

ECEn 370

Quiz 4 Solutions

Friday, February 5, 2010.

You have the following joint PMF of random variables X and Y :

		$y \uparrow$			
	3	1/10	0	1/10	0
	2	1/10	0	1/10	1/10
	1	1/10	1/10	2/10	1/10
		1	2	3	4
		$x \rightarrow$			

1. Find the marginal PMFs of
- X
- and
- Y
- ,
- $p_X(x)$
- and
- $p_Y(y)$
- .

$$p_X(x) = \sum_y p_{X,Y}(x,y) = \begin{cases} \frac{3}{10}, & x = 1 \\ \frac{1}{10}, & x = 2 \\ \frac{4}{10}, & x = 3 \\ \frac{2}{10}, & x = 4 \\ 0, & \text{otherwise} \end{cases} \quad p_Y(y) = \sum_x p_{X,Y}(x,y) = \begin{cases} \frac{5}{10}, & y = 1 \\ \frac{3}{10}, & y = 2 \\ \frac{2}{10}, & y = 3 \\ 0, & \text{otherwise} \end{cases}$$

2. Find the expectation and variance of
- X
- and
- Y
- ,
- $\mathbf{E}[X]$
- and
- $\mathbf{E}[Y]$
- , and
- $\text{var}(X)$
- and
- $\text{var}(Y)$
- .

$$\begin{aligned} \mathbf{E}[X] &= 1 \cdot \frac{3}{10} + 2 \cdot \frac{1}{10} + 3 \cdot \frac{4}{10} + 4 \cdot \frac{2}{10} = \frac{3+2+12+8}{10} = \frac{25}{10} = \frac{5}{2} \\ \mathbf{E}[Y] &= 1 \cdot \frac{5}{10} + 2 \cdot \frac{3}{10} + 3 \cdot \frac{2}{10} = \frac{5+6+6}{10} = \frac{17}{10} \\ \mathbf{E}[X^2] &= 1 \cdot \frac{3}{10} + 4 \cdot \frac{1}{10} + 9 \cdot \frac{4}{10} + 16 \cdot \frac{2}{10} = \frac{3+4+36+32}{10} = \frac{75}{10} = \frac{15}{2} \\ \mathbf{E}[Y^2] &= 1 \cdot \frac{5}{10} + 4 \cdot \frac{3}{10} + 9 \cdot \frac{2}{10} = \frac{5+12+18}{10} = \frac{35}{10} = \frac{7}{2} \\ \text{var}(X) &= \mathbf{E}[X^2] - (\mathbf{E}[X])^2 = \frac{75}{10} - \frac{625}{100} = \frac{750-625}{100} = \frac{125}{100} = \frac{5}{4} \\ \text{var}(Y) &= \mathbf{E}[Y^2] - (\mathbf{E}[Y])^2 = \frac{35}{10} - \frac{289}{100} = \frac{350-289}{100} = \frac{61}{100} \end{aligned}$$

3. Find the conditional PMFs
- $p_{X|Y}(x|3)$
- and
- $p_{Y|X}(y|3)$
- .

$$p_{X|Y}(x|3) = \begin{cases} \frac{1}{2}, & x = 1 \\ 0, & x = 2 \\ \frac{1}{2}, & x = 3 \\ 0, & x = 4 \\ 0, & \text{otherwise} \end{cases} \quad p_{Y|X}(y|3) = \begin{cases} \frac{1}{2}, & y = 1 \\ \frac{1}{4}, & y = 2 \\ \frac{1}{4}, & y = 3 \\ 0, & \text{otherwise} \end{cases}$$

4. Compute
- $\mathbf{E}[Z]$
- , where
- $Z = 2X + 3Y + 10$
- .

$$\mathbf{E}[Z] = 2\mathbf{E}[X] + 3\mathbf{E}[Y] + 10 = 2 \cdot \frac{25}{10} + 3 \cdot \frac{17}{10} + \frac{100}{10} = \frac{50+51+100}{10} = \frac{201}{10}$$

5. Given the event
- $A = \{X = 2\}$
- , compute
- $\mathbf{E}[Z|A]$
- , where
- $Z = 2X + 3Y + 10$
- .

$$\text{Since } \mathbf{P}(Y = 1|X = 2) = 1, \mathbf{E}[Z] = 2(2) + 3(1) + 10 = 17$$