Consider the IVP describing vibration of the spherical membrane.

a) Find the solution of the Wave Equation $u(\vartheta,t)$

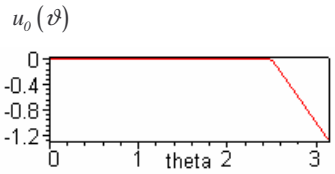
$$\frac{1}{r^2}\frac{\cos\vartheta}{\sin\vartheta}\frac{\partial u}{\partial\vartheta}+\frac{1}{r^2}\frac{\partial^2 u}{\partial\vartheta^2}=a^2\frac{\partial^2 u}{\partial t^2} \qquad t>0$$

in the domain $D:\quad r=const\ ,\ 0\le\phi\le 2\pi\ ,\ 0\le\vartheta\le\pi$

subject to initial conditions:

$u(\vartheta,0)=u_0(\vartheta)$ (*initial shape of the membrane*)

$\dfrac{\partial u(\vartheta,0)}{\partial t}=u_1(\vartheta)$ (*initial velocity of the membrane*)



b) Show the solution for:

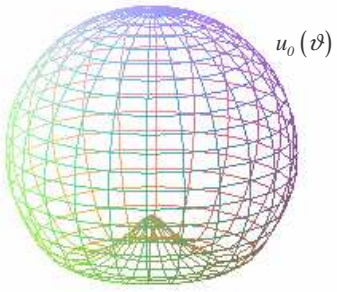
$r=2$

$a=0.5$

$u_0(\vartheta)=2(2.5-\vartheta)H(\vartheta-2.5)$

where $H(\vartheta-2.5)=\begin{cases}1 & if \quad \vartheta>2.5\\ 0 & if \quad \vartheta<2.5\end{cases}$

$u_1(\vartheta)=0$



c) Find an interesting setting of the problem:

$r=$

$a=$

$u_0(\vartheta)=$

$u_1(\vartheta)=$

and illustrate it (the best animations will be posted on the class web-site - submit animation on a disk – do not send by e-mail !!!).

ΠΥΘΑΓΟΡΗΣ