Name:

ECEn 370

Quiz 3 Solutions

Friday, January 22, 2010.

1. Three students invite six people to an event tonight. The probability that any one of the six people will show up is $\frac{1}{2}$, independent of the others.

a) What is the probability that not all three students will have a date tonight, i.e. less than three of the six people show up.

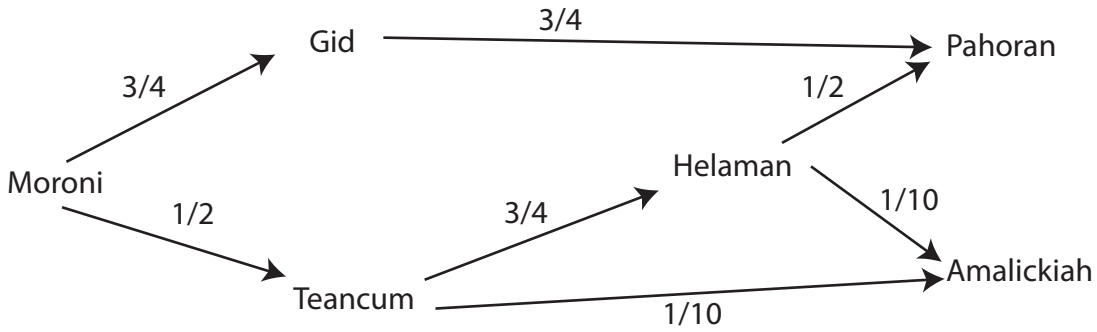
$\mathbf{P}(k<3) = \sum_{k=0}^{2} \binom{6}{k} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{6-k} = \binom{6}{0}\left(\frac{1}{2}\right)^6 + \binom{6}{1}\left(\frac{1}{2}\right)^6 + \binom{6}{2}\left(\frac{1}{2}\right)^6 = (1+6+15)\frac{1}{64} = \frac{22}{64} = \frac{11}{32}$

b) What is the probability that more than three people show up?

$\mathbf{P}(k>3) = \sum_{k=4}^{6} \binom{6}{k} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{6-k} = \binom{6}{4}\left(\frac{1}{2}\right)^6 + \binom{6}{5}\left(\frac{1}{2}\right)^6 + \binom{6}{6}\left(\frac{1}{2}\right)^6 = (15+6+1)\frac{1}{64} = \frac{22}{64} = \frac{11}{32}$

You might note that the binomial formula, like Pascal's triangle, is symmetric. Thus, it is not surprising that the answers for these two cases are the same.

2. Moroni is sending a message to Pahoran. He makes two copies and sends one to Gid and one to Teancum. They then pass the message on as follows in the figure until it reaches Pahoran. The probabilities of the message reaching the next person are also in the figure. Amalickiah's spies can copy the message with the probabilites indicated in the figure from Teancum and Helaman. The probabilities of success for each leg of the journey are independent of the others.



a) What is the probability that Pahoran receives the message?

$\mathbf{P}(MGP) = \frac{3}{4}\cdot\frac{3}{4} = \frac{9}{16}$

$\mathbf{P}(MTHP) = \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{2} = \frac{3}{16}$

$\mathbf{P}(MP) = 1-(1-\frac{9}{16})(1-\frac{3}{16}) = 1-\frac{7}{16}\cdot\frac{13}{16} = 1-\frac{91}{256} = \frac{165}{256} \approx 0.6445$

b) What is the probability that Amalickiah receives the message?

$\mathbf{P}(THA) = \frac{3}{4}\cdot\frac{1}{10} = \frac{3}{40}$

$\mathbf{P}(TA) = 1-(1-\frac{3}{40})(1-\frac{1}{10}) = 1-\frac{37}{40}\cdot\frac{9}{10} = 1-\frac{333}{400} = \frac{67}{400}$

$\mathbf{P}(MA) = \frac{1}{2}\cdot\frac{67}{400} = \frac{67}{800} \approx 0.0838$

c) What is the probability that Amlickiah receives the message and Pahoran does not?

It turns out that these events are not completely independent. If you made the assumption that they are independent then you would have:

$\mathbf{P}(MA \cap MP^C) = \frac{67}{800}\cdot(1-\frac{165}{256}) = \frac{67}{800}\cdot\frac{91}{256} = \frac{6097}{204800} \approx 0.0298$

Note for future years - I will not ask a question like part c.