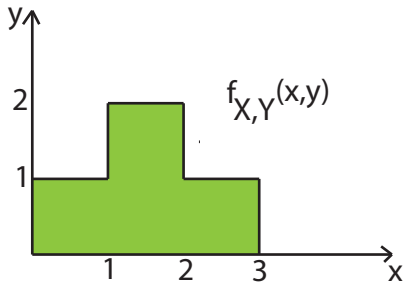
Name:

ECEn 370

Quiz 7 Solutions

Friday, February 26, 2010.

The random variables X and Y have a joint distribution given by:



$$f_{X,Y}(x,y) = \begin{cases} c, & \text{if } (x,y) \text{ is in shaded region} \\ 0, & \text{otherwise} \end{cases}$$

1. What is the value of c that makes this pdf legitimate?

$\int\int f_{X,Y}(x,y)dxdy = 1$
$\therefore c = \frac{1}{4}$

2. What is the probability of the event $\{X \leq Y\}$?

This is all of the area that is above the line $y = x$. It forms two box halves, which summed together give an area of 1. Since the density is 1/4, then the the probability of this event is 1/4.

3. What is the pdf associated with $f_{X|Y}\left(x|\frac{3}{2}\right)$?

$$f_{X|Y}\left(x|\tfrac{3}{2}\right) = \begin{cases} 1, & \text{if } 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

4. What is the pdf associated with $f_{Y|X}\left(y|\frac{5}{2}\right)$?

$$f_{Y|X}\left(y|\tfrac{5}{2}\right) = \begin{cases} 1, & \text{if } 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

5. Suppose you have the event $\{Y \geq 1\}$. What is the joint PDF associated with $f_{X,Y|A}(x,y)$?

$$f_{X,Y|A}(x,y) = \begin{cases} 1, & \text{if } 1 \leq x \leq 2 \text{ and } 1 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

6. Find E$[X]$.

$\mathrm{E}[X] = \int\int x f_{X,Y}(x,y)dxdy = \int_0^1 cxdx + \int_1^2 2cxdx + \int_2^3 cxdx$
$= c\left(\frac{1}{2} + 3 + \frac{5}{2}\right) = \frac{3}{2}$

7. Find E$[X|A]$.

$\mathrm{E}[X|A] = \int\int x f_{X,Y|A}(x,y)dxdy = \int_1^2 xdx = \frac{3}{2}$