Name:

ECEn 370

Quiz 6 Solutions

Friday, February 19, 2010.

1. Many locations need to fill up their reservoirs. Suppose that the annual snowfall in the mountains is modeled as a normal random variable with a mean of $\mu = 50$ inches and a standard deviation of $\sigma = 10$. What is the probability that this year's snowfall will be greater than 45 inches and less than 60 inches? Put your answer in terms of the standard normal CDF, denoted by $\Phi(y)$.

$$P(45 \leq X \leq 60) = P\left(\frac{45-50}{10} \leq \frac{X-80}{10} \leq \frac{60-50}{10}\right) = P\left(-0.5 \leq Y \leq 1\right) = \Phi(1) - \Phi(-0.5) = \Phi(1) - (1 - \Phi(0.5))$$

2. The joint pdf of two random variables, X and Y, is given by

$$f_{X,Y}(x,y) = \begin{cases} ke^{-(ax+by)}, & x > 0, \ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

where a and b are positive constants and k is a constant. Determine the value of k to make this a legitimate pdf.

We keep in mind that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dxdy = 1$$

This means that

$$k \int_{0}^{\infty} \int_{0}^{\infty} e^{-(ax+by)}\,dxdy = k \int_{0}^{\infty} e^{-ax}\,dx \int_{0}^{\infty} e^{-by}\,dy = \frac{k}{ab} = 1$$