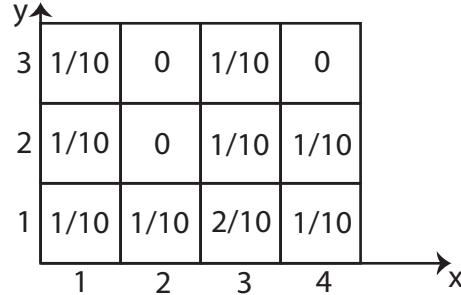


ECEn 370**Quiz 4 Solutions**

Friday, February 5, 2010.

You have the following joint PMF of random variables X and Y :



1. Find the marginal PMFs of X and Y , $p_X(x)$ and $p_Y(y)$.

$$p_X(x) = \sum_y p_{X,Y}(x,y) = \begin{cases} \frac{3}{10}, & x=1 \\ \frac{1}{10}, & x=2 \\ \frac{4}{10}, & x=3 \\ \frac{2}{10}, & x=4 \\ 0, & \text{otherwise} \end{cases} \quad p_Y(y) = \sum_x p_{X,Y}(x,y) = \begin{cases} \frac{5}{10}, & y=1 \\ \frac{3}{10}, & y=2 \\ \frac{2}{10}, & y=3 \\ 0, & \text{otherwise} \end{cases}$$

2. Find the expectation and variance of X and Y , $\mathbf{E}[X]$ and $\mathbf{E}[Y]$, and $\text{var}(X)$ and $\text{var}(Y)$.

$$\mathbf{E}[X] = 1 \cdot \frac{3}{10} + 2 \cdot \frac{1}{10} + 3 \cdot \frac{4}{10} + 4 \cdot \frac{2}{10} = \frac{3+2+12+8}{10} = \frac{25}{10} = \frac{5}{2}$$

$$\mathbf{E}[Y] = 1 \cdot \frac{5}{10} + 2 \cdot \frac{3}{10} + 3 \cdot \frac{2}{10} = \frac{5+6+6}{10} = \frac{17}{10}$$

$$\mathbf{E}[X^2] = 1 \cdot \frac{3}{10} + 4 \cdot \frac{1}{10} + 9 \cdot \frac{4}{10} + 16 \cdot \frac{2}{10} = \frac{3+4+36+32}{10} = \frac{75}{10} = \frac{15}{2}$$

$$\mathbf{E}[Y^2] = 1 \cdot \frac{5}{10} + 4 \cdot \frac{3}{10} + 9 \cdot \frac{2}{10} = \frac{5+12+18}{10} = \frac{35}{10} = \frac{7}{2}$$

$$\text{var}(X) = \mathbf{E}[X^2] - (\mathbf{E}[X])^2 = \frac{75}{10} - \frac{625}{100} = \frac{750-625}{100} = \frac{125}{100} = \frac{5}{4}$$

$$\text{var}(Y) = \mathbf{E}[Y^2] - (\mathbf{E}[Y])^2 = \frac{35}{10} - \frac{289}{100} = \frac{350-289}{100} = \frac{61}{100}$$

3. Find the conditional PMFs $p_{X|Y}(x|3)$ and $p_{Y|X}(y|3)$.

$$p_{X|Y}(x|3) = \begin{cases} \frac{1}{2}, & x=1 \\ 0, & x=2 \\ \frac{1}{2}, & x=3 \\ 0, & x=4 \\ 0, & \text{otherwise} \end{cases} \quad p_{Y|X}(y|3) = \begin{cases} \frac{1}{2}, & y=1 \\ \frac{1}{4}, & y=2 \\ \frac{1}{4}, & y=3 \\ 0, & \text{otherwise} \end{cases}$$

4. Compute $\mathbf{E}[Z]$, where $Z = 2X + 3Y + 10$.

$$\mathbf{E}[Z] = 2\mathbf{E}[X] + 3\mathbf{E}[Y] + 10 = 2 \cdot \frac{25}{10} + 3 \cdot \frac{17}{10} + \frac{100}{10} = \frac{50+51+100}{10} = \frac{201}{10}$$

5. Given the event $A = \{X = 2\}$, compute $\mathbf{E}[Z|A]$, where $Z = 2X + 3Y + 10$.

$$\text{Since } \mathbf{P}(Y=1|X=2) = 1, \mathbf{E}[Z] = 2(2) + 3(1) + 10 = 17$$