Feedforward Control

So far, most of the focus of this course has been on feedback control. In certain situations, the performance of control systems can be enhanced greatly by the application of feedforward control. What you need to look for are two key characteristics:

1. An identifiable disturbance is affecting significantly the measured variable, in spite of the attempts of a feedback control system to regulate these effects, and

2. This disturbance can be measured, perhaps with the addition of instrumentation.

Also, we would be interesting in controlling the source of the disturbance locally, before it affects our main process, if that were possible. If it is possible, we would usually implement cascade control, not feedforward control.
Examples of Feedforward Control

• **Shower**
  – Hear toilet flush (measurement)
  – Adjust water to compensate
  – Feedback is when you wait for the water to turn hot before changing the setting

• **Car approaching hill**
  – See how steep the hill is (measurement)
  – Push on pedal to keep steady speed
  – Feedback is to wait for slowing before adjusting pedal

• **Chemical system**
  – Measure something in feed stream
    • like $T_{\text{water,return}}$ in heating plant
  – Change heat to reactor
Example

- Goal: Cool down hot water with cold water stream
- Controlled & measured variable: $T_{\text{out}}$
- Manipulated variable: flow rate of hot stream ($q_{\text{hot}}$)
How do we manipulate $U(s)$ to cancel the effect that $D(s)$ will have on $Y(s)$?
Derivation

1. Write an algebraic equation for the block diagram
   \[ Y(s) = D(s)\cdot G_d(s) + U(s)\cdot G_p(s) \]

2. If \( Y(s) \) is to be unaffected by \( D(s) \), then we want \( Y(s) = 0 \)

3. Solve for \( U(s) \) in terms of \( D(s) \)
   \[ U(s) = \left[ -\frac{G_d(s)}{G_p(s)} \right] \cdot D(s) \]

   So \( G_{ff} = -\frac{G_d(s)}{G_p(s)} \)
If $G_d$ and $G_p$ are first order

$$G_p(s) = \frac{K_p e^{-\theta_p s}}{\tau_p s + 1} \quad \text{and} \quad G_d(s) = \frac{K_d e^{-\theta_d s}}{\tau_d s + 1}$$

Therefore,

$$G_{ff}(s) = -\frac{\left(\frac{K_d e^{-\theta_d s}}{\tau_d s + 1}\right)}{\left(\frac{K_p e^{-\theta_p s}}{\tau_p s + 1}\right)} = -\frac{K_d}{K_p} \frac{\tau_p s + 1}{\tau_d s + 1} e^{-(\theta_d - \theta_p)s}$$

\(\text{dynamic}\)

$$= -\frac{K_d}{K_p}$$

\(\text{static}\)
When is $G_{ff}$ not feasible?

$$G_{ff}(s) = -\frac{K_d}{K_p} \frac{\tau_p s + 1}{\tau_d s + 1} e^{-(\theta_d - \theta_p)s}$$

When $\theta_p > \theta_d$, the function will grow without bounds.
Modify block diagram
Feed Forward with Feedback Trim
• Measure the disturbance (fluctuating cold water inlet flow rate)
• Adjust controller through model (G_{ff})
Seborg’s version

Figure 15.11
Disturbance Rejection Performance

Disturbance Rejection Performance of Single Loop PI Controller

Disturbance Rejection Performance of PI With Feed Forward

- Reactor exit temperature
- Constant set point
- Disturbance variable steps
- Rapid control action from feed forward

Feedback (PI) Feedforward with Feedback (PI)
Practice

\[ G_p(s) = \frac{0.6e^{-37s}}{39s + 1} \quad G_d(s) = \frac{0.25e^{-57s}}{31s + 1} \]

\[ \frac{K_d}{K_p} = \frac{0.25}{0.60} = 0.417 \]

\[ \theta_d - \theta_p = 57 - 37 = 20 \]

\[ G_{ff} = -0.417\frac{39s + 1}{31s + 1}e^{-20s} \]
Physically Realizable $G_{ff}$

1. The exponential term must be negative
   - $\theta_p < \theta_d$

2. The order of the numerator must be less than or equal to that of the denominator

\[
\frac{(s+1)(s+2)(s+3)}{(s+4)(s+5)} \quad \text{Physically unrealizable}
\]

\[
\frac{(s+1)(s+3)}{(s+4)(s+5)} \quad \text{Physically realizable}
\]

\[
\frac{(s+1)(s+3)}{s^3 + s^2 + s + 1} \quad \text{Physically realizable}
\]
### Comparison of Feedforward & Cascade

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<th>Cascade</th>
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<td>Valve</td>
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<tr>
<td>Model</td>
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<tr>
<td>Restrictions</td>
<td>$t_{\text{settling}}$ small for inner loop</td>
<td>$\theta_p &lt; \theta_d$</td>
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Recommendation

• Use cascade first
• Use feedforward when
  – Disturbance can be isolated
  – There is no “inner loop” variable that responds to the manipulated variable
  – Cannot use the same valve to control the disturbance
Feed Forward vs Cascade

Jacketed Reactor

FeedForward with Feedback

Cascade
Feed Forward vs Cascade

Jacketed Reactor

FeedForward with Feedback

Disturbance Rejection Performance of PI With Feed Forward

- Process: Single Loop Jacketed Reactor
- Controller: PID with Feed Forward

Tuning:
- Gain = -2.70, Reset Time = 1.60, Deriv Time = 0.0, Sample Time = 1.00
- Process Model: Gain(Kp) = -0.36, T1 = 1.58, T2 = 0.0, TD = 0.88, TL = 0.0
- Disturbance Model: Gain(Kd) = 0.95, T1 = 1.92, T2 = 0.0, TD = 1.30, TL = 0.0

- Rapid control action from feed forward
- Constant set point

Cascade

Disturbance Rejection Performance of Cascade Architecture

- Process: Cascade Jacketed Reactor
- Pri. PID (P= RA, I= ARW, D= off, F= off)
- Sec. PID (P= DA, I= off, D= off, F= off)

Tuning:
- Gain = 0.61, Reset Time = 0.55, Sample Time = 1.00

- P-Only control offset
- Constant set point for primary PV

- Disturbance
- Variable steps

Tuning: Gain = -6.40, Sample Time = 1.00
Control Station Example

Jacketed Reactor

Volunteer needed