Chemical Engineering 374

Fluid Mechanics

Fluid Statics
Key Points

- Statics – No fluid motion
- Pressure characteristics
  - Scalar, all directions, normal to surfaces, units
- Barometric Equation
- Derivation
  - Constant properties
  - Non-constant properties
- Examples
Pressure

• \( P = \frac{F}{A} \)

• Isotropic – same in all directions
  – Force depends on area – next lecture

• Units
  – \( \frac{F}{A} = \frac{N}{m^2} = \text{Pa} = \frac{kg}{m/s^2} \text{ or } \frac{\text{lbf}}{\text{in}^2} = \text{psi} \)

• Absolute

• Gauge
Pressure Derivation

- Start with slice of fluid; FORCE BALANCE!
- x-direction
- y-direction
- how many Forces? 3

\[ \Sigma F_x = 0 \Rightarrow \text{some on both sides} \]
\[ \Sigma F_y = 0 \Rightarrow \text{weight} \]
\[ \text{top} \ p \]
\[ \text{bottom} \ p \]
\[ 0 = -P_1 A + P_2 A - \rho g h A \]

\[ P_1 - P_2 = -\rho g h \]
\[ \Delta P = P_2 - P_1 = \rho g h \]
What if $\rho$ isn’t constant?

- Same equation, but shrunk to infinitesimal point
- $\Delta P = \pm \rho g h$
- $h = z_1 - z_2$
  
  $= -\Delta z$
- $\Delta P = - \rho g \Delta z$
- $\lim \Delta z \to 0$
Pressure Intuition
Examples

 Turns out there are aliens at the bottom of the marina trench (10,994 m)! We of course want to go say “hi”, Abyss-style, but need to make sure our sub can get there. What should the design pressure of our sub be?

\[ \Delta P = \rho gh = (1000 \, \text{kg/m}^3)(9.8 \, \text{m/s}^2)(10,994 \, \text{m}) \]

\[ \Delta P = 1063 \text{ atm} \]
Example 2

• Develop and expression for the pressure in the atmosphere as a function of elevation:
  - $P(z)$
  - Where $P(0) = P_{\text{atm}}$.
  - Assume constant temperature $T$.
  - Use $\frac{dP}{dz} = -\rho g$

• Strategy
  - Go with what you know:
  - What do you expect to happen?
  - Draw a picture
  - What is an expression or law that relates the physics?
  - What additional information or assumptions are needed.
  - Talk to your neighbor 😊

Hint: Use $\rho = \frac{MP}{RT}$

$$\frac{dP}{dz} = -\rho g \Rightarrow \frac{dP}{dz} = -\frac{MP}{RT}g$$

$$\begin{aligned}
\int_{P_{\text{atm}}}^{P} \frac{dP}{P} &= -\alpha \int_0^z dz \\
\Rightarrow \ln \frac{P}{P_{\text{atm}}} &= -\alpha z \\
\Rightarrow P &= P_{\text{atm}} e^{-\alpha z}
\end{aligned}$$

$$P = P_{\text{atm}} e^{\left(-\frac{Mg}{RT} z\right)}$$
Temperature, Pressure in Atmosphere

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