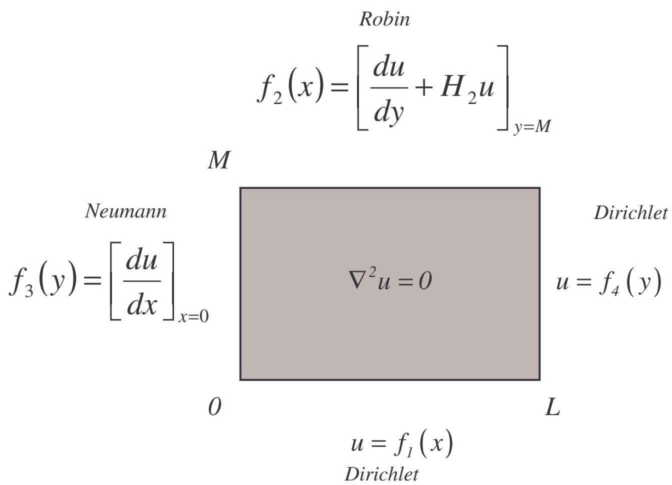
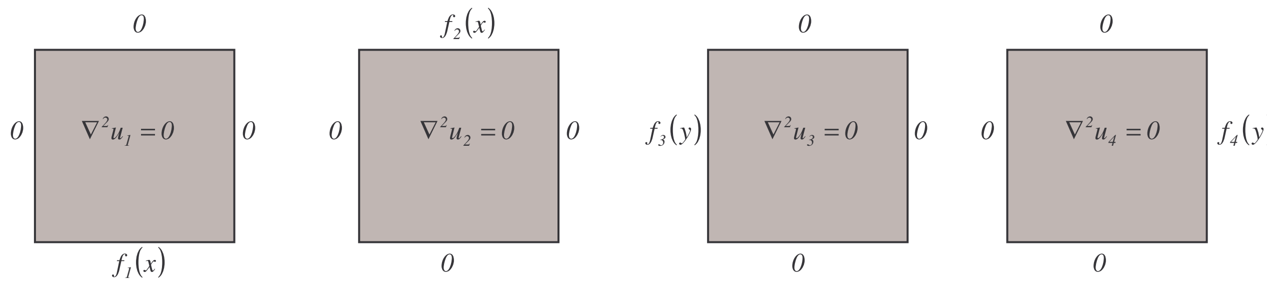
The Laplace Equation: 04 – DRND (Dirichlet- Robin-Neumann-Dirichlet)



Supplemental problems: ↓



<u>Solution of supplemental problems</u>:

$$u_1(x,y) = \sum_{n=0}^{\infty} a_n [\cosh(\lambda_n (y-M)) - \frac{H_2}{\lambda_n}\sinh(\lambda_n (y-M))]\cos(\lambda_n x) \qquad a_n = \frac{\frac{2}{L}\int_0^L f_1(x)\cos(\lambda_n x)dx}{\cosh(\lambda_n M) + \frac{H_2}{\lambda_n}\sinh(\lambda_n M)}$$

Where $$\lambda_n = (n + \frac{1}{2})\frac{\pi}{L}$$

$$u_2(x,y) = \sum_{n=0}^{\infty} b_n \sinh(\lambda_n y)\cos(\lambda_n x) \qquad b_n = \frac{\frac{2}{L}\int_0^L f_1(x)\cos(\lambda_n x)dx}{\lambda_n \cosh(\lambda_n M) + H_2 \sinh(\lambda_n M)}$$

Where $$\lambda_n = (n + \frac{1}{2})\frac{\pi}{L}$$

$$u_3(x,y) = \sum_{n=1}^{\infty} c_n \sinh(\lambda_n (x-L))\sin(\lambda_n y) \qquad c_n = \frac{\int_0^M f_3(y)\sin(\lambda_n y)dy}{\left(\frac{M}{2} - \frac{\sin(2\lambda_n M)}{4\lambda_n}\right)\lambda_n \cosh(\lambda_n L)}$$

Where $\lambda_n \to$ positive roots of : $\lambda_n \cos(\lambda_n M) + H_2 \sin(\lambda_n M) = 0$

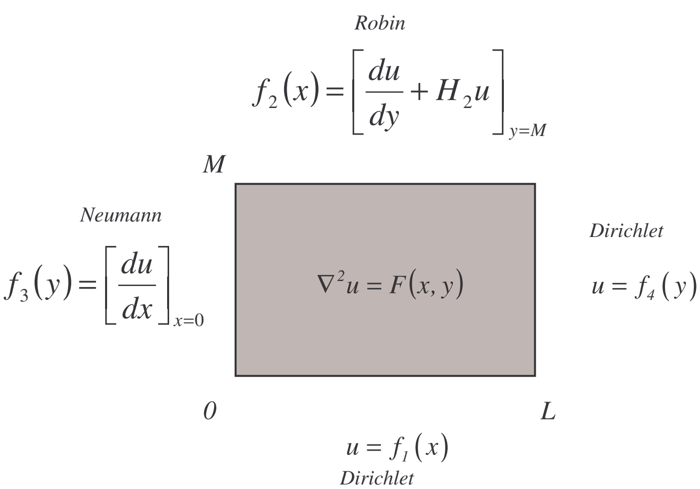
$$u_4(x,y)=\sum_{n=1}^{\infty} d_n \cosh(\lambda_n x)\sin(\lambda_n y) \qquad d_n=\frac{\int_0^M f_4(y)\sin(\lambda_n y)dy}{\left(\frac{M}{2}-\frac{\sin(2\lambda_n M)}{4\lambda_n}\right)\cosh(\lambda_n L)}$$

Where $\lambda_n \rightarrow$ positive roots of: $\lambda_n \cos(\lambda_n M)+H_2\sin(\lambda_n M)=0$

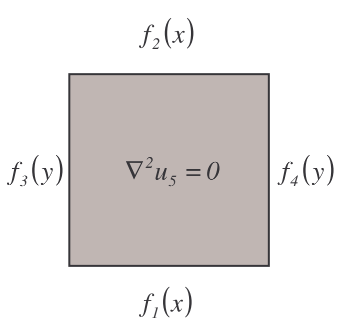
Solution of BVP problem:

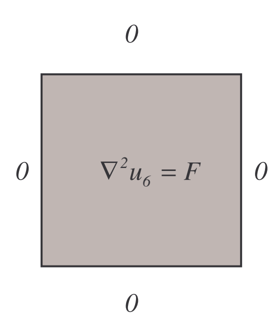
$$u(x,y)=u_1(x,y)+u_2(x,y)+u_3(x,y)+u_4(x,y)$$

<u>POISSON'S EQUATION: 04 – DRND</u>



Supplemental problems: ↓





Solution of supplemental problems:

Solution of Laplace's homogeneous equation with non-homogeneous b.c.'s:

$$u_5(x, y) = u_1(x, y) + u_2(x, y) + u_3(x, y) + u_4(x, y)$$

Solution of Poisson's equation with homogeneous boundary conditions

$$u_6(x, y) = \sum_{n=1}^{\infty} \sum_{m=0} A_{nm} \sin(\lambda_n y) \cos(\mu_m x)$$

$$A_{nm} = \frac{-1}{(\lambda_n{}^2 + \mu_m{}^2)\left(\frac{M}{2} - \frac{\sin(2\lambda_n M)}{4\lambda_n}\right)\left(\frac{L}{2}\right)} \int_0^L \int_0^M F(x, y) \sin(\lambda_n y) \cos(\mu_m x) dy dx$$

Where $\lambda_n \rightarrow$ positive roots of: $\lambda_n \cos(\lambda_n M) + H_2 \sin(\lambda_n M) = 0$ and $\mu_m = (m + \frac{1}{2})\frac{\pi}{L}$

Solution of BVP for Poisson's Equation (superposition principle):

$$u(x, y) = u_5(x, y) + u_6(x, y)$$