Homework

Equation 19.22 is our best current prediction method for the concentration in steady-state plumes far above the ground. However, we are generally most interested in concentrations at ground level because people and property are exposed at ground level.

The blind application of Eq. 19.22 at or near ground level gives misleadingly low results. The reason is that it indicates that the pollutants continue to disperse at any value of z, even at z less than zero. (Using it alone, we could continue Example 19.9 and compute the concentration underground; the result would bear no relation to what we would observe in nature.) For this reason, it is necessary to account for the effect of the ground.

The ground damps out vertical dispersion. The upward and downward turbulent eddies that spread the plume in the vertical direction cannot penetrate the ground. Thus, the vertical spreading terminates at ground level. The method commonly used to account for this in calculations is to assume that the pollutants that would have carried below $z = 0$ if the ground were not there are "reflected" upward as if the ground were a mirror. Thus, the concentration at any point is that due to the plume itself, plus that reflected upward from the ground. This is equivalent to assuming that there is a mirror-image plume below the ground that transmits as much up through the ground surface as the above-ground plume would transmit down through the ground surface if the ground were not there.

The concentrations due to the "mirror-image" plume are exactly the same as those shown by Eq. 19.22, except that the $(z - H)^2$ term is replaced by $(z + H)^2$. At the ground, $z = 0$, both the main plume and the mirror-image plume have identical values. High in the air, for example, at $z = H$, the main plume has a high concentration [exp − (0) = 1] while that for the mirror-image plume [exp − $(2H)^2$ etc.] is a small number. The combined contribution of both plumes is obtained by writing Eq. 19.22 and the analogous equation for the mirror-image plume, adding the values for the two plumes and factoring out the common terms to find

(a) Show the form that this equation takes for a point directly downwind of the source ($y = 0$) and at ground level ($z = 0$). This form is the most widely used simple point-source air-pollution modeling equation.

(b) Using that equation, estimate the concentration at ground level, directly under the plume centerline, at $x = 1$ km, for $H = 100$ m, $Q = 10$ g / s, $u = 3$ m / s, and C stability.

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(this is Eq. 19.22)

$$c = \frac{Q}{2\pi u \sigma_y \sigma_z} e^{-\left(\frac{y^2}{2\sigma_y^2} + \frac{(z-H)^2}{2\sigma_z^2}\right)}$$

$$c = \frac{Q}{2\pi u \sigma_y \sigma_z} e^{\left[-0.5\left(\frac{y}{\sigma_y}\right)^2\right]} \cdot \left[e^{-0.5\left(\frac{z-H}{\sigma_z}\right)^2} + e^{-0.5\left(\frac{z+H}{\sigma_z}\right)^2}\right]$$