Name:

ECEn 370

Quiz 9 Solutions

Friday, March 12, 2010.

1. At BYU the probability that any random person on campus will accept a date tonight is a uniform random variable, Y, distributed from 0.1 to 0.2. The probability that he/she will accept is independent of previous and other requests. Suppose you pick a person on campus and you ask them for a date repeatedly *until* they accept. The number of times that you ask this person is given by the random variable X.
What is $\mathrm{E}[X|Y]$?

$\mathrm{E}[X|Y=y] = 1/y$
$\therefore \mathrm{E}[X|Y] = 1/Y$

What is $\mathrm{E}[X]$, i.e. the number of times you expect to ask tonight?

$\mathrm{E}[X] = \mathrm{E}\left[\mathrm{E}[X|Y]\right] = \int_{0.1}^{0.2} \frac{10}{y} dy = [10 \ln y]_{0.1}^{0.2} = 10\left[\ln(0.2) - \ln(0.1)\right] \approx 6.93$

2. Suppose you have X and Y be independent Poisson random variables with mean of 2 and 3 respectively. Let $Z = X + Y$. What is the distribution of Z?
Note: For a Poisson random variable with parameter λ, the transform is $M_X(s) = e^{\lambda(e^s-1)}$, $(s < \lambda)$.

$M_X(s) = e^{\lambda_X(e^s-1)}$
$M_Y(s) = e^{\lambda_Y(e^s-1)}$
$M_Z(s) = M_X(s)M_Y(s) = e^{(\lambda_X+\lambda_Y)(e^s-1)}$
Therefore, by "pattern matching" we see that Z has a Poisson distribution with a mean of 5.

3. Suppose I have the following transform:

$$M_X(s) = \frac{1}{2}e^{2s} + \frac{1}{6}e^{3s} + \frac{1}{3}e^{5s}$$

What is $\mathrm{E}[X]$?
$\mathrm{E}[X] = \frac{d}{ds}M_X(s)\big|_{s=0} = 2 \cdot \frac{1}{2}e^{2s} + 3 \cdot \frac{1}{6}e^{3s} + 5 \cdot \frac{1}{3}e^{5s}\big|_{s=0} = 1 + \frac{1}{2} + \frac{5}{3} = \frac{6+3+10}{6} = \frac{19}{6}$

What is $\mathrm{E}[X^2]$?
$\mathrm{E}[X] = \frac{d^2}{ds^2}M_X(s)\Big|_{s=0} = 4 \cdot \frac{1}{2}e^{2s} + 9 \cdot \frac{1}{6}e^{3s} + 25 \cdot \frac{1}{3}e^{5s}\big|_{s=0} = 2 + \frac{3}{2} + \frac{25}{3} = \frac{12+9+50}{6} = \frac{71}{6}$

What is $p_X(3)$?
This is the coefficient of the e^{3s} term, so $\frac{1}{6}$.