Consider the boundaries $x = 0$ and $x = L$ exposed to convective and radiative environment.

**Energy Balance:**

$$q_{\text{cond}} = q_{\text{conv}} + q_{\text{rad}}$$

**Linear boundary conditions for the Heat Equation at $x = 0$ and $x = L$:**

**Dirichlet**

\[ T_{x=0} = T_{0,0} \]
\[ T_{x=L} = T_{L,L} \]

**Neumann**

\[ -k \frac{dT}{dx}_{x=0} = q_{x=0}^* \]
\[ k \frac{dT}{dx}_{x=L} = q_{x=L}^* \]

**Robin**

\[ -k \frac{dT}{dx} + h_0 T_{x=0} = h_0 T_{0,0} \]
\[ k \frac{dT}{dx} + h_0 T_{x=L} = h_0 T_{L,L} \]

**Control Surface:**

**Linearization of the boundary conditions:**

$$
\begin{align*}
&
\text{non-linear boundary condition:} \\
&
\text{linear convective boundary condition with} \\
&
h = h_{\text{conv}} + h_{\text{rad}} \text{ effective convective coefficient} \\
&\text{when it is assumed that } T_{\text{in}} = T_{\text{out}} \\
&
\text{ prescribed temperature at the boundary} \\
&\quad \text{(thermostated boundary with the} \\
&\quad \text{surface temperature } T_{\text{surf}}) \quad \text{when } h \gg k \\
&
\text{prescribed heat flux at the boundary} \\
&\quad \text{insulated boundary} \quad \text{when } k \gg h \\
&
\text{convective boundary condition} \\
\end{align*}
$$

**INITIAL CONDITION**

$$T(x, y, z, 0) = T_0(x, y, z)$$

for all interior points of the domain $(x, y, z) \in D$