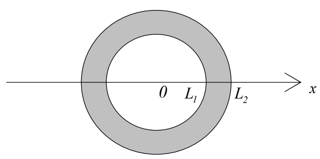
Sturm-Liouville Problem in annular domain



Consider BE in the **annular domain**

$$x^2 y'' + xy' + \left(\lambda^2 x^2 - \nu^2\right)y = 0, \quad x \in \left(L_1, L_2\right)$$

with homogeneous boundary conditions:

$$\left[-k_1 \frac{dy}{dx} + h_1 y\right]_{x=L_1} = 0 \qquad H_1 = \frac{h_1}{k_1}$$

$$\left[k_2 \frac{dy}{dx} + h_2 y\right]_{x=L_2} = 0 \qquad H_2 = \frac{h_2}{k_2}$$

The general solution is given by

$$y(x) = c_1 J_\nu(\lambda x) + c_2 Y_\nu(\lambda x)$$

The derivative of the general solution:

$$y'(x) = c_1 \lambda\left[-J_{\nu+1}(\lambda x) + \frac{\nu}{\lambda x} J_\nu(\lambda x)\right] + c_2 \lambda\left[-Y_{\nu+1}(\lambda x) + \frac{\nu}{\lambda x} Y_\nu(\lambda x)\right]$$

1	**Dirichlet-Dirichlet**	$[y]_{x=L_1} = 0$	$[y]_{x=L_2} = 0$
2	**Neumann-Dirichlet**	$\left[\frac{dy}{dx}\right]_{x=L_1} = 0$	$[y]_{x=L_2} = 0$
3	**Dirichlet-Neumann**	$[y]_{x=L_1} = 0$	$\left[\frac{dy}{dx}\right]_{x=L_2} = 0$
4	**Neumann-Neumann**	$\left[\frac{dy}{dx}\right]_{x=L_1} = 0$	$\left[\frac{dy}{dx}\right]_{x=L_2} = 0$
5	**Dirihlet-Robin**	$[y]_{x=L_1} = 0$	$\left[k_2 \frac{dy}{dx} + h_2 y\right]_{x=L_2} = 0$
6	**Neumann-Robin**	$\left[\frac{dy}{dx}\right]_{x=L_1} = 0$	$\left[k_2 \frac{dy}{dx} + h_2 y\right]_{x=L_2} = 0$
7	**Robin-Dirichlet**	$\left[-k_1 \frac{dy}{dx} + h_1 y\right]_{x=L_1} = 0$	$[y]_{x=L_2} = 0$
8	**Robin-Neumann**	$\left[-k_1 \frac{dy}{dx} + h_1 y\right]_{x=L_1} = 0$	$\left[\frac{dy}{dx}\right]_{x=L_2} = 0$
9	**Robin-Robin**	$\left[-k_1 \frac{dy}{dx} + h_1 y\right]_{x=L_1} = 0$	$\left[k_2 \frac{dy}{dx} + h_2 y\right]_{x=L_2} = 0$