Chemical Engineering 374

Fluid Mechanics

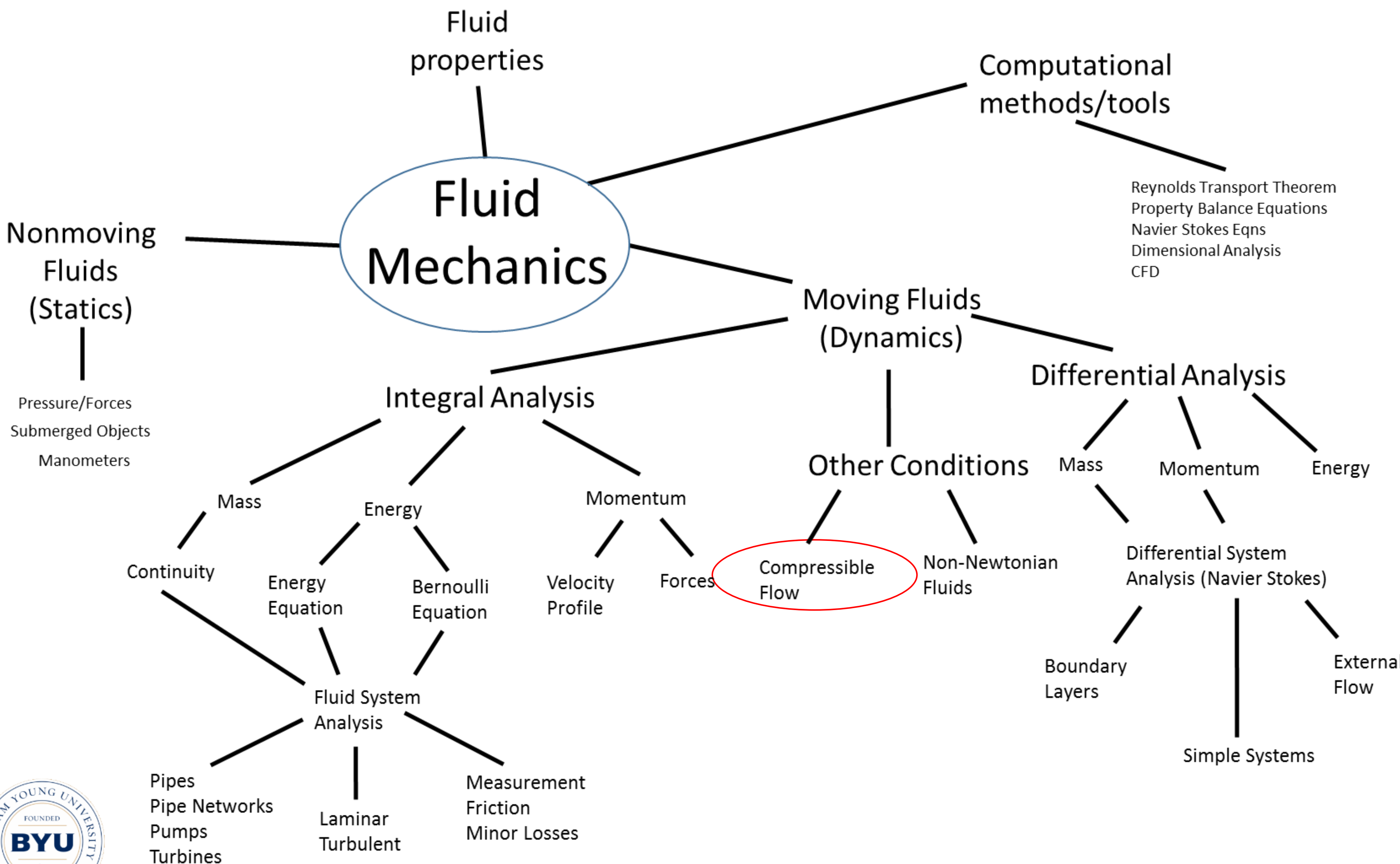
Compressible Flow

Spiritual Thought

John 11:35

Jesus wept.

Fluids Roadmap



Key Points

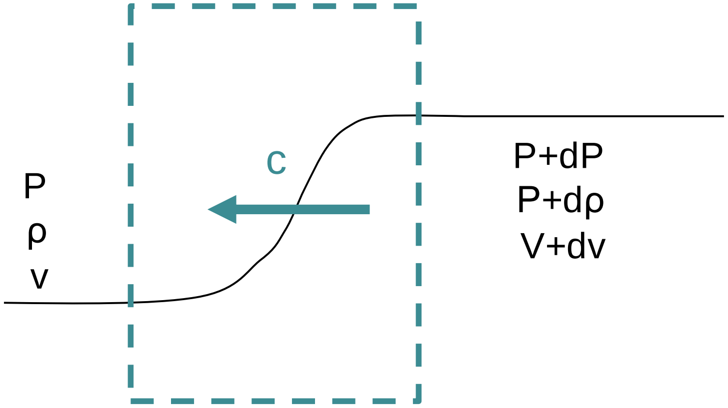
- Characteristics
  - Converts u to kinetic energy
  - T drops, ρ drops
  - v greater than in non-compressible flow
  - M
- Equations
- Nozzles
- Choked Flow

Compressible Flows

- Previously, ρ was constant
- Now, ρ varies
  - High speed flow, large ΔP, ΔT, Δv, etc.
  - Internal energy is converted to kinetic energy
    - T Drops; ρ drops
    - v > v of Bernoulli Eqn. where ρ is constant
  - Mach number, M = v/c; c is ***local*** value, [c(T)]
  - Compressible for M > 0.3, ρ/ρ ~ 0.95,

Sound Speed

- Pressure Pulse Analysis



- Mass Balance
- Momentum Balance
- Expression for c
- Error

- Mass Balance

$$\dot{m}_{in} = \dot{m}_{out} \Rightarrow \rho A v = (\rho + \Delta\rho) A (v + \Delta v)$$

$$\cancel{\rho v} = \cancel{\rho v} + \rho dv + v d\rho + \underbrace{\cancel{d\rho dv}}_{\sim 0}$$

$$\rho dv = -v d\rho$$

- Momentum Balance → SS, only Pressure Forces

$$AP - A(P + dP) = \dot{m}(v + dv) - \dot{m}V \Rightarrow (\dot{m} = \rho A v)$$

$\dot{m} = \rho A v$, A cancels

$$-dP = \rho v dv \Rightarrow \boxed{\rho dv = -\frac{dP}{v}}$$

$$\frac{dP}{d\rho} = v^2 = c^2 \rightarrow c = \sqrt{\frac{dP}{d\rho}}$$

$$P = \frac{\rho R T}{M}\,;\quad \frac{dP}{d\rho} \rightarrow \left(\frac{\partial P}{\partial \rho}\right)_T \text{ or } \left(\frac{\partial P}{\partial \rho}\right)_s$$

$$\boxed{c = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s} = \sqrt{\frac{kRT}{M}}}$$

- Sound waves compress
  - No heat transfer
  - Isentropic
  - NOT isothermal

$$c = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s} = \sqrt{\frac{kRT}{M}}$$

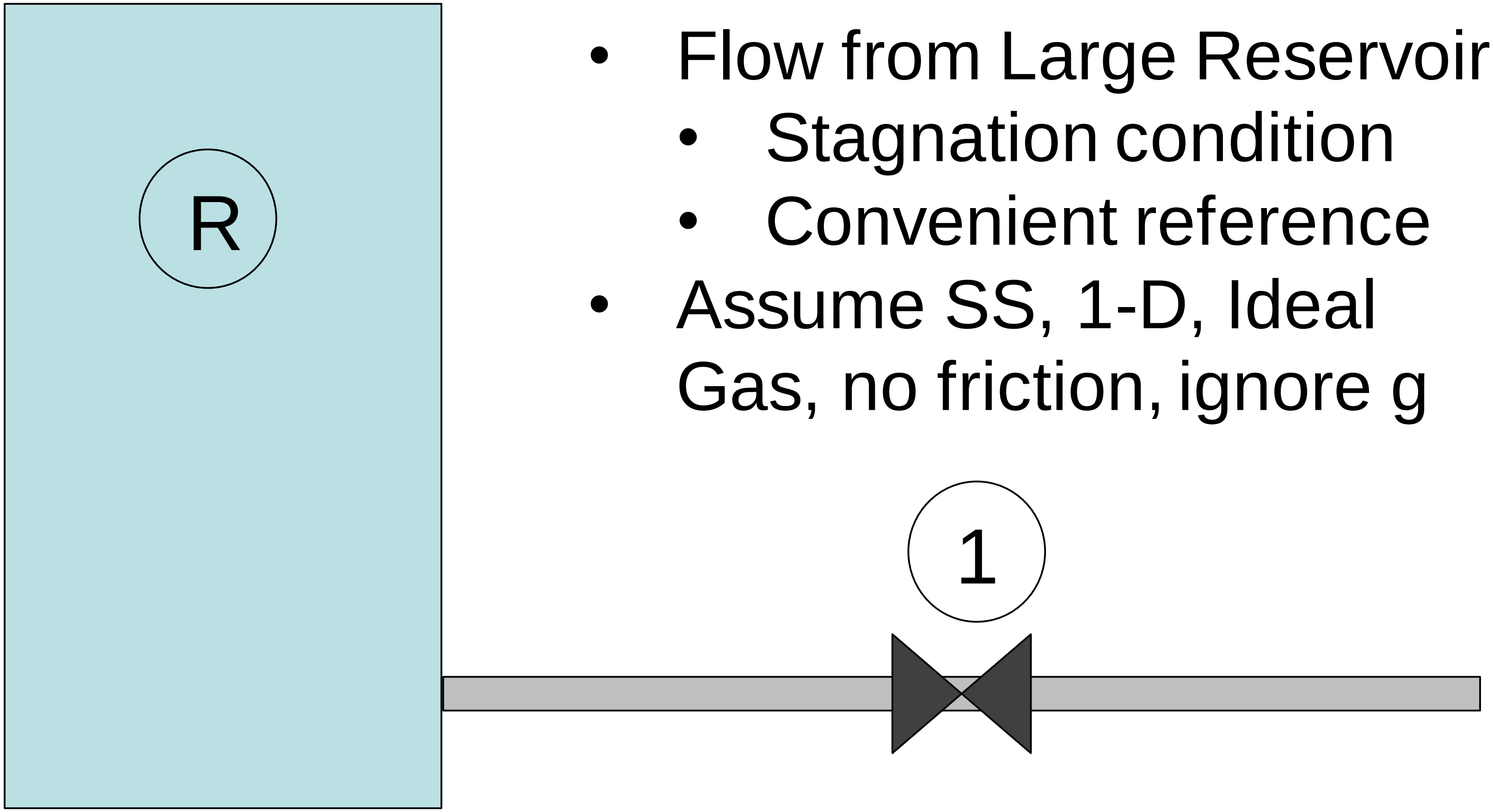
$$k = {}^{C_p}\!/_{C_v} = 1.4 \text{ for air}$$

Common Values of c

	c (m/s)	c (mph)
Air @ 298 K	346	774
Air @ 2000 K	896	2000
H_2O @ 288 K	1490	3333
Steel @ 188 K	5060	11318

Compressible Flow

- Flow from Large Reservoir
  - Stagnation condition
  - Convenient reference
- Assume SS, 1-D, Ideal Gas, no friction, ignore g

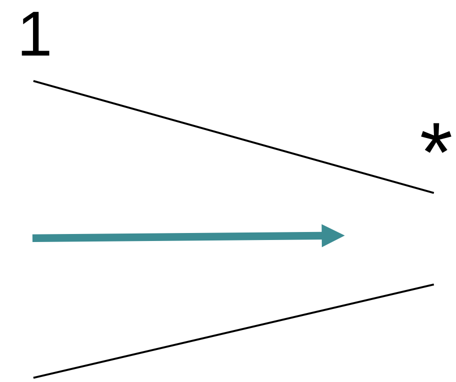


- Energy Balance
- $\cancelto{0}{\dot{Q}} + \cancelto{0}{\dot{W}_s} = \dot{m}\left(\overbrace{\frac{P}{\rho} + u}^{h} + \cancelto{0}{\frac{v^2}{2}} + \cancelto{0}{gz}\right)_R - \dot{m}\left(\overbrace{\frac{P}{\rho} + u}^{h} + \frac{v^2}{2} + \cancelto{0}{gz}\right)_1$
- $v_1 = 2(h_r - h_1) = 2C_p(T_r - T_1)$
- Given: $C_p = C_v + \frac{R}{M} \rightarrow k = {}^{C_p}/_{C_v} \rightarrow C_v = {}^{C_p}/_{k}$
- $C_p = \frac{Rk}{M(k-1)}$
- $c^2 = \frac{kRT}{M}$
- $\frac{v_1^2}{c_1^2} = M^2 = \frac{2}{k-1}\left(\frac{T_r}{T_1} - 1\right)$
- $\boxed{\frac{T_r}{T_1} = \frac{M^2(k-1)}{2} + 1}$

- Ideal Gases: (thermo, next semester…)
- $\frac{P_r}{P_1} = \left(\frac{T_r}{T_1}\right)^{k/(k-1)}; \frac{\rho_r}{\rho_1} = \left(\frac{T_r}{T_1}\right)^{1/(k-1)}$
- Thus,
- $\frac{P_r}{P_1} = \left(M^2 \frac{k-1}{2} + 1\right)^{k/(k-1)}$
- $\frac{\rho_r}{\rho_1} = \left(M^2 \frac{k-1}{2} + 1\right)^{1/(k-1)}$

Nozzles

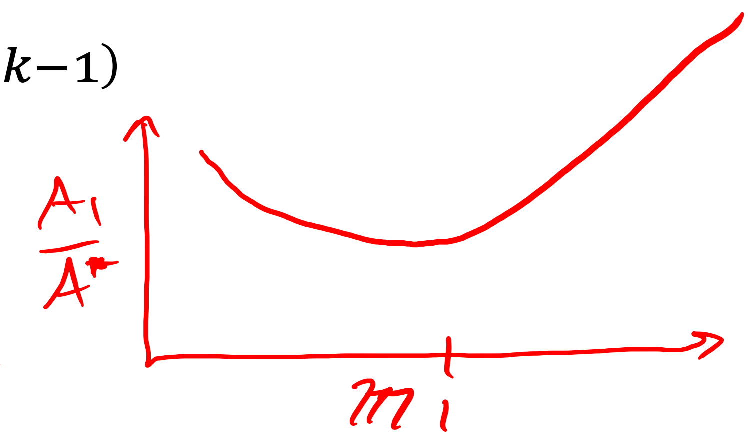
- Mass flow = constant
- As A decreases, v increases
- Eventually v = c at (*)
  - $A_1 \rho_1 v_1 = A^* \rho^* v^*$
  - $A_1 / A^* = \rho^* v^* / \rho_1 v_1$



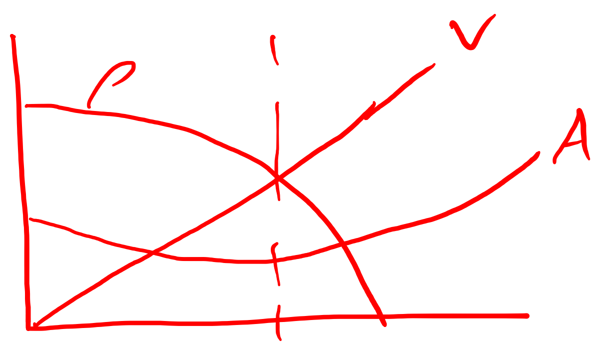
- $$\frac{A_1}{A^*} = \frac{1}{M_1}\left[\frac{M_1^2\frac{(k-1)}{2}+1}{\frac{(k-1)}{2}+1}\right]^{(k+1)/2(k-1)}$$

$M = \frac{v}{c}$

plot



- For M<1, as velocity increases, A decreases
- For M > 1, as velocity increases, A increases!
  - Because ρ is decreasing faster than A is increasing → v increase to keep $\dot{m} = \rho A v$ constant



Choked Flow

- Flow Changes “communicated” at speed of sound
- i.e. pressure waves – travel at c
- If flow speed increases to c, then communication wave can’t get back to reservoir
  - Shouting in wind
  - “Communication” doesn’t occur
- Flow doesn’t increase not matter how low P_2 becomes
  - Choked flow
  - Gas Control valves, orifices, vacuum systems, bike tires
  - Choke point is minimum area in the nozzle/valve
- Flow is constant until ΔP decreases below Critical pressure