ChEn 374
Fluid Mechanics

Bernoulli Applications

Spiritual Thought

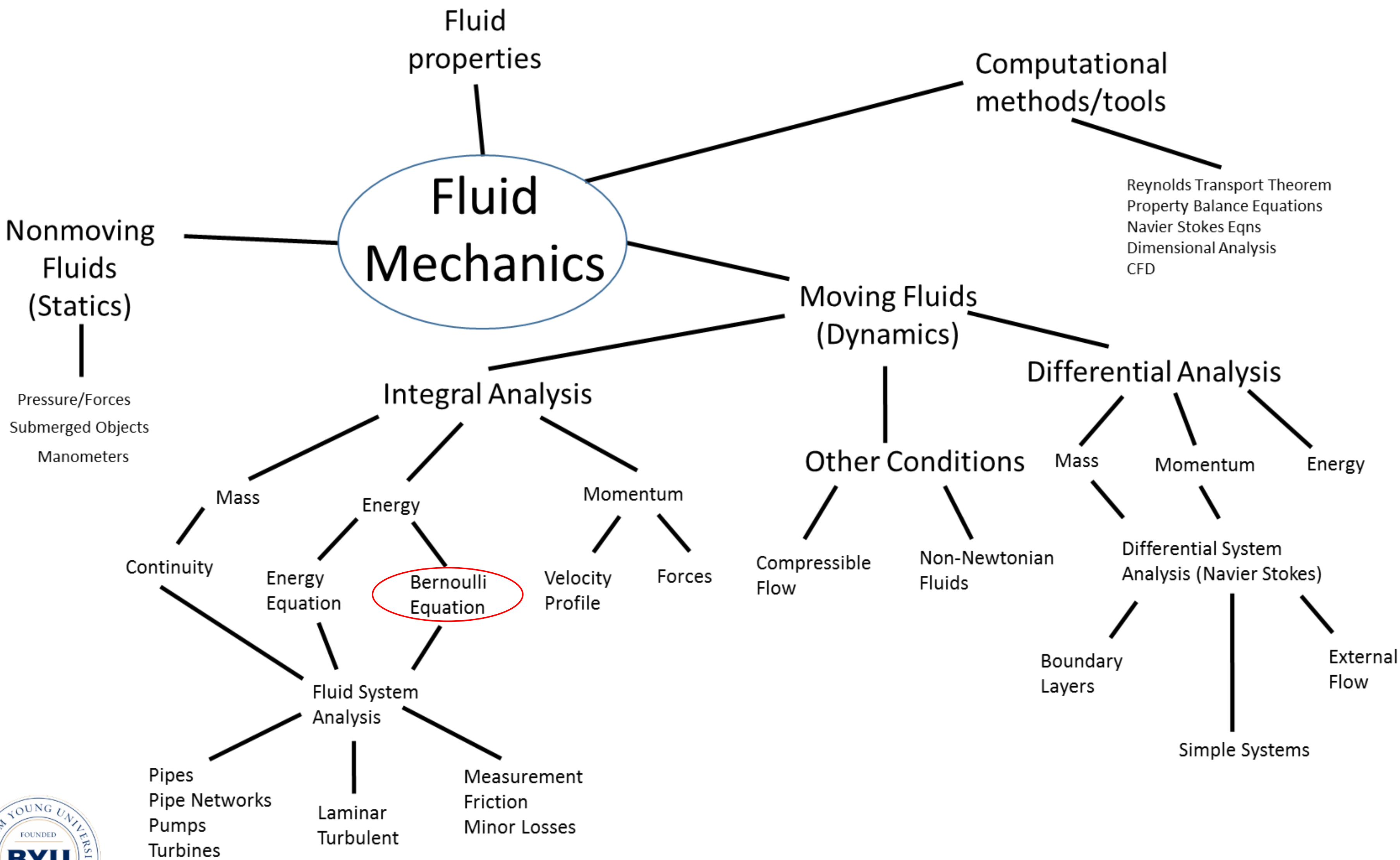
"Many of you think you are failures. You feel you cannot do well, that with all of your effort it is not sufficient. We all worry about our performance. We all wish we could do better. But unfortunately we do not realize, we do not often see the results that come of what we do. You never know how much good you do… Get on your knees and ask for the blessings of the Lord; then stand on your feet and do what you are asked to do. Then leave the matter in the hands of the Lord. You will discover that you have accomplished something beyond price."

-President Gordon B. Hinkley

Exam

- Take Home Exam
- Friday to Wednesday (turned in at start of class)
- 4 hour exam (only need 2) – ONE SITTING!
- Closed book, closed notes
- You can use 1 sheet (one side) of handwritten notes – stapled to back of exam
- Book info (like tables, units, properties) are provided.

Fluids Roadmap



Bernoulli Applications

BE:

$$\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)_1=\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)_2$$

Units:

(J/kg)
* ρ → (N): Pressure form
/ g → (m): Head form

Assumptions:

SS	Ws = 0
Const ρ	Q = 0
No friction	Streamline

Interactions among terms:

Const z:	If v up, P down If v down, P up	Stagnation
Const v:	If z up, P down If z down, P up	Statics, weight
Const P:	If z up, v down If z down, v up	Falling fluid

Problem Solving

- B.E. → apply widely
  - How?
- Ask and answer questions
  - What do I know?
  - What is wanted?
  - What is happening physically
  - What relationships do I have?
  - Write them down
  - Draw a picture

Problem → Equations → Solve → Assess

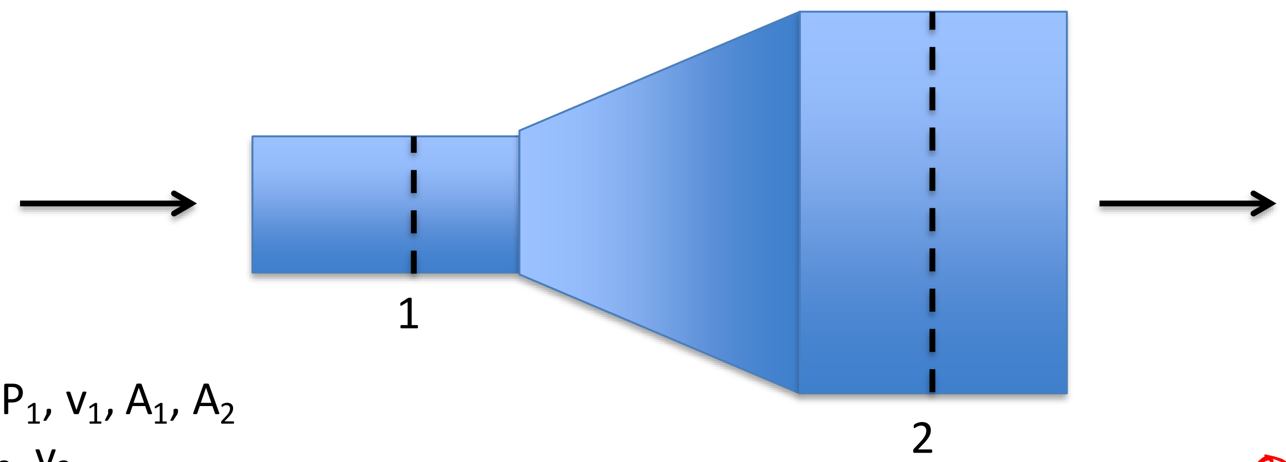
Bernoulli Equation

$$\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_1 = \left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_2$$

- Write Equation
- Pick points for application
- May require more than one application
- May need an auxiliary relation:
  - $\Delta P = \rho g h$,
  - $m = \rho A v$
  - $v_2 = v_1(A_1/A_2)$
- Practice!
  - Do your homework
  - Do book examples (try yourself before reading solution)
- Reading and understanding a solution is NOT the same as being able to come up with a solution yourself.

Sample Problem

Diffuser



Given P_1, v_1, A_1, A_2
Find P_2, v_2

Ask:

What's happening?
What do I expect? For P? for v?
What equations?

$\Delta z = 0$

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 + g\cancel{z_1} = \frac{P_2}{\rho} + \frac{1}{2}v_2^2 + g\cancel{z_2}$$

0 0

↳ 2 unknowns

use continuity: $v_1 A_1 = v_2 A_2$

$v_2 = v_1 \frac{A_1}{A_2}$

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2 \Rightarrow \left(v_1 \frac{A_1}{A_2}\right)^2$$

$$\frac{P_2}{\rho} = \frac{P_1}{\rho} + \frac{1}{2}v_1^2 - \frac{1}{2}v_1^2 \frac{A_1^2}{A_2^2}$$

$$\boxed{P_2 = P_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_1^2 \frac{A_1^2}{A_2^2}}$$

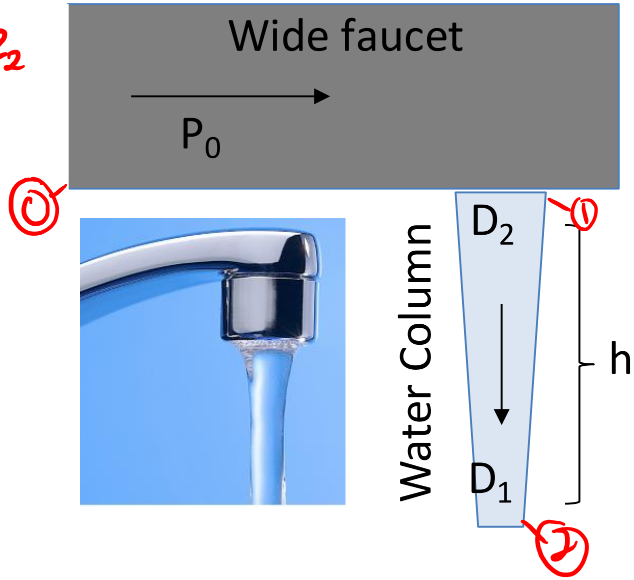
Sample Exam Problem

Your friend gives you a photograph of a water faucet (with an assumed large area).
To impress your friend with your Fluids skills, you **COMPUTE the volumetric flow rate** (L/min) of the water, **and the gage pressure** (Pa) of the water in the faucet.
As the water falls, it accelerates, and the diameter of the water column decreases.
From the photo, you measure the diameter of the column at two points, and its height.
Take h=0.3 m, D_2=1 cm, D_1 = 0.85 cm.

$$\frac{P_0}{\rho} + \frac{1}{2}\cancel{v_0^2}^{\,0} + gz_0 = \cancel{\frac{P_2}{\rho}}^{\,0} + \frac{1}{2}v_2^2 + gz_2$$

$$\frac{P_0}{\rho} = \frac{1}{2}v_2^2 + gz_2 - gz_1$$

$$\boxed{P_0 = \frac{1}{2}\rho v_2^2 + \rho g(z_2 - z_1)}$$



$$\cancel{\frac{P_1}{\rho}}^{\,0} + \frac{1}{2}v_1^2 + gz_1 = \cancel{\frac{P_2}{\rho}}^{\,0} + \frac{1}{2}v_2^2 + g\cancel{z_2}^{\,0}$$

$$\frac{1}{2}v_1^2 + gh = \frac{1}{2}v_2^2$$

Continuity

$$V_1 A_1 = V_2 A_2 \Rightarrow V_2 = V_1\frac{A_1}{A_2}$$

$$\frac{1}{2}v_1^2 + gh = \frac{1}{2}v_1^2\frac{A_1^2}{A_2^2}$$

$$V_1^2\left(1 - \frac{A_1^2}{A_2^2}\right) = -2gh$$

$$V_1^2 = \frac{-2gh}{\left(1 - \frac{A_1^2}{A_2^2}\right)}$$

Sample OEP Exam Problem

Sample OEP Exam Problem

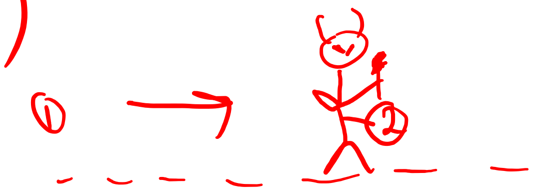
When the ***one ring*** was cut from the Dark Lord Sauron's finger, his power created a powerful vacuum inside his armor, which drew in the surrounding air. Several elves slid or stumbled due to the force of the air.

1) What is the velocity of this inwardly flowing air?

2) Was the resulting force sufficient to cause an elf to slide across the rock ground?

Find velocity of air

1)



Assume: steady state
constant density
single elevation
(No W_s, Q, Δu, on streamline, constant ρ, ss)

$$\frac{P_1}{\rho} + \frac{1}{2}v_1^2 + \cancel{g z_1}^{\,0} = \frac{P_2}{\rho} + \cancel{\frac{1}{2}v_2^2}^{\,0} + \cancel{g z_2}^{\,0}$$

$$P_1 - P_2 + \frac{1}{2}\rho v_1^2 = 0$$

$$(P_1 - P_2) = -\frac{1}{2}\rho v_1^2$$

$P_2 \cong 0$ atm $P_1 = 14.7$ atm

$V_2 = 0$ m/s

2) 

What F is produced just as the elf is starting to slide by the air? Is V_1 enough to produce this?

$$\Sigma F = \cancel{\frac{d}{dt}\int_{cv} \rho \vec{v}\, dV}^{\,0} + \int_{cs} \vec{v} \rho \vec{v}\cdot\vec{n}\, dA$$

Uniform Properties
Steady State
no y (flow splits up/down)

$\mu = 0.2$
$M = 50\ kg$
$\rho_{air} = 1.2\ kg/m^3$

$$-\Sigma F = +\mu M g = +\rho A v^2$$

$F = \rho A v^2$ Is $F > \mu M g$?

Give quick reason assesment about answer...