Name:

ECEn 370

Quiz 5 Solutions

Friday, February 12, 2010.

1. You are meeting your date at the Wilkinson Center. You forget whether your date is coming from home, from class, or from work. The probability that your date is coming from home is 0.5, coming from class is 0.25, and coming from work is 0.25. For your date, the expected arrival time from home is 20 minutes, the expected arrival time from class is 10 minutes, and the expected arrival time from work is 10 minutes. Given that you have no other information and arrival time random variables are independent, what is the expected arrival time of your date?

Answer: 15 minutes

$\mathbf{E}[X] = \mathbf{E}[X|A]P(A) + \mathbf{E}[X|B]P(B) + \mathbf{E}[X|C]P(C) = 20(0.5) + (10)(0.25) + (10)(0.25) = 15$

2. Suppose that X, Y, and Z are independent random variables. $\mathbf{E}[X] = 3$, $\mathbf{E}[Y] = 2$, and $\mathbf{E}[Z] = 1$. $\text{var}(X) = 3$, $\text{var}(Y) = 2$, and $\text{var}(Z) = 1$. Compute the following expressions.

a) $\mathbf{E}[2X + X^2] = \mathbf{E}[2X] + \mathbf{E}[X^2] = 2(3) + \text{var}(X) + (E[X])^2 = 6 + 3 + (3)^2 = 18$

b) $\mathbf{E}[3XYZ] = 3 \cdot 3 \cdot 2 \cdot 1 = 18$

c) $\text{var}(X + Y) = 3 + 2 = 5$

d) $\text{var}(2X + 3Z) = 4(3) + 9(1) = 21$

e) $\text{var}(Y - X) = 2 + 3 = 5$

f) $\mathbf{E}[XY^2] = \mathbf{E}[X]\mathbf{E}[Y^2] = (3)\left(\text{var}(Y) + (\mathbf{E}[Y])^2\right) = 3(2 + 2^2) = 18$

3. Suppose that a continuous random variable X has a probability density function given by

$$f_X(x) = \begin{cases} ce^{-2x}, & \text{if } 0 < x < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

a) Find c so that $f_X(x)$ is a legitimate PDF.

$1 = \int_0^\infty ce^{-2x}dx = \left[\frac{-c}{2}e^{-2x}\right]_0^\infty = 0 + \frac{c}{2} \therefore c = 2$

b) Find $\mathbf{E}[X]$.

$\mathbf{E}[X] = \int_0^\infty xf_X(x)dx = \int_0^\infty 2xe^{-2x}dx$

By integration by parts, $u = x$, $dv = 2e^{-2x}$. Then $du = dx$ and $v = e^{-2x}$. This means that

$\mathbf{E}[X] = \left[xe^{-2x} - \int e^{-2x}dx\right]_0^\infty = \left[xe^{-2x} + \frac{1}{2}e^{-2x}\right]_0^\infty = \frac{1}{2}$.

Or we may just recognize this as an exponential random variable with $\mathbf{E}[X] = \frac{1}{\lambda}$ where $\lambda = 2$.