1. A velocity vector is given by \( \mathbf{v} = \begin{bmatrix} 10.0 \\ 20.0 \\ 30.0 \end{bmatrix} \)

Given

\[
\begin{bmatrix}
0.866 & -0.500 & 0 & 11.0 \\
0.500 & 0.866 & 0 & -3.0 \\
0 & 0 & 1 & 9.0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

compute \( \mathbf{v} \).

2. A vector \( \mathbf{p} \) must be mapped through three rotation matrices:

\[
\mathbf{p} = \mathbf{A}_B \mathbf{R}_D \mathbf{C}_B \mathbf{R}_C \mathbf{D}_B \mathbf{p}
\]

One choice is to first multiply the three rotation matrices together to form the expression:

\[
\mathbf{p} = \mathbf{A}_D \mathbf{R}_D \mathbf{p}
\]

Another choice is to transform the vector through the matrices one at a time, that is,

\[
\mathbf{p} = \mathbf{A}_B \mathbf{R}_D \mathbf{C}_B \mathbf{R}_C \mathbf{D}_B \mathbf{p}
\]

\[
\mathbf{p} = \mathbf{A}_B \mathbf{R}_D \mathbf{C}_B \mathbf{p}
\]

\[
\mathbf{p} = \mathbf{A}_B \mathbf{R}_D \mathbf{p}
\]

Because \( \mathbf{D} \mathbf{p} \) is changing at 100 Hz, we must calculate \( \mathbf{A} \mathbf{p} \) at this rate. However, the three rotation matrices are also changing as determined by a vision system which gives us new values for \( \mathbf{A}_B \mathbf{R} \), \( \mathbf{B}_D \mathbf{R} \), and \( \mathbf{C}_B \mathbf{R} \) at 30 Hz. What is the best way to organize the computation to minimize the calculation effort (multiplication and additions)?

3. Determine the direction \( \mathbf{k} \) and the angle of rotation \( \theta \) about that direction which cause a gripper to relocate from an initial point \( \mathbf{p}_0 = [0 \ 0 \ 4 \ 1]^T \) to a final location of \( \mathbf{p} = [2 \ 2 \ 8^{1/2} \ 1]^T \).
4. The following frame definitions are given. C initially locates an object relative to a world frame. Frame D locates the object after a move and D should be interpreted relative to the object in frame C. Determine the screw axis (rotation angle, lead, and point on screw axis) that will move an object in frame C to its final location in frame D.

\[
C = \begin{bmatrix}
0.866 & -0.500 & 0 & 11.0 \\
0.500 & 0.866 & 0 & -1.0 \\
0 & 0 & 1 & 8.0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
1 & 0 & 0 & 0.0 \\
0 & 0.866 & -0.500 & 10.0 \\
0 & 0.500 & 0.866 & -20.0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]