Chemical Engineering 374

Fluid Mechanics

Exam 1 Review

Spiritual Thought

The Lord did not people the earth with a vibrant orchestra of personalities only to value the piccolos of the world. Every instrument is precious and adds to the complex beauty of the symphony. All of Heavenly Father’s children are different in some degree, yet each has his own beautiful sound that adds depth and richness to the whole.

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Exam Review, By Content/Lectures

- Classes 1-9 (plus review)
- Chapter 1 Introduction/Basics
- Chapter 2.1-2.6 Fluid Properties
- Chapter 3.1-3.6 Pressure/Fluid Statics
- Chapter 4.1, 4.6 RTT, Conservation Laws
- Chapter 5.1-5.5 M.B., E.B., Bernoulli
- Chapter 6 Momentum Balance
- Homeworks 1-10

Class 1--Introduction

- Syllabus
- Schedule
- Policies
- Competencies/Goals
- Info Sheets
- Units, esp. lbm vs. lbf
- Sig. Figs.
- Problem Solving

Class 2 – Fluid Properties

- Fluid definition in terms of stress
- Liquids vs gases
- Continuum assumption
- Density – gases, liquids (T, P variations)
  - Coefficient of compressibility (P, V)
  - Coefficient of volume expansion (T, V)
- Viscosity– in terms of stress ($\tau=-\mu\ du/dy$)
  - Force per area, also a momentum flux.
- Non-Newtonian Fluids
  - Newtonian
  - Bingham plastic
  - Pseudoplastic
  - Dilitant

Class 3 – Pressure/Fluid Statics

- Pressure (F/A), a normal stress, isotropic/scalar.
  - Units
  - Absolute vs gage (which/when/specify)
- Barometric equation $\frac{dP}{dz} = -\rho g$
  - Force balance on a body, divergence theorem

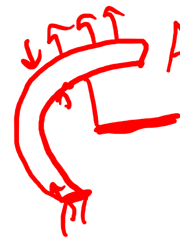
$$\sum \vec{F} = 0 = \vec{F}_p + \vec{F}_b \qquad \vec{F}_p + \vec{F}_b = -\int_{SA} P\vec{n}dA + \int_V \rho\vec{a}dV \qquad \vec{F}_p + \vec{F}_b = -\int_V \nabla P dV + \int_V \rho\vec{a}dV$$

  - ΔP = ρgh (for constant density).
  - Know what to do for variable ρ
    - Ideal Gas: Isothermal → ideal gas law
    - Ideal Gas: Adiabatic → ideal gas law with T, P relation, where $\gamma = c_p/c_v \sim$ const

$$T = \frac{T_1}{P_1}^{\alpha} P^{\alpha} \qquad \alpha = \frac{\gamma - 1}{\gamma}$$

  - Liquid P variation with depth, gas variation with depth.

Class 4—Pressure Measurement/Surface Forces



$AP = F$

- Barometer (P_{atm})
- Bourdon Tube (gage pressure)
- Manometer (pressure differences)
  - Pressure is the same at the same height in a continuous fluid.
  - Travel from one end to the other. “down” increases pressure, “up” decreases pressure.
    - Gas/Liquid calculation: delta P = = ρ * g * h
    - Liquid/Liquid calculation: delta P = $\Delta\rho$ * g * h

Class 4—Pressure Measurement/Surface Forces

- Forces on surfaces
  - F = P*A → dF = P*dA → (often) dF = P*W*dh → Integrate to get net force
  - Net force is Area * P at the centroid (area-weighted average depth).
  - Point of exertion of that force is through the "center of pressure" (force-weighted average depth, or the centroid of the "pressure prism").
  - For a flat plate, centroid is the middle of the plate, and the pressure center is 2/3 down the plate.
  - Atmospheric pressure cancels when acting on both sides
  - Inclined surfaces are just like vertical, but have an angle between dA and dh
- Bouyancy
  - Net pressure force on a submerged body.
  - Force is upward equal to the weight of displaced fluid.
  - Can neglect bouyancy if account for forces (body/pressure) as before (or rather, we are accounting for it directly).

Class 5—Math/RTT

- Scalar, vector, tensor (basic idea only).
- Dot product
- Mass flux = ρA**v . n** = ρAv*cos θ
- Lagrangian system: follow some fixed mass: moves/deforms
  - No mass crosses boundary.
  - Conservation laws defined for Lagrangian systems
- Eulerian control volume: consider some (fixed) space
  - Convenient for engineering analysis
- Material derivative/substantial derivative
  - (Rate of change following the flow) = (rate of change at a fixed point) + (rate of change due to change with position)

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}$$

Class 5—Math/RTT

- Reynolds Transport Theorem
  - Relates a Lagrangian system for which a conservation law applies, to an Eulerian Control Volume, which we want a conservation law for.

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b dV + \int_{A} \rho b \vec{v} \cdot \vec{n} dA$$

  - B is mass, b = 1 for mass conservation.
  - B is total energy E, b = total energy per mass for energy conservation.
  - When in doubt (especially when things are not simple/uniform), start with this equation and simplify.
  - Usually apply for fixed control volumes, but works for variable control volumes too.
  - Related to the substantial derivative, but the S.D. is not finite like the integral form, so it has no deformation component.

Class 6—Mass Balance

- RTT → B = M, b = 1.
- Conservation of mass → dB/dt = 0.
- RTT → $$0 = \frac{d}{dt}\int_{CV}\rho dV + \int_{A}\rho\vec{v}\cdot\vec{n}dA$$
- Know how to use this equation in terms of various assumptions.
  - Steady, constant volume, constant density, single streams.
- Steady state → drop the first term
- Uniform within the control volume: first term → dM/dt
- Single inlet and outlet, uniform ρ, average v across outlet: second term → $\dot{m}_{out} - \dot{m}_{in}$
- Density for ideal gases: ρ=MP/RT
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Class 7—Integral Energy Balance

- RTT → B=E, b=e, e = u+v^2/2+gz
- Conservation law: dE/dt = dQ/dt + dW/dt
  - 1st law of thermodynamics

$$\frac{dQ}{dt} + \frac{dW}{dt} = \frac{d}{dt}\int_{CV} \rho(u + \frac{1}{2}v^2 + gz)dV + \int_{CS} \rho(u + \frac{1}{2}v^2 + gz)\vec{v}\cdot\vec{n}dA$$

  - Work is shaft and flow work. Split these up, and also assume uniform properties and single streams. u+P/ρ = h

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\rho(u + \frac{1}{2}v^2 + gz)V\right] + \left[\rho vA(u + \frac{P}{\rho} + \frac{1}{2}v^2 + gz)\right]_{out} - [\,]_{in}$$

  - Know how to simplify the problem for various assumptions
    - Steady state, constant density, shaft work, etc.
- Mechanical energy
- Efficiency

Class 8—Bernoulli Equation

$$\Delta\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)=0$$

- Mechanical energy is conserved
- Recall assumptions
  - SS, frictionless, const ρ, no W_s, no Q, streamline
- Energy form, pressure form, head form
- Pitot tubes → velocity measurement
- Grade lines (head form)
- Version across streamlines, compressible

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Class 9—Bernoulli Applications

- Problem solving approach
- Tank problem
- Water faucet problem
- Convert between P, v, z
- Often mix mass and energy balances to find desired quantities.
- How to pick the control volume.

Sample Problem #1

$$\rho = \rho_0 t^2$$

You are Kaladin Storm-blessed, and you, above all odds, are standing above the unconscious body of Elhokar Kholin, ready to save his life! All hope seems lost, until you realize that you ***know*** the words of the second windrunner oath! As you utter the words (causing readers everywhere to stand up and cheer) you realize that Stormlight will be surging into your body. The greater the time of Stormlight that is in you, the greater the density of the Stormlight (it's square the time passed times original density). The Stormlight (S_L) enters your body through your mouth in a uniform velocity profile (v) perpendicular to your mouth plane, (your mouth is open fully wide throughout), and is evenly distributed throughout your body. Derive an expression for the rate of change of Stormlight in your body.

RTT $\quad \dfrac{dB_{sys}}{dt} = \dfrac{d}{dt}\int_{cv} \rho \dfrac{S_L}{m}\, dV + \int \rho \dfrac{S_L}{m} \cdot \vec{V} \cdot \vec{n}\, dA$

$$B_{sys} = S_L \quad b = \frac{S_L}{m} \quad \rho_0 t^2$$

$$-\rho V \frac{S_L}{m} A$$

Sample Problem #1 (cont.)

$$\frac{dS_L}{dt} = \rho \frac{s_L}{m} \int dV$$

$$\boxed{0 = \frac{d\,\rho_0 t^2 \frac{s_L}{m} V}{dt} + -\rho v \frac{s_L}{m} A}$$

Sample Problem #2

You are James Bond, international Superspy, and you have discovered the location of the secret uranium enrichment plant in Russia, and need to signal the airstrike. You are unable to ignite anything, so you have the bright idea of piercing one of three external horizontal pipelines on the ground where you are to create a vertical stream of fluid to signal the low-flying aircraft. Which of the following 3 pipes do you puncture, and why?

- Bromine Pipe; stagnant at 10 MPa, density of 3 Mg/m^3
- UF_6 Pipe; flowing at 20 m/s, 5 MPa, density of 509 kg/m^3
- Water Pipe; flowing at 5 m/s, 500 kPa

$\frac{P}{\rho} = gz$

Example problem #3

Turns out Jupiter has a HUGE amount of hydrogen, and you want to mine the planet to make some money. You set up a teleportation system that operates based on pressure forces; so long as the pressure exerted on the system is >9,050,000 atm, you can beam hydrogen to earth. If the Temperature (250K), gravity coefficient (24.79m/s^2), density (1.326g/cm^3), and atmospheric composition (hydrogen) of Jupiter is constant, how far below top of the atmosphere would you need to establish your system?

$$\frac{dP}{dz} = -\rho g \qquad \Delta P = -\rho g \Delta z \qquad \frac{\Delta P}{-\rho g} = \Delta z$$

$$\int dP = -\frac{MP}{RT} g \, dz$$

Sample Exam Problem

- What are the four conditions (cases or assumptions) needed to take the RTT with mass as the property to the Continuity equation we derived in class?

SS

const ρ

Uniform prop

fixed CV