Problem Statement:
The robot in the figure below has three revolute joints. The 3rd joint rotates the cylindrical mass at the end of the robot arm by 90 degrees as the mass is moved between P1 and P2. Neglect the terminal motor mass for the 3rd joint motor. Assume that the end mass rotation is proportional to the path distance moved. At P1 the mass x is aligned with the path; at P2 the mass x is perpendicular to the path. The links not including the motor and end mass are made of titanium and assumed to be solid. Size the motors which would act at each joint (direct drive) to make the robot capable of moving and rotating an end mass of 100 kg along the line from P1 to P2 using an S-curve profile, where the rise and fall acceleration and max jerk parameters are: \( a_r = 2000 \text{ mm/s}^2 \), \( a_f = 2000 \text{ mm/s}^2 \), and \( J_m = 20,000 \text{ mm/s}^3 \). The end mass is treated as a uniformly solid cylinder of radius 100 mm. Consider its rotational inertia. Assume that motor 2 is a mass of 50 kg placed at the second joint and mounted to the second link (ground link is the 0th link). Assume that it is a uniformly solid cylinder of radius 40 mm. Assume that the joint speed maximums are: joint 1: 250°/sec; joint 2: 300°/sec; joint 3: 400°/sec and that the acceleration capabilities of the joints are never exceeded. Note that \( a_r \) and \( a_f \) refer to the max accelerations possible for the rise and fall profiles, respectively. Each link must include the motor or end mass as appropriate to determine the mass properties. Design would normally place joint motors on the inner link.

For the path from point P1 to point P2 shown in the figure, apply the Newton-Euler recursion relationships to determine the torque and horsepower as the robot moves the TCF along the path from P1 to P2, starting and ending with zero speed. List any assumptions made and plot the horsepower (in hp) requirements for each motor as the robot moves along the path starting and ending with zero speed, while attempting to maintain the maximum linear speed of 400 mm/sec along the path. Determine the maximum horsepower and torque along the path for each motor. Include gravity, but neglect any viscous friction in the motors. **Assume that the zero state of the robot places both links vertically up along the Y-axis**, fully extended. Also use the elbow up configuration for the robot in your analysis (as shown), if it is possible to use this configuration throughout the path motion.
Suggestion: This problem is efficiently solved by a recursive C/C++ program using the CODE API library. It is not advisable to use manual means like a calculator. You may also use Excel, MathCAD, MATLAB, etc.

Warning: This exam can take up anywhere from 4 to 20 hours, depending on your approach and organization. It effectively is a final exam, because it integrates most of the course material in one problem. In general the plots should be as a function of time if not so stated.

Part 1 requirements

1. Determine mass properties for each link (mass, mass moments of inertia). (10 pts)
2. Determine at 100 Hz the position, velocity, and acceleration components (X, Y, and Z) of the path motion from P1 to P2 using S-curve trajectory generation. (40 pts)
3. Plot the path velocity and acceleration magnitude along the path from P1 to P2 as a function of time. (10 pts)
4. Derive and show the inverse kinematics for the robot and plot the $\theta_1$, $\theta_2$ and $\theta_3$ joint values along the path as a function of time. (40 pts)

Part 2 requirements:

5. Derive and show the joint rate equations for joint speed and joint acceleration as a function of the path position, speed, and acceleration between P1 to P2. (40 pts)
6. After determining the joint rates (speed and acceleration) as a function of the path position, speed, and acceleration between P1 to P2, plot them as a function of time. (40 pts)
7. Configure and simplify the forward and backward recursion equations for the three link robot (link 3 is the end mass), ensuring that the vectors are referenced consistently in each equation. (20 pts)
Part 3 requirements:

8. Given the joint motion, apply the forward recursion equations to determine the cm (center of mass) acceleration at each link at each trajectory point from P1 to P2. (30 pts)

9. Given the joint motion, forward recursion results, and the cm acceleration, apply the backward recursion equations to determine the forces and torques on each link along the path from P1 to P2 as a function of time. (30 pts)

10. Now apply the torque equations to generate the horsepower about each joint as a function of position along the path from P1 to P2. Plot the torque, and horsepower for each joint as a function of time, and then size each joint motor in horsepower. Identify the maximum absolute value of the torque and horsepower experienced by each motor. (40 pts)