EngT 502   FALL 04

FINAL   Part I

Due Tuesday Dec 14,   3 pm   (under the door of the room CB 133)

1. a) Reduce the following BVP to a Sturm-Liouville problem:

$$x^2u'' + 3xu' + \mu u = 0$$
$$u(1) = 0$$
$$u'\left(\sqrt{e}\right) = 0$$

and find eigenvalues and eigenfunctions.

b) Use the obtained set of eigenfunctions for generalized Fourier series representation of the function

$$f(x) = xe^{-x} \qquad \text{in the interval } \left(1, \sqrt{e}\right)$$

Sketch the graph for $n = 5$ and $n = 10$.
(Do not forget about the weight function in the inner product, see section 4.5.0 7)

2. a) Find the solution of the Wave Equation $u(x, y, t)$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + F(x, y) = a^2 \frac{\partial^2 u}{\partial t^2} \qquad t > 0$$

in the domain $D = (0, L) \times (0, M)$ subject to boundary conditions:

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0, \quad \left.u\right|_{x=L} = f_4(y), \quad \left.-\frac{\partial u}{\partial y} + H_1 u\right|_{y=0} = 0, \quad \left.\frac{\partial u}{\partial y}\right|_{y=M} = 0$$

and initial conditions:

$$u(x, y, 0) = u_0(x, y)$$
$$\frac{\partial u(x, y, 0)}{\partial t} = u_1(x, y)$$

b) Sketch the graph of the solution for $L = 4$, $M = 2$, $a = 2.5$, $H_1 = 1.5$ and

$f_4(y) = 0.5$ $\qquad$ $u_0(x) = x(L - x)y(M - y)$

$F(x, y) = -1$ $\qquad$ $u_1(x) = 0$

c) Construct interesting example with your choice of

$L =$ $\quad$, $M =$ $\quad$, $a =$ $\quad$, $H_1 =$ $\quad$ and

$f_4(y) =$ $\qquad$ $u_0(x) =$

$F(x, y) =$ $\qquad$ $u_1(x) =$

Comments for those who use Maple:

1. Use just *4-5* terms in the truncated series solution.

2. If double integration is difficult for Maple use numerical integration.

3. Maple performs integration with some methods which yield small imaginary residue. In this case function will not be plotted. To resolve this problem use only the real part of the result.

For example, in calculation of the Fourier coefficients by double integration:

```
> B[n,m]:=Re(eval(Int(Int(U0(x,y)*X[n]*Y[m],x=0..L),y=0..M)/NX[n]/NY[m]));
```

(note that the integration command is capitalized)