Chemical Engineering 374

Fluid Mechanics

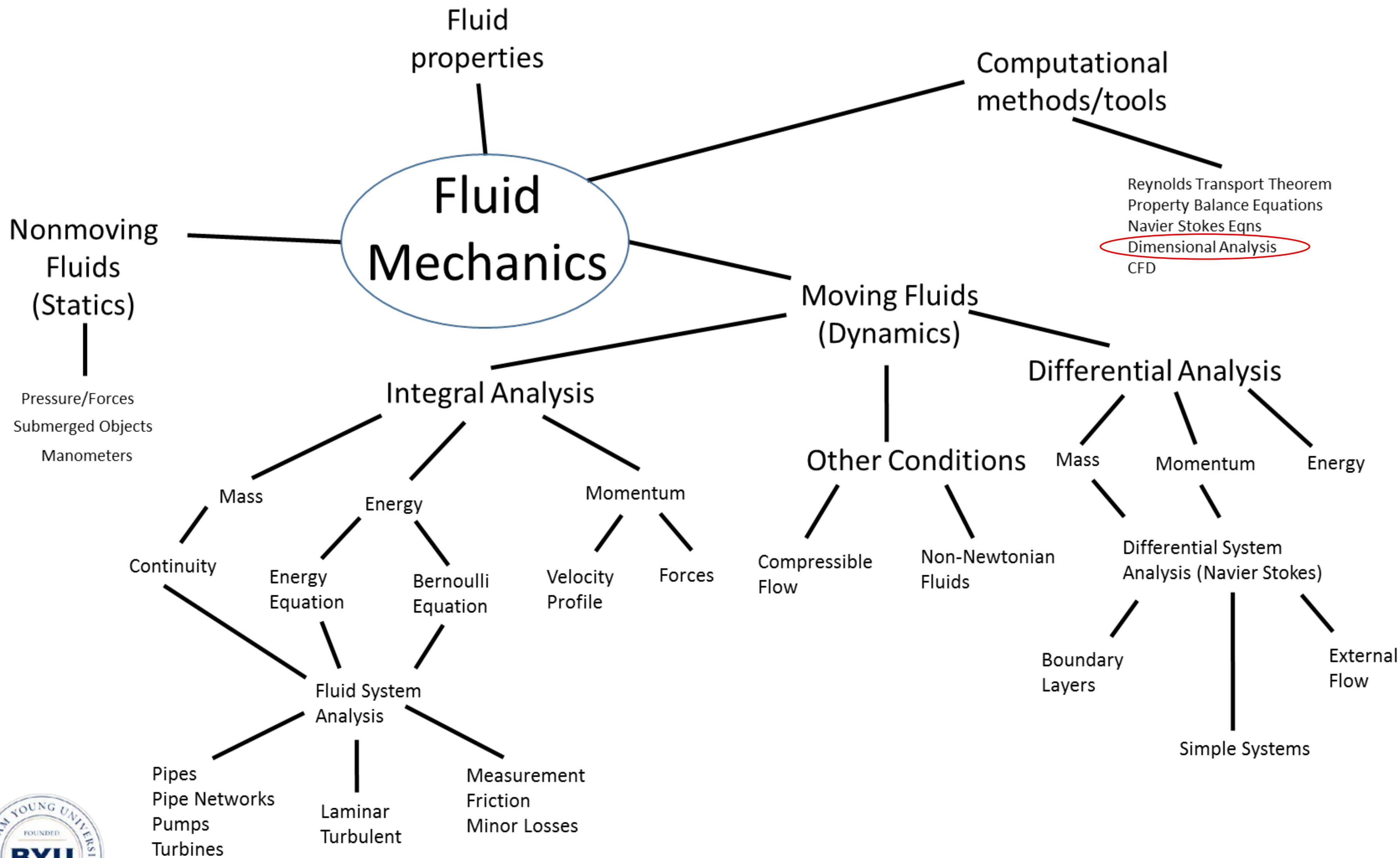
Dimensional Analysis

Spiritual Thought

"It's not enough, however, merely to believe in Him, and his mission. We need to work and learn and search and pray, ***repent, and improve***."

President Thomas S. Monson

Fluids Roadmap

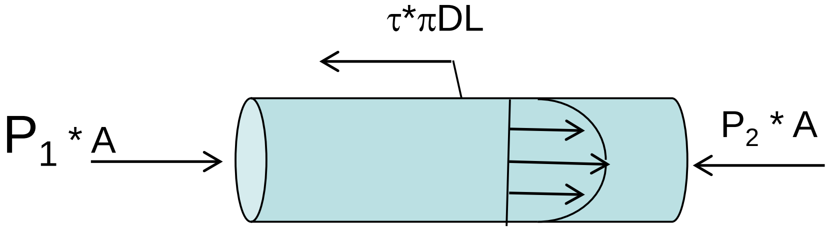


Key Points

- Introduction and Motivation
  - Theory
    - Limitations
  - Experiments
    - Cost, practicality, efficiency
    - Qualitative understanding
- Dimensionless groups
  - Eliminate the units
  - Methods
    - Governing Equations
    - Force Ratios
    - **Π Method**
  - Examples
- Similarity / Scale models

Pipe Flow

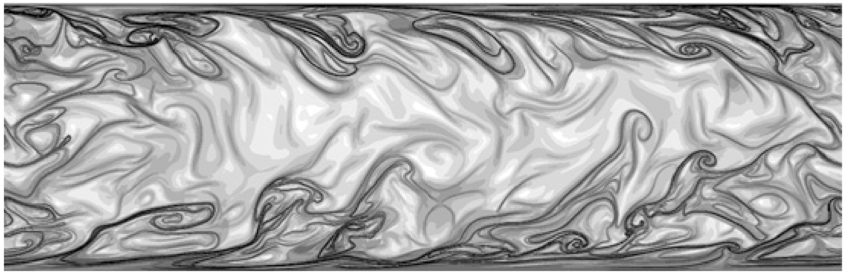
- Consider flow in a pipe
  - (We'll do lots of this!)
- So far, mostly frictionless flow
  - (Friction next week)
  - But we've discussed friction on a plate for Newton's law of viscosity: $\tau=\mu*du/dr$.
  - Friction force is balanced by pressure forces:
    - Pressure drop Δp balanced by friction

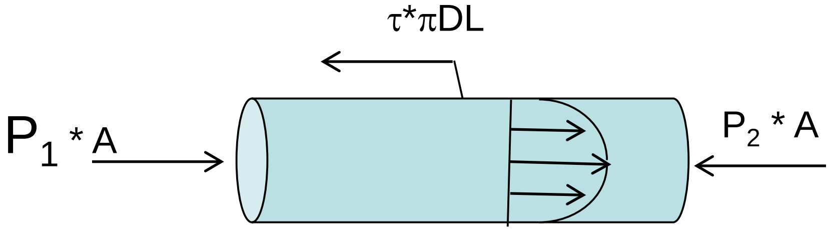


- Most flows are turbulent (consider averages)
- **Question/Goal:** How to characterize pipe flow?

The Challenge

- Turbulent flow is complex
  - Random, Chaotic
- How to measure and relate the important properties...
  - List them:





- v, ΔP, L, D, μ, ρ

Or

- v, ΔP/L, D, μ, ρ

Flow Examples (I)

Flow Examples (II)

Solutions

- Solve Governing Equations?

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} &= 0 \\
\frac{\partial \rho u}{\partial t} + \frac{\partial (\rho u^2)}{\partial x} + \frac{\partial (\rho u v)}{\partial y} + \frac{\partial (\rho u w)}{\partial z} &= -\frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\
\frac{\partial \rho v}{\partial t} + \frac{\partial (\rho u v)}{\partial x} + \frac{\partial (\rho v^2)}{\partial y} + \frac{\partial (\rho v w)}{\partial z} &= -\frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\
\frac{\partial \rho w}{\partial t} + \frac{\partial (\rho u w)}{\partial x} + \frac{\partial (\rho v w)}{\partial y} + \frac{\partial (\rho w^2)}{\partial z} &= -\frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \\
\frac{\partial \rho E}{\partial t} + \frac{\partial (\rho u E)}{\partial x} + \frac{\partial (\rho v E)}{\partial y} + \frac{\partial (\rho w E)}{\partial z} &= -\frac{\partial p u}{\partial x} - \frac{\partial p v}{\partial y} - \frac{\partial p w}{\partial z} + S
\end{aligned}
$$

  - (Yes, but its expensive!)
- Experiments?
  - Design Parameters?
  - Span the Space.
  - How many experiments to do?
    - Measure $\Delta P/L$, for a range of D, v, μ, ρ?
    - If 10 points in each of D, v, μ, ρ then have 10,000 experiments to do
  - Lets be smart! (be efficient)
- Dimensional analysis

Dimensional Analysis

- Dimensional analysis is a powerful tool
  - Insight into equations
  - Relationships among parameters
  - Reduce the number of parameters in a system
  - Obtain scaling laws
- Concept: Nature does not "know" about our units.
  - kilogram, pounds
  - eV, Joules
  - seconds, hours
- Nature only knows about fundamental dimensions, and **relative sizes** with those dimensions.
  - When we assign values to those dimensions or combinations of those dimensions, we do so relative to our own contrived scales.

Find Dimensionless Groups

- Not all parameters are independent
- Pipe flow:
  - A flow with v=4, D=1 is the SAME, as a flow with v=2, D=$\sqrt{2}$
    - (all else equal).
  - A flow with μ=20, ρ=1.2 is the SAME as a flow with μ=40, ρ=2.4
- Find the independent parameters, the dimensionless groups.
- 3 Methods
  - Governing Equations
  - Force Ratios
  - Π method

Governing Equations

- Find relevant dimensionless groups from a governing equation.
- Bernoulli equation: $$\frac{P}{\rho} + \frac{v^2}{2} + gz = C$$
  - Divide by one of the terms like $v^2/2$

$$\underbrace{\frac{P}{\rho v^2/2}}_{C_p} + 1 + \underbrace{\frac{gz}{v^2/2}}_{2/Fr^2} = \frac{C}{v^2/2}$$

Force Ratios

- Take important forces in a problem and form ratios of them
- B.E. → pressure, gravity, momentum (velocity) forces.
  - Pressure → P*A
  - Gravity → mg = ρAzg
  - Momentum → $\rho Av^2/2$
    - (recall stagnation flow converts pressure force to velocity)
  - Form ratios → same as for method of governing equations

Π Method

- General and systematic approach
- Simple to do, but lots of conditions and rules to be general
  - WARNING, your book is very confusing here.
- Method for pressure drop in a pipe.
  1) List the parameters and variables, along with their symbols & units:
     - $\Delta P/L$ kg/m³*s²
     - D m
     - ρ kg/m³
     - μ kg/m*s
     - v m/s

     $\rightarrow$ n_{var}=5 variables
  2) Count the number of dimensions $\rightarrow$ $j_{dim} = 3$ (kg, m, s)
  3) # of Π's $\rightarrow$ $k_\Pi = n_{var} - j_{dim} = 5-3 = 2$
     - That is, I have 5 vars, but how many are independent? Nature doesn't regard our units, so, the problem has to be non-dimensional, so I have 5 variables, but I have to get rid of 3 dimensions (units), so I have 3 constraints $\rightarrow$ 5-3 = 2 independent variables $\rightarrow$ 2 Π's.
  4) Now find the Π's
     - Need to include all variables among the Π's
     - The Π's need to be independent
     - Usually can find the Π's by trial (and error)
     - Be smart: if you want to find a relationship for ΔP, then don't put ΔP in all the Π's.

Get the Π's

- Vars:
  - ΔP/L kg/m²*s²
  - D m
  - ρ kg/m³
  - μ kg/m*s
  - v m/s
- Π_1:
  - Start with ΔP/L and get rid of its units using the other variables:

$$\frac{\Delta P}{L}\,(=)\,kg/m^2s^2 \mid\ \div\rho \rightarrow \frac{\Delta P}{\rho L}\,(=)\,m/s^2 \mid\ \div v^2 \rightarrow \frac{\Delta P}{\rho v^2 L}\,(=)\,1/m \mid\ \times D \rightarrow \frac{D\Delta P}{\rho v^2 L}$$

- Now try Π_2:

$$\mu\,(=)\,kg/ms \mid\ \div\rho \rightarrow \frac{\mu}{\rho}\,(=)\,m^2/s \mid\ \div v \rightarrow \frac{\mu}{\rho v}\,(=)\,m \mid\ \div D \rightarrow \frac{\mu}{\rho v D}$$

- The Π's are nondimensional so they are not unique:
  - $\Pi_2 \leftarrow \Pi_1\Pi_2$ is okay (that is, replace Π_2 with $\Pi_1\Pi_2$)
  - Π to any power is okay → our $1/\Pi_2$ above IS VERY SPECIAL: **Re = ρDv/μ**
  - Π_2 is also special (but we'll talk about it later).

More General

- We found Π by inspection, (pretty easy).
- "Repeating variables" approach (book gives lots of rules):
  - $n_{var} = 5$, $j_{dim}=3$, $k_{\Pi}=2$
  - Pick j_{dim} repeating vars that will show up in all Πs
    - D, ρ, v
  - Form the two Πs using the two leftover vars (one var in each Π)
    - $\Delta P/L$ and μ
- Π_1:

$$\frac{\Delta P}{L} \cdot D^a \rho^b v^c \;(=)\; kg m^{-2} s^{-2} \cdot m^a kg^b m^{-3b} \cdot m^c s^{-c}$$

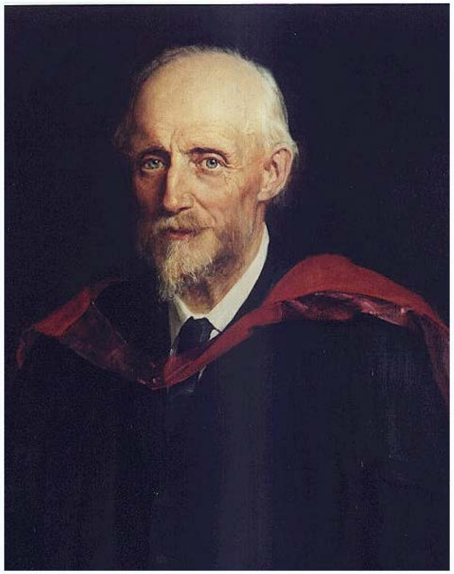
$$(=)\; kg^{1+b} \cdot m^{a+c-3b-2} \cdot s^{-2-c}$$

  - Now, select a, b, c so units cancel (powers=0) → $kg^0 = 1$
  - kg: $1+b = 0$
  - m: $a+c-3b-2 = 0$
  - s: $-2-c = 0$

→ 3 eqn in 3 unknowns → a=1, b=-1, c=-2
→ $\Pi_1 = D\Delta P/\rho L v^2$ as before.

Reynolds Number

- Dimensionless groups reprent the ratio of two physical phenomena.
- Reynolds number
  - Most important in fluid mechanics
  - Re = $\rho Lv/\mu$.
  - Ratio of inertial and viscous forces
    - (or timescales or lengthscales).
- Osborne Reynolds: 1842-1912
  - British engineer.
  - Many advances in fluid mechanics
  - Pipe flow: laminar/turbulent transition.



Applications—Similarity

- Why do animals have the shape and size they do?
- Are giants are practical?

Similarity

- As chemical engineers, we design processes and plants.
  - Use governing equations.
  - These equations are often inadequate
    - Can’t always solve.
    - Don’t always know the equations.
    - Reality is often too complex.
  - Do experiments.
    - Cost is high → small scale, then scale up.
    - Make sure consistent at the two scales.
    - “Have your failures on a small scale, in private; have your successes on a large scale, in public!”
  - 3 similarity requirements
    - Geometric (shape)
    - Kinematic (velocities)
    - Dynamic (Forces)
  - Find the dimensionless groups, and make sure they are the same for the model and the scale versions.
    - Dimensionless groups don’t need the full model, only the important parameters.