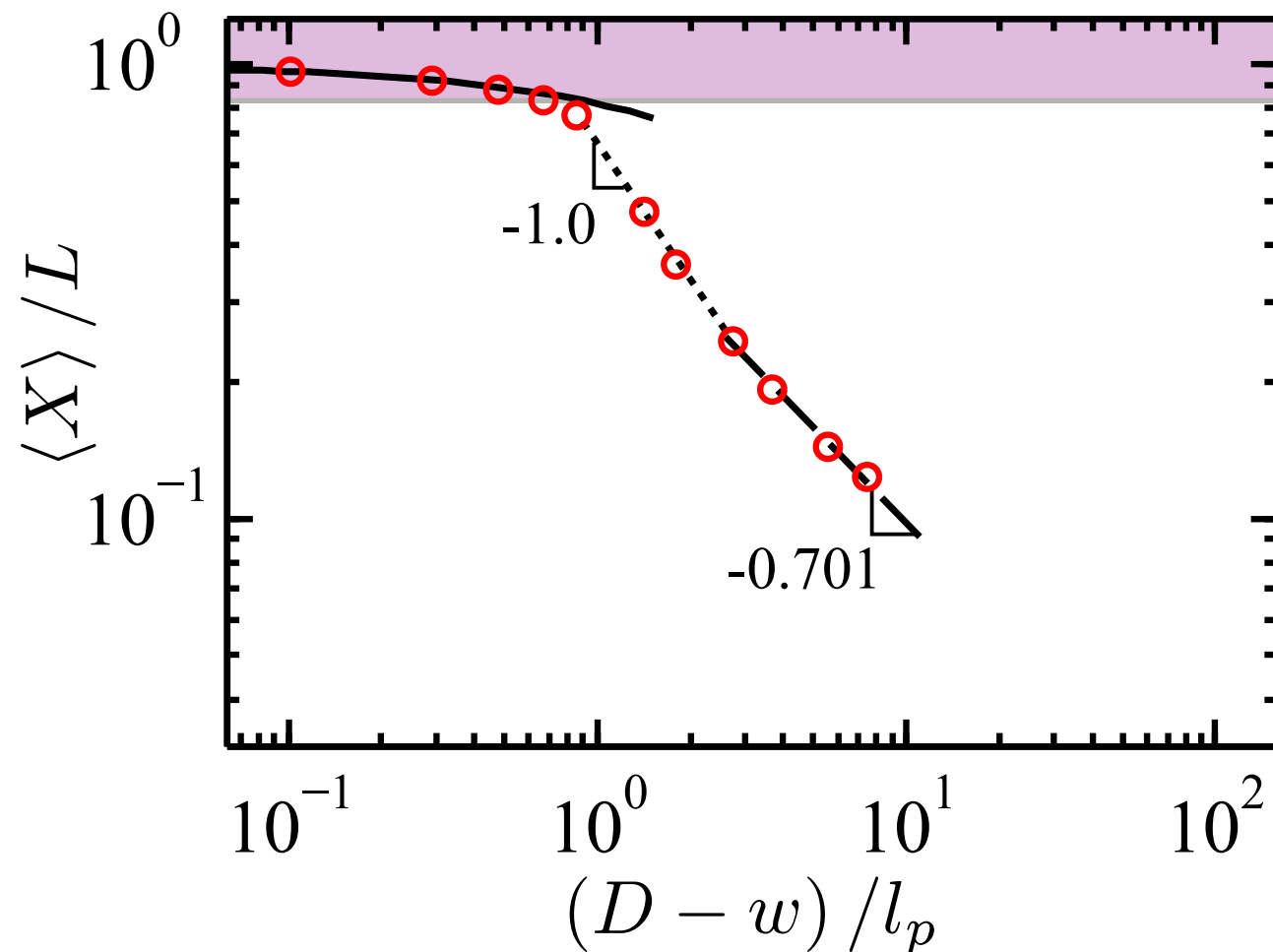
# Connecting Mobility to Conformation



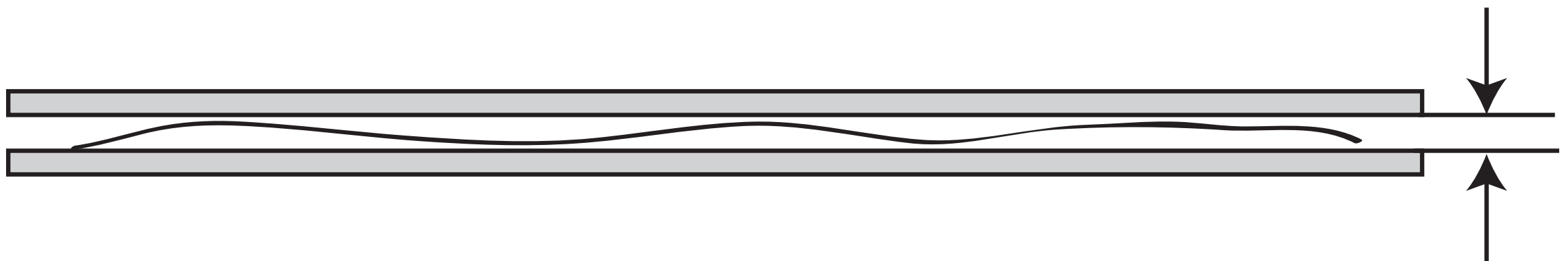
Friction dominated by segment-fluid interactions



# Connecting Mobility to Conformation



Friction dominated by segment-wall interactions



# Conclusions

- Monomer anisotropy also affect the hydrodynamic interactions and the chain mobility
- Predict that the hydrodynamic mobility will be proportional to chain length for DNA in nanochannels.