Name:

ECEn 370

Quiz 8 Solutions

Friday, March 5, 2010.

1. Suppose that you walk to class at a speed which is uniformly distributed from 4 to 5 miles per hour (maybe because of snow, like today?). The distance of a single trip to class is 20 miles. What is the pdf of the duration of the trip?

$X = \text{speed}$
$Y = g(X) = 20/X$

$$f_X(x) = \begin{cases} 1, & \text{if } 4 \leq x \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

$$F_X(x) = \begin{cases} 0, & \text{if } x \leq 4 \\ x-4, & \text{if } 4 < x < 5 \\ 1, & \text{if } 5 \leq x \end{cases}$$

$$F_Y(y) = P(Y \leq y) = P\left(\tfrac{20}{X} \leq y\right) = P\left(\tfrac{20}{y} \leq X\right) = 1 - F_X\left(\tfrac{20}{y}\right)$$

$$F_Y(y) = \begin{cases} 1, & \text{if } \frac{20}{y} \leq 4 \\ 5 - \frac{20}{y}, & \text{if } 4 < \frac{20}{y} < 5 \\ 0, & \text{if } 5 \leq \frac{20}{y} \end{cases} = \begin{cases} 0, & y \leq 4 \\ 5 - \frac{20}{y}, & 4 < y < 5 \\ 1 & y \geq 5 \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{20}{y^2}, & 4 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

2. Suppose you are given random variables X and Y which have their values determined by the the joint PMF given by the following points: (1,0), (0,1), (-1,0), (0,-1) each with probability 1/4. Find the following: $\text{cov}(X,Y)$ and correlation coefficient, ρ. Are X and Y independent random variables?

$E[X] = 0$
$E[Y] = 0$
$E[XY] = 0$
$\text{var}(X) = 1(\frac{1}{4}) + 1(\frac{1}{4}) = \frac{1}{2}$
$\text{var}(Y) = 1(\frac{1}{4}) + 1(\frac{1}{4}) = \frac{1}{2}$
$\text{cov}(X,Y) = E[XY] - E[X]E[Y] = 0$
$\rho = \frac{\text{cov}(X,Y)}{\sqrt{\text{var}(X)\text{var}(Y)}} = 0$

X and Y are not independent random variables. For example, if you know that X is 1, then you know that Y is 0.

3. Suppose you know that $\text{var}(X) = 2$, $\text{var}(Y) = 3$, $\text{cov}(X,Y) = 1$, and $Z = 2X - 3Y + 3$. Find $\text{var}(Z)$.

Let $A = 2X$
Let $B = -3Y$
$Z = A + B + 3$
$\text{var}(Z) = \text{var}(A) + \text{var}(B) + 2\text{cov}(A,B)$
$\text{cov}(A,B) = \text{cov}(A,-3Y) = -3\text{cov}(A,Y) = -3 \cdot \text{cov}(2X,Y) = -3 \cdot 2\text{cov}(X,Y) = -6$
$\text{var}(A) = 4\text{var}(X) = 4(2) = 8$
$\text{var}(B) = 9\text{var}(Y) = 9(3) = 27$
$\text{var}(Z) = 8 + 27 + 2(-6) = 23$