OEP (I)

Buzz Lightyear, the heroic Space Ranger from Star Command, is on an urgent quest to find and capture the evil Emperor Zurg before he can utilize his world-destroying tembler bomb, thus ending life in the Universe as we know it. He chased Zurg to a mystical and somewhat backward world known as Hyrule, but was there held up by some kid with a sword who was named after part of a chain. After convincing the youth that he was a friend, and promising that he had no idea why 3 triangles were so important, Buzz discovered that Zurg's hideout could be reached through a vent at the bottom of a deep chasm in the ocean. By following a strong current within this vent, Buzz is able to float quickly to the center of the planet. Just as Buzz is about to reach Zurg's hidden base, the tembler bomb activates, and will destroy the planet within 2 minutes. Buzz has 4 paths to choose from in order to reach Zurg's hideout at the planet's watery center in time to diffuse the bomb, as indicated by the drawing below. Which path provides the best chance of reaching Zurg's base in time, and why? (i.e. prove it...)

OEP (II)

- What is the problem asking for?

The shortest time to get to Zurg's base. i.e. $t = L/V \Rightarrow$ shortest pathlength to velocity ratio

- What assumptions are you making in order to solve the problem?

ρ = constant, (liquid), No shaft work (No pumps or turbines)

Picture is to scale...

$D_1 = 0.25$ cm $D_2 = 0.75$

$D_3 = 0.5$ $D_4 = 1$ cm (from part d,

1 cm = 1000 m

$K_1 = 1.5$ $K_2 = 2.35$ $K_3 = 3.5$ $K_4 = 9.2$

Fully turbulent flow, $f \sim$ constant

$P_{surface} = 1$ atm $g = 9.8\, m/s^2$

Buzz has plenty of O_2

SS, uniform properties, $D \sim$ constant in each pipe, etc.

OEP (III)

- Provide an answer based on your assumptions

4 pipes - $D_1 = 250m$, $D_2 = 750m$, $D_3 = 500m$, $D_4 = 1000m$

$L_1 = 6000m$, $L_2 = 2500m$, $L_3 = 3000m$, $L_4 = 6500m$

$$\Delta P_1 = \Delta P_2 = \Delta P_3 = \Delta P_4 = \Delta P_{total} \therefore \left(f\frac{L_1}{D_1} + K_1\right)\frac{\rho v_1^2}{2} = \left(f\frac{L_2}{D_2} + K_2\right)\frac{\rho v_2^2}{2} = \left(f\left(\frac{L_3}{D_3}\right) + K_3\right)\frac{\rho v_3^2}{2}$$

$$= f\left(\frac{L_4}{D_4} + K_4\right)\frac{\rho v_4^2}{2}$$

(plug in values)

$$782.88 v_1^2 = 1179.6 v_2^2 = 1758.22 v_3^2 = 4608.9 v_4^2$$

Now, since we don't have absolute values, but we can do some comparisons... let's normalize these values using v_1

$$\frac{V_1}{V_1} = \sqrt{\frac{782.88}{782.88}} = 1 \quad \frac{V_2}{V_1} = \sqrt{\frac{1179.5}{782.88}} = 1.22 \quad \frac{V_3}{V_1} = 1.498 \quad \frac{V_4}{V_1} = 2.43$$

V_1 is fastest, V_4 is slowest

but, L is significant too!

L relative to $L_1 \Rightarrow \frac{L_1}{L_1} = 1$, $\frac{L_2}{L_1} = .42$, $\frac{L_3}{L_1} = .5$, $\frac{L_4}{L_1} = 1.08$

$t_1 = \frac{L_1}{V_1}$ $\quad t_2 = \frac{L_2}{V_2}$, $\quad t_3 = \frac{L_3}{V_3}$, $\quad t_4 = \frac{L_4}{V_4}$

$t_1 = 1$, $\underline{t_2 = 0.5}$, $t_3 = 0.75$, $t_4 = 2.63$

Path 2 is fastest!

- Assuming the drawing is 100,000 scale, how fast does the water in your chosen path need to be flowing in order for Buzz to make it with 30 seconds left to diffuse the bomb?

$$\Delta P = f\frac{L}{D}\frac{\rho v^2}{2} \qquad t_2 = \frac{L_2}{v_2} \qquad L_2 = 2500\,m \qquad K = 3.5$$

$$D_2 = 750\,m$$

$$v_2 = \frac{L_2}{t_2} = \frac{2500\,m}{90\,s} = \boxed{27.78\,m/s}$$

Water flows from a reservoir, into a **sharp-edged pipe** (K_L=0.5), through a couple of **90° miter bends** (K_L=1.1) to a lower reservoir as shown in the figure. The difference in the levels of the reservoirs is **H=100 m**. The water flows through a pipe of length **L=200 m** and relative roughness **ε/D=0.002**. The velocity in the pipe is **v=10 m/s**, giving a very **high Re**. If the power generated by the turbine is **W_t=4,437,500 J/s, Find the required pipe diameter.**

$$\dot{m}\left(\cancel{\frac{\Delta P}{\rho}}^{0} + \cancel{\frac{\Delta v^2}{2}}^{0} + \Delta g z\right) = \dot{W}_s - \dot{F} \qquad K_{L_{tot}} = 2.7$$

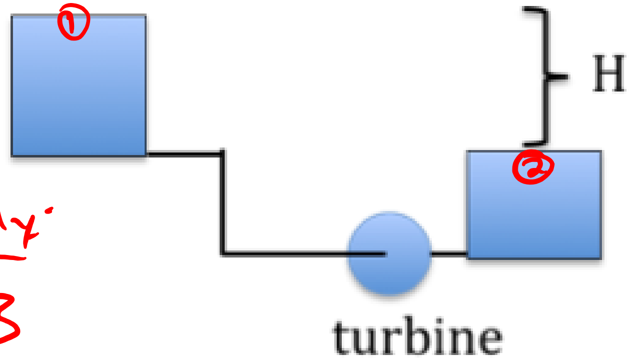
$$\Delta g z = \frac{\dot{W}_s}{\dot{m}} - \left(f\frac{L}{D} + K_L\right)\frac{v^2}{2}$$

Using Moody:

$f = .023$

solve for D →

$$\boxed{= 0.61\,m}$$



We have seen how turbulence consists of large eddies breaking into smaller eddies until the scales are small enough that viscosity dissipates the kinetic energy (mechanical energy) into internal energy (friction). A most remarkable property of turbulence was found by Kolmogorov using dimensional analysis. He argued that the small scales are determined only by the **kinematic viscosity ν (m^2/s)** and the rate that energy is fed to the small scales by the large scales—the **dissipation rate ε (m^2/s^3).**

(a) Using some combination of these two parameters, **form a length scale η (m)**, and a **time scale τ (s)** for the small scales (the Kolmogorov (or small eddy) scales).

$$\eta \Rightarrow \varepsilon = \frac{m^2}{s^3} \quad \nu^3 = \frac{m^6}{s^3} \quad \frac{\nu^3}{\varepsilon} = \frac{\frac{m^{\cancel{6}\,4}}{\cancel{s^3}}}{\frac{\cancel{m^2}}{\cancel{s^3}}} = m^4 \quad \frac{\nu^{3/4}}{\varepsilon^{1/4}} = m \quad \therefore \boxed{\eta = \sqrt[4]{\frac{\nu^3}{\varepsilon}}}$$

$$\tau \Rightarrow \frac{\nu}{\varepsilon} = \frac{\frac{\cancel{m^2}}{s}}{\frac{\cancel{m^2}}{s^{\cancel{3}\,2}}} = s^2 \quad \sqrt{\frac{\nu}{\varepsilon}} = s \quad \therefore \tau = \sqrt{\frac{\nu}{\varepsilon}}$$

(b) At high Re, the rate of energy transfer at large scales (big eddies breaking into small ones, etc.) does not depend on viscosity. **Write the dissipation rate ε as a combination of the large length scale L and the large scale velocity v.**

$$\varepsilon = \frac{m^2}{s^3} = \frac{v^3}{L} = \frac{m^3}{m \cdot s^3} = \frac{m^2}{s^3} \Rightarrow \boxed{\varepsilon = \frac{v^3}{L}}$$

(c) Now, find η in a 10 cm diameter pipe (that is, L=D=10 cm) at Re = 5,000?

$$Re = \frac{\rho v D}{\mu} = \frac{vD}{\nu} = 5000 \Rightarrow \frac{v}{\nu} = 500 \qquad \nu^{3/4} = \frac{v^{3/4}}{500^{3/4}}$$

$$\eta = \frac{\nu^{3/4}}{\varepsilon^{1/4}} \qquad \varepsilon^{1/4} = \frac{v^{3/4}}{L^{1/4}} \qquad \frac{\nu^{3/4}}{L^{1/4}} = \frac{\frac{\cancel{v^{3/4}}}{500^{3/4}}}{\frac{\cancel{v^{3/4}}}{L^{1/4}}} = \frac{L^{1/4}}{500^{3/4}}$$

$$\boxed{\eta = 0.017}$$

- An orifice with a 1.8in-diameter opening is used to measure the mass flow rate of water at 60 ˚F (ρ = 62.38 lbm/ft^3 and μ = 7.536E-4 lbm/ft-s) through a horizontal 4 in-dimeter pipe. A mercury manometer is used to measure the pressure difference across the orifice. If the differential height of the manometer is 7 in, determine the volume flow rate of water through the pipe, the average velocity and the head loss caused by the orifice meter.

$\beta = \frac{d}{D} = \frac{1.8}{4} = 0.45$ $\quad A_0 = \pi d^2/4 = \pi\left(\frac{1.8}{12}ft\right)^2/4 = 0.0177 ft^2$

Guess $C_d = 0.61$

$\Delta P = P_1 - P_2 = (\rho_{Hg} - \rho_f) g h$

$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1-P_2)}{\rho(1-\beta^4)}} = 0.2393\ ft^3/s$ $\quad V = 2.742\ ft/s$

$h_L = \frac{\Delta P}{\rho g} - \frac{\Delta V^2}{2g} = 5.2\ ft\ H_2O$ $\quad Re = \frac{\rho V D}{\mu} = 7.56\times10^4$

$C_d = 0.6042 \rightarrow$ Iterate

$\boxed{\dot{V} = 0.237\ ft^3/s,\ V = 2.72\ ft/s,\ h_L = 5.26\ ft\ H_2O}$

In the pipe network shown in the figure, the total flow rate is 15.
Find the flow rate and pressure drop through pipe c.
The system demand curves for each pipe INDIVIDUALLY are shown in the figure.
Show your work for full/partial credit.