9. Hermite's differential equation with parameter λ is

$$y'' - 2xy' + \lambda y = 0 \qquad x \in (-\infty, \infty),\ \lambda \in \mathbb{R} \qquad \text{(HE)}$$

a) Solve the HE by the power series method

b) Consider two linearly independent solutions
$y_1(x) = \ldots$
$y_2(x) = \ldots$
which include parameter λ

c) If λ is a non-negative even integer, $\lambda = 0, 2, 4, \ldots, 2n, \ldots$,
then the series terminates, and one obtains, alternating for y_1 and y_2,
polynomials of degree n, which are multiples of so called Hermitian polynomials $H_n(x)$.

d) Rewrite HE in self-adjoint form and determine the weight function $w(x)$

e) Check if the HP are orthogonal with the weight function $w(x)$ over $(-\infty, \infty)$:

$$\int_{-\infty}^{\infty} H_m(x) H_n(x) w(x)\, dx = 0 \quad \text{if} \quad m \neq n$$

f) Give an example of function representation into Fourier-Hermite series