**Differentiation**

**Derivative**

\[ f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \]

**Approximations of the Derivative**

- **forward difference**
  \[ f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x} \]

- **back difference**
  \[ f'(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x} \]

- **central difference**
  \[ f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} \]

**Linear Ordinary Differential Equations**

**1\textsuperscript{st} order ODE :**

\[ \frac{dy}{dx} + P(x)y = Q(x) \]

**Initial Value Problem:**

\[ y(x_0) = y_0 \]

**Integrating Factor :**

\[ \mu(x) = e^{\int P(x)dx} \text{ or } \mu(x) = e^{\nu} \]

**General Solution :**

\[ y = \frac{c}{\mu(x)} + \frac{1}{\mu(x)} \int \mu(x)Q(x)dx \]

**Solution of IVP :**

\[ y = \frac{\mu(x_0)}{\mu(x)} y_0 + \frac{1}{\mu(x)} \int_{x_0}^{x} \mu(x')Q(x')dx' \]

**Constant Coefficient P(x) = a :**

\[ y = y_0 e^{(a-x)} + e^{-ax} \int x e^{ax} Q(x')dx' \]

**Homogeneous ODE (Q = 0) :**

\[ \frac{dy}{dx} + P(x)y = 0 \]

**General Solution :**

\[ y = \frac{c}{\mu(x)} \]

**Solution of IVP :**

\[ y = \frac{\mu(x_0)}{\mu(x)} y_0 \]

**Constant Coefficient P(x) = a :**

\[ y = y_0 e^{(a-x)} \]

**2\textsuperscript{nd} order homogeneous ODE with constant coefficients :**

\[ \frac{d^2y}{dx^2} - m^2y = 0 \quad x \in (0,L) \]

**General Solution can be in one of the following forms :**

\[ y = c_1 e^{mx} + c_2 e^{-mx} \]

\[ y = c_1 e^{-m(x-L)} + c_2 e^{m(x-L)} \]

\[ y = c_1 \sinh(mx) + c_2 \cosh(mx) \]

\[ y = c_1 \sinh[m(x-L)] + c_2 \cosh[m(x-L)] \]

**Boundary Value Problem** (there are 3 types of the boundary conditions):

\[ y(0) = y_0 \quad y'(0) = f_0 \quad -ky'(0) + hy(0) = f_0 \]

\[ y(L) = y_L \quad y'(L) = f_L \quad ky'(L) + hy(L) = f_L \]

(one boundary condition has to be set at x=0, and one boundary condition at x=L)

**Solution of BVP :**

find the coefficients \(c_1\) and \(c_2\) such that solution satisfies the boundary conditions