6-7 THE CARNOT CYCLE

$\eta_c = 1 - \frac{T_L}{T_H}$

T_L

T_H

Sadi Carnot (1822-1888)

The Carnot Cycle *(Heat Engine)*

P

① T_H
Q_H
②
$T_H = const$
$Q = 0$ $W_{net,out}$ $Q = 0$
④
③ $T_L = const$
V_1 V_4 Q_L V_2 V_3 V
T_L

The Reversed Carnot Cycle *(Refrigerator)*

P

① T_H
Q_H
④
$T_H = const$
$Q = 0$ $W_{net,in}$ $Q = 0$
②
③ $T_L = const$
V_1 V_2 Q_L V_4 V_3 V
T_L

The Carnot Cycle consists of 4 reversible processes

6-6 Reversible Processes

Q
$-W$
$-Q$
W
(1) (2)
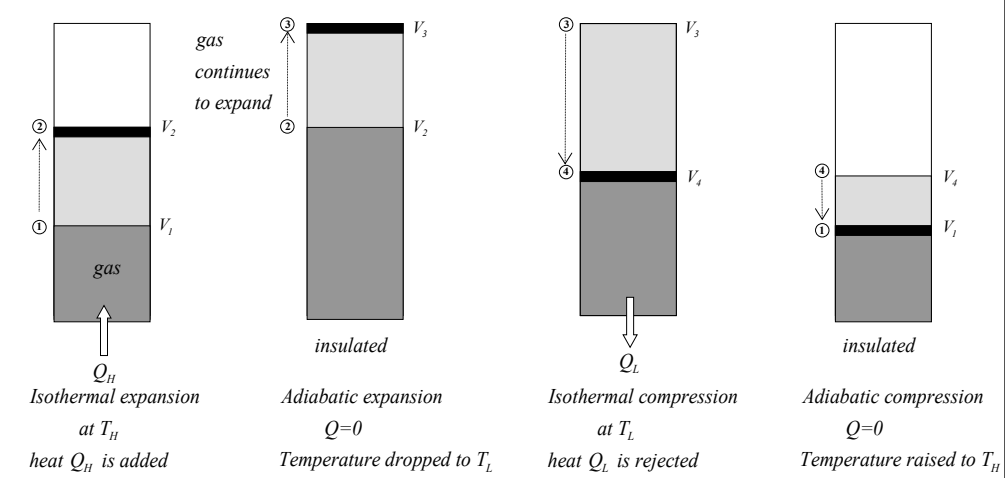
- *at thermal equilibrium*
- *the paths coinside*
- *heat transfer due to infinitely small difference in temperature*

Irreversible Processes

$\Delta T > 0$

- *heat transfer due to $\Delta T > 0$*
- *heat generation by friction*

gas continues to expand

② V_2
① V_1
gas
Q_H
Isothermal expansion
at T_H
heat Q_H is added

③ V_3
② V_2
insulated
Adiabatic expansion
Q=0
Temperature dropped to T_L

③ V_3
② V_2
④ V_4
Q_L
Isothermal compression
at T_L
heat Q_L is rejected

③ V_3
④ V_4
① V_1
insulated
Adiabatic compression
Q=0
Temperature raised to T_H

6-8 CARNOT PRINCIPLES

Efficiency of two Heat Engines operating between the same two reservoirs at T_L and T_H

Violation of Carnot Principles yields violation of the 2nd Law of Thermodynamics

C1 $\eta_{irreversible} < \eta_{reversible}$

C2 $\eta_{reversible\ 1} = \eta_{reversible\ 2}$

T_H
$\eta_{IR} < \eta_R$ $\eta_{R1} = \eta_{R2}$
IR   **R1**   **R2**
T_L

6-9 THE THERMODYNAMIC TEMPERATURE SCALE

For reversible heat engine operating between T_L and T_H :

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

Kelvin Scale of Absolute Temperature

6-10 THE CARNOT HEAT ENGINE

The efficiency of heat engine operating on a reversible Carnot cycle between T_L and T_H :

$$\eta_C = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

Carnot Efficiency — *η_C is the highest possible efficiency of the heat engine operating between two reservoirs at T_L and T_H*

6-11 THE CARNOT HEAT PUMP

Carnot Heat Pump:

$$COP_{CHP} = \frac{1}{1 - \frac{Q_L}{Q_H}} = \frac{1}{1 - \frac{T_L}{T_H}}$$

Carnot Refrigerator:

$$COP_{CR} = \frac{1}{\frac{Q_H}{Q_L} - 1} = \frac{1}{\frac{T_H}{T_L} - 1}$$

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COMMENTS AND ADDITIONAL NOTES ON CARNOT CYCLE

Carnot Cycle consists of 4 reversible processes which operate between two temperature recervoirs at T_H and T_L.

Where T_H and T_L are temperatures according to not specified temperature scale (empirical).
We know only that ordering of this temperature scale is such that $T_H > T_L$
to be consistent with the 2nd Law of Thermodynamics (Clausius statement).

Carnot Principles

CP1	$\eta_{any} \leq \eta_{reversible}$	Assume the opposite, $\eta_{any} > \eta_{reversible}$, to demonstrate that violation of the CP1 yields violation of the 2nd Law.	
CP2	$\eta_{reversible,1} = \eta_{reversible,2}$	From CP1, we have	$\eta_{reversible,1} \leq \eta_{reversible,2}$ and $\eta_{reversible,2} \leq \eta_{reversible,1}$

Derivation of the **Thermodynamic Temperature Scale** (from Carnot Cycle with intermediate temperature reservoir at T_0)
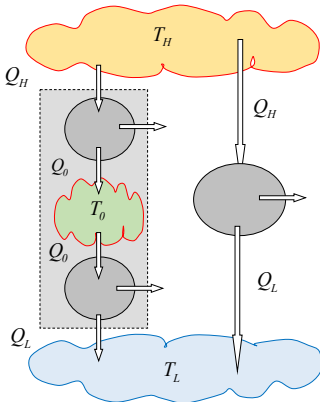
for any two Carnot cycles
between the same T_H and T_L

$$\eta_{C_1} = 1 - \frac{Q_L^1}{Q_H^1} \quad = \quad \eta_{C_2} = 1 - \frac{Q_L^2}{Q_H^2}, \quad \text{therefore,}$$

$$\frac{Q_L}{Q_H} = f(T_L, T_H) \quad \text{depends only on } T_H \text{ and } T_L \text{, but not on the engine}$$

$$\frac{Q_H}{Q_0} = f(T_0, T_H) \quad \text{where } T_0 \text{ is any such that } T_H > T_0 > T_L$$

$$\frac{Q_L}{Q_0} = f(T_0, T_L)$$

$$\frac{Q_L}{Q_H} = \frac{Q_L}{Q_0} \cdot \frac{Q_0}{Q_H} \quad = f(T_L, T_H) = \frac{f(T_0, T_L)}{f(T_0, T_H)} \quad = \frac{\theta(T_L)}{\theta(T_H)} \quad \overset{\substack{Kelvin \\ chosed}}{=} \quad \frac{T_L}{T_H}$$

Demonstrate that the temperature of the **Thermodynamic Temperature Scale** is the same as
Absolute Temperature of the Ideal Gas equation of state

$$PV = mRT$$

This absolute temperature T can be measured by the Ideal Gas thermometer by measuring V of the fixed mass at constant pressure:

$$T = \frac{P}{mR} \cdot V$$

or by measuring pressure P in the rigid tank of constant volume V :
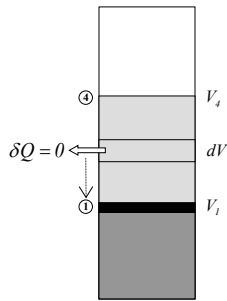
$$T = \frac{V}{mR} \cdot P$$

(formally, then zero absolute temperature corresponds to zero volume or zero pressure (no molecules in the tank))

Consider reversible process without heat transfer (adiabatic) expansion/compression of ideal gas (process $4 \to 1$):

$$\boxed{\delta\!\!\!/Q - PdV = m \cdot du}$$

differential energy balance

$$-\frac{1}{V}mRTdV = mc_v \cdot dT$$

from ideal gas equation of state $P = \dfrac{1}{V}mRT$

$$-\frac{1}{V}dV = \frac{c_v}{R} \cdot \frac{1}{T}dT$$

$$-\frac{1}{V}dV = \frac{1}{k-1} \cdot \frac{1}{T}dT$$

$$c_p = c_v + R \;\Rightarrow\; \frac{c_v}{R} = \frac{c_v}{c_p - c_v} = \frac{1}{\dfrac{c_p}{c_v} - 1} = \frac{1}{k-1}$$

$$(k-1)\cdot\int\frac{1}{V}dV + \int\frac{1}{T}dT = \ln C$$

$$\ln V^{k-1} + \ln T = \ln C$$

$$\boxed{T V^{k-1} = C}$$

$\dfrac{PV}{mR}V^{k-1} = C$, from ideal gas equation, $T = \dfrac{PV}{mR}$

$$\boxed{PV^{k} = C}$$

polytropic process with $n = k$

$$\boxed{PT^{\frac{k}{1-k}} = C}$$

$TV^{k-1} = C \Rightarrow V = \left(CT^{-1}\right)^{\frac{1}{k-1}} = CT^{\frac{1}{1-k}}$

Carnot Cycle

$$\frac{V_2}{V_3} = \left(\frac{T_L}{T_H}\right)^{\frac{1}{k-1}}$$

Carnot Cycle $\;2 \to 3$

$$\frac{V_4}{V_1} = \left(\frac{T_H}{T_L}\right)^{\frac{1}{k-1}}$$

Carnot Cycle $\;4 \to 1$

$$\frac{V_1}{V_4} = \left(\frac{T_L}{T_H}\right)^{\frac{1}{k-1}} = \frac{V_2}{V_3} \qquad\Rightarrow\qquad \frac{V_3}{V_4} = \frac{V_2}{V_1}$$

$$Q_H = mRT_H \ln\frac{V_2}{V_1} = mRT_H \ln\frac{V_3}{V_4}$$

Carnot Cycle $\;1 \to 2$

$$Q_L = mRT_L \ln\frac{V_3}{V_4}$$

Carnot Cycle $\;3 \to 4$

Divide second equation by the first, then the **Thermodynamic temperature scale** of Kelvin for Carnot cycle is obtained:

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \qquad \text{where } T_L \text{ and } T_H \text{ are } \textbf{absolute temperature} \text{ of ideal gas equation.}$$

Dependence of the temperature of the atmosphere on the height above the sea level
(application of adiabatic expansion of a gas)

When air rises to the upper regions of lower pressure, it EXPANDS.
This expansion can be considered ADIABATIC,
because air is a poor conductor of heat.

control mass

m

Pascal's Law

$$\frac{dP}{dz} = -\rho g = -\frac{1}{v}g$$

ideal gas eqn

$$\frac{1}{v} = \frac{P}{RT}$$

$$\frac{dP}{dz} = -\frac{P}{RT}g$$

separate variables

$$z$$

$$m$$

$$\frac{1}{P}dP = -\frac{1}{RT}gdz$$

polytropic process
for ideal gas

$$PT^{\frac{k}{1-k}} = C \qquad \text{then differential}$$

$$d\left(PT^{\frac{k}{1-k}}\right) = 0 \qquad \text{use product rule}$$

$$T^{\frac{k}{1-k}}dP + P\frac{\partial}{\partial T}\left(T^{\frac{k}{1-k}}\right)dT = 0, \quad \text{where} \quad \frac{\partial}{\partial T}\left(T^{\frac{k}{1-k}}\right) = \frac{k}{1-k}T^{\frac{k}{1-k}-1} = \frac{k}{1-k}T^{\frac{2k-1}{1-k}}$$

$$\frac{1}{P}dP = -\frac{k}{1-k}\frac{1}{T}dT$$
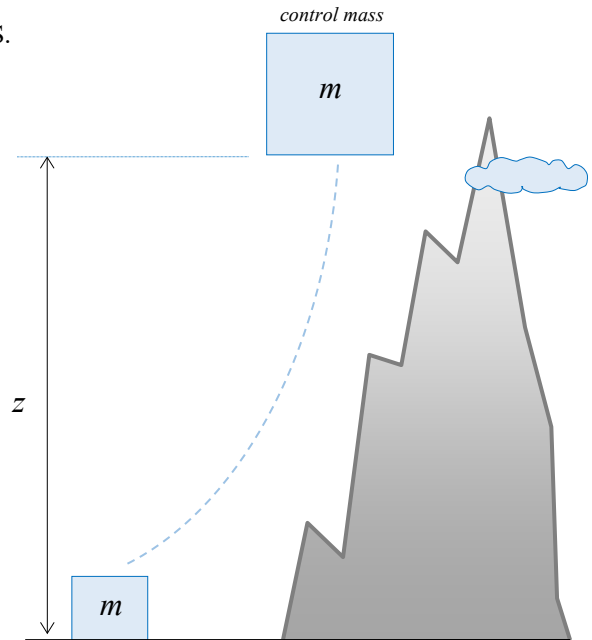
$$-\frac{k}{1-k}\frac{1}{T}dT = -\frac{1}{RT}gdz$$

$$dT = -\frac{k-1}{k}\frac{g}{R}dz$$

$$dT = -\frac{g}{c_p}dz$$

integrate, using initial temperature $T(0)$, to solve for

$$T(z) = T(0)e^{-\frac{g}{c_p}z}$$

Then temperature gradient $\dfrac{dT}{dz} = -\dfrac{g}{c_p}dz \approx -9.8 \left[\dfrac{^oC}{km}\right]$

Another derivation:

Consider the **Carnot Cycle** operating between two temperature reservoirs (for which both T_L and T_H are the absolute temperature of ideal gas equation of state) for the fixed mass m of **ideal gas**, and show that $\dfrac{Q_L}{Q_H} = \dfrac{T_L}{T_H}$.

Energy balance for

Isothermal process **1 -> 2** :

$$Q_H - W_{12} = \ m \cdot \cancel{(u_2 - u_1)}$$
$\qquad\qquad\qquad T_2 = T_1 = T_H = const$

$$Q_H - mRT_H \ln\frac{V_2}{V_1} = 0$$
$\qquad\qquad\qquad$ boundary work for isothermal process

$$Q_H = mRT_H \ln\frac{V_2}{V_1}$$

Isothermal process **3 -> 4** :

$$-Q_L - W_{34} = \ m \cdot \cancel{(u_4 - u_3)}$$
$\qquad\qquad\qquad T_3 = T_4 = T_L = const$

$$Q_L + mRT_L \ln\frac{V_4}{V_3} = 0$$
$\qquad\qquad\qquad$ boundary work for isothermal process

$$Q_L = - mRT_L \ln\frac{V_4}{V_3}$$

$$Q_L = mRT_L \ln\frac{V_3}{V_4}$$

Adiabatic process **2 -> 3**:

$$\cancel{Q} - W_{23} = m \cdot (u_3 - u_2)$$

$$-PdV = m \cdot du$$
$\qquad\qquad\qquad$ differential balance

$$-PdV = m\, c_v \cdot dT$$
$\qquad\qquad\qquad$ ideal gas

$$-\frac{1}{V}mRTdV = mc_v \cdot dT$$
$\qquad\qquad\qquad$ from ideal gas equation of state $P = \dfrac{1}{V}mRT$

$$-\frac{1}{V}dV = \frac{c_v}{R}\cdot\frac{1}{T}dT$$

$$-\frac{1}{V}dV = \frac{c_v}{c_p - c_{c_v}}\cdot\frac{1}{T}dT$$
$\qquad\qquad\qquad$ from $c_p = c_v + R$

$$-\frac{1}{V}dV = \frac{1}{\dfrac{c_p}{c_v}-1}\cdot\frac{1}{T}dT$$

$$-\frac{1}{V}dV = \frac{1}{k-1}\cdot\frac{1}{T}dT$$
$\qquad\qquad\qquad k = \dfrac{c_p}{c_v}$ specific heats ratio

$$-\int_2^3 \frac{1}{V} dV = \frac{1}{k-1} \cdot \int_2^3 \frac{1}{T} dT$$

$$(k-1) \cdot \ln \frac{V_2}{V_3} = \ln \frac{T_3}{T_2}$$

$$\ln \frac{V_2^{k-1}}{V_3^{k-1}} = \ln \frac{T_3}{T_2}$$

$$\frac{V_2^{k-1}}{V_3^{k-1}} = \frac{T_L}{T_H} \qquad\qquad \text{can be obtained faster for } \textit{isentropic} \text{ process}$$

$$\frac{V_2}{V_3} = \left(\frac{T_L}{T_H} \right)^{\frac{1}{k-1}}$$

Adiabatic process **4 -> 1**:
$$\frac{V_4^{k-1}}{V_1^{k-1}} = \frac{T_H}{T_L}$$

$$\frac{V_4}{V_1} = \left(\frac{T_H}{T_L} \right)^{\frac{1}{k-1}}$$

Consider
$$\frac{V_2}{V_3} = \left(\frac{T_L}{T_H} \right)^{\frac{1}{k-1}} = \frac{1}{\left(\frac{T_H}{T_L} \right)^{\frac{1}{k-1}}} = \frac{V_1}{V_4} \qquad \text{then}$$

$$\frac{V_2}{V_3} = \frac{V_1}{V_4}$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Consider
$$Q_H = mRT_H \ln \frac{V_2}{V_1} \qquad\qquad = mRT_H \ln \frac{V_3}{V_4}$$

Carnot Cycle 1->2

$$Q_L = mRT_L \ln \frac{V_3}{V_4}$$

Divide second equation by the first, then

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \qquad\qquad \text{where both } T_L \text{ and } T_H \text{ are absolute temperature of ideal gas equation of state.}$$

Conclusion: the absolute thermodynamic scale coincides with the absolute temperature scale of the ideal gas thermometer.

Derivation of

$$\eta_C = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

with the help of entropy:

$$Q_L = -T_L \cdot (S_4 - S_3)$$

$$Q_H = T_H \cdot (S_2 - S_1)$$

Because adiabatic processes 2->3 and 4->1 are isentropic, $S_1 = S_4$ and $S_2 = S_3$,

$$Q_L = -T_L \cdot (S_4 - S_3)$$

$$Q_H = T_H \cdot (S_3 - S_4)$$

Division yields

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

COP in terms of $\eta = 1 - \frac{Q_L}{Q_H}$:

$$COP_{HP} = \frac{1}{1 - \frac{Q_L}{Q_H}} = \frac{1}{\eta}$$

$$COP_{CR} = \frac{1}{\frac{Q_H}{Q_L} - 1} = \frac{1}{\eta} - 1$$

Negative absolute temperature

Definition of the thermodynamic temperature scale does not prohibit to choose the negative absolute temperature scale, because

$$\frac{Q_L}{Q_H} = \frac{-T_L}{-T_H} = \frac{T_L}{T_H}$$

Show that choice of the negative absolute temperature scale will yield violation of the 2nd law of thermodynamics.

Zero absolute temperature – is it allowed by this definitions?

Entropy

Equation $\dfrac{Q_L}{Q_H} = \dfrac{T_L}{T_H}$ looks like the …basic equation for derivation of the results for reversible machines.

Yes, this equation, in fact, is fundamental for the further development of classical thermodynamics, if rewritten as

$$\frac{Q_L}{T_L} = \frac{Q_H}{T_H}$$

Quantity $\dfrac{Q}{T} = S$ is called **entropy**.   'loose definition of entropy"

It is equal to transferred heat per unit temperature $\left[\dfrac{kJ}{K}\right]$.

Change of entropy during the Carnot cycle is zero: $\dfrac{Q_H}{T_H} - \dfrac{Q_L}{T_L} = 0$.

Entropy is a property of the system and depends on its state like other properties $P, T, V, ...$

Nernst's postulate: entropy of any system at zero absolute temperature is zero.

Differential change of entropy during the differential part of reversible process:

$\dfrac{\delta Q}{T} = dS$ ratio of amount of heat transferred to the system

                     to temperature of the system during this differential reversible process

$\displaystyle\int_1^2 \dfrac{\delta Q}{T} = S_2 - S_1$ change of entropy does not depend on path of reversible process