Chemical Engineering 374

Fluid Mechanics

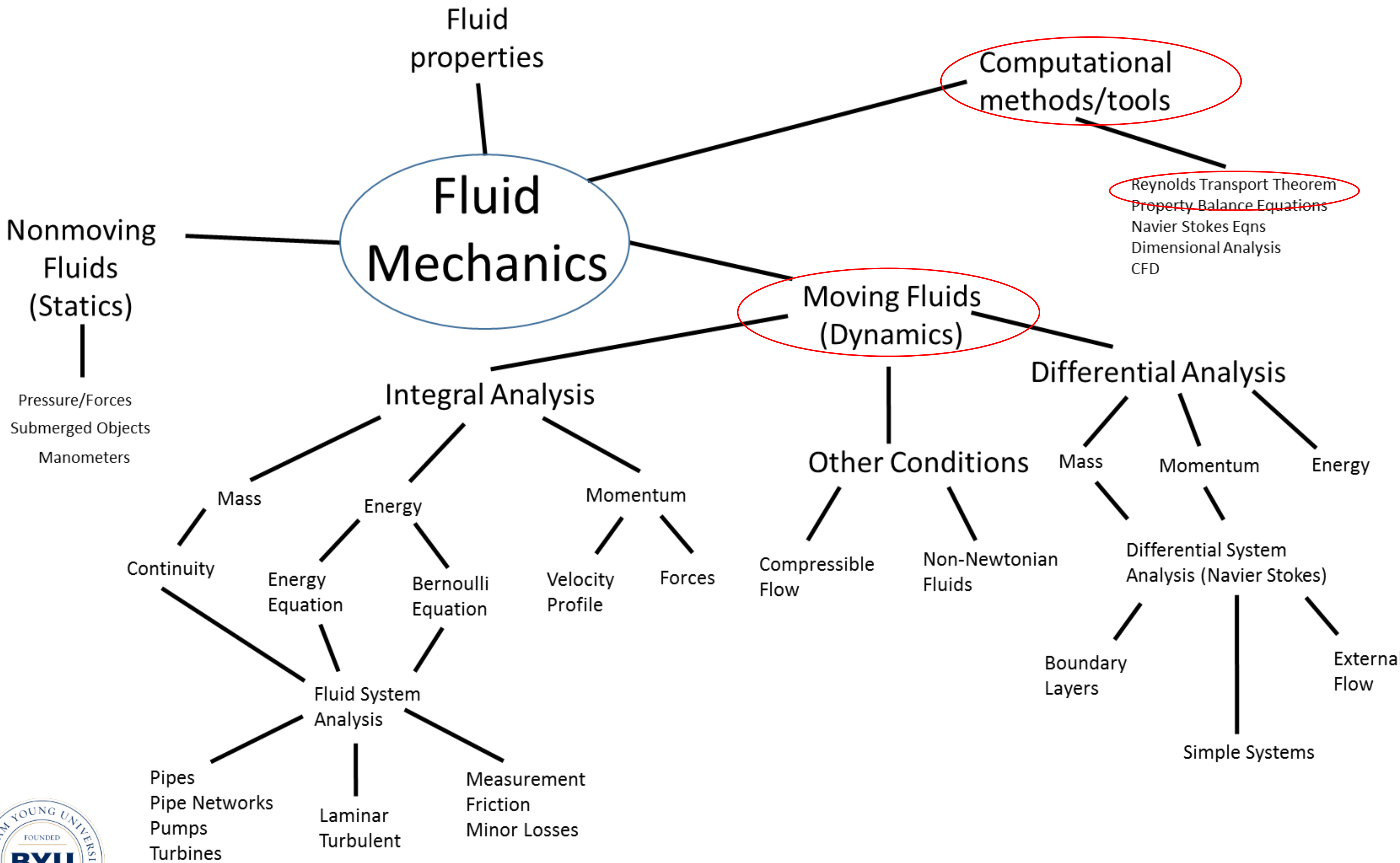
Math Tools for Moving Fluids

Spiritual Thought

"That is the second lesson of the spirit of revelation. After you have gotten the message, after you have paid the price to feel his love and hear the word of the Lord, "go forward." Don't fear, don't vacillate, don't quibble, don't whine. You may, like Alma going to Ammonihah, have to find a route that leads an unusual way, but that is exactly what the Lord was doing here for the children of Israel. Nobody had ever crossed the Red Sea this way, but so what? There's always a first time. With the spirit of revelation, dismiss your fears and wade in with both feet. In the words of Joseph Smith, "Brethren [and, I would add, sisters], shall we not go on in so great a cause? Go forward and not backward. Courage, brethren; and on, on to the victory!"

-"Elder Jeffery R. Holland

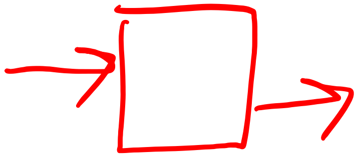
Fluids Roadmap



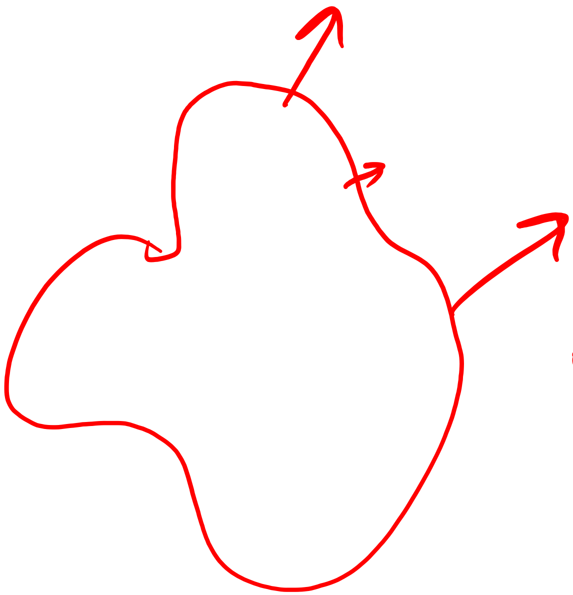
Key Points

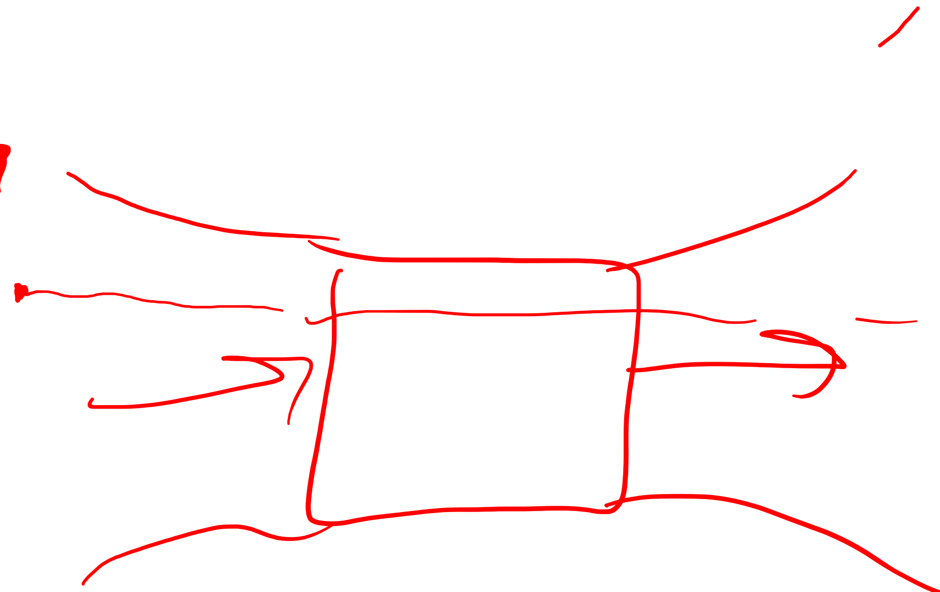
- Math 302 Review
  - Dot product (physical meaning)
  - Tensor, Flux, Vector
- Lagrangian vs. Eulerian
- Substantial or Material Derivative
- Reynold’s Transport Theorem

Balances



$$\dot{m}_{in} - \dot{m}_{out} + gen/cons = acc$$





Math Review and Goals

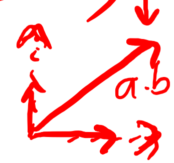
- Scalar
- Vector
- Tensor
- Dot Product
- Flux ~ next page
- Gradient operator

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{bmatrix}$$

$$\vec{v} = u\hat{i} + v\hat{j} + z\hat{k} \quad (3\ \text{comp})$$

$$\begin{bmatrix} ii & ij & ik \\ ji & jj & jk \\ ki & kj & kk \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = (ae) + (bf) + (cg) + (dh)$$



$$a \cdot b \neq |a|\,|b| \cos\theta$$

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

- Shifting Perspective: going from stagnant "system" to a moving, flowing system, or "control volume"
- Need some tools to analyze this new "moving" system or a control volume in which property of interest enters and exits

Flux

$-\frac{(\vec{v}\cdot\vec{n})}{A}$

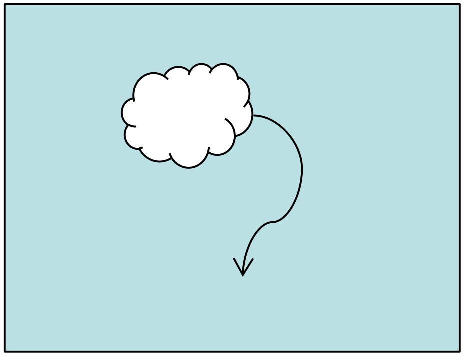
- Flux = quantity per unit area per unit time
  - Heat flux : J/m^2*s
  - Mass flux: kg/m^2*s
  - Neutron flux: neutrons/m^2*s
  - Momentum flux: $kg*m/s*m^2*s = kg/ms^2$
  - $\rho v = kg/m^3 * m/s = kg/m^2*s$ = mass flux
- Then the flux of any quantity per unit mass (q) is
  - .ρq**v** $q\cdot\rho v$ $q = \frac{mass}{mass} = 1$ $\cancel{q}^{1}\rho v$
    - q = h → J/m^2*s Heat flux
    - q=1 → kg/m^2*s Mass flux
    - q=v → $kg/s*m^2*s = kg/m*s^2$ Momentum flux
- -ρ*q***v·n***A is the rate of quantity through surface A

Lagrangian/Eulerian

- As we go to moving fluids (not fluid statics)…
- GOAL: Write balance equations to describe and solve problems
- Conservation Laws:
  - Mass "The mass of an object is converved (not created/destroyed)"
  - Momentum "Acceleration of an object = net force on the object"
  - Energy "Energy of a given mass is conserved"
- All these laws are written in terms of some object, or some fixed mass
- In Engineering, we don't normally care about some object, but some fixed region in space.
  - We care about the pump, not the "piece" of fluid flowing through it.
  - While the mass of a "piece" of fluid is conserved, the mass inside a pump can change.
  - The conservation law is written for the piece of fluid, NOT the pump.
- So how do we get a conservation law for a piece of fluid in terms of a region of space?
  - Two frames of reference: **Lagrangian** (piece of fluid) ←→ **Eulerian** (region of space)

Lagrangian/Eulerian

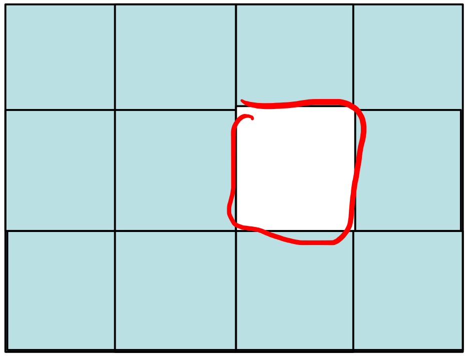
Lagrangian



- Motion of system of **fixed mass**
- CONSERVATION LAWS
- Fluid elements move around and deform

Differential

Eulerian

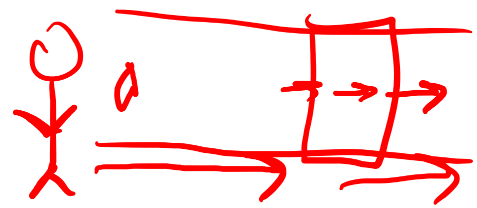


- Some **fixed control volume**
- CONVENIENT FOR ENGINEERING
- Don't care about fluid elements
- Want pressure and velocity fields at a point.
  - Pressure on a wing
  - Drag on a car
  - Not the pressure of a chunk of fluid as it moves along

Integral

Material Derivative (I)

- Start with a property (like P or velocity) – call it Ø
  - Ø= Ø(x,y,t)



- Eulerian:
  - $\left(\frac{\partial Ø}{\partial t}\right)_{x,y}$ – change in Ø in time, x & y constant
  - $\left(\frac{\partial Ø}{\partial x}\right)_{t,y}$ – change in Ø in x direction, t & y constant
- Lagrangian:
  - Change in system as it moves

Material or Substantial Derivative

- Start w/ property: ∅= ∅(x,y,t)
- Take total derivative:
  - $d\emptyset = \frac{\partial \emptyset}{\partial x}dx + \frac{\partial \emptyset}{\partial y}dy + \frac{\partial \emptyset}{\partial t}dt$
- Divide by dt:
  - $\frac{D\emptyset}{Dt} = \frac{\partial \emptyset}{\partial x}\frac{dx}{dt} + \frac{\partial \emptyset}{\partial y}\frac{dy}{dt} + \frac{\partial \emptyset}{\partial t}$
- What is dy/dt? dx/dt? Identify gradient…
  - $\frac{D\emptyset}{Dt} = \frac{\partial \emptyset}{\partial x}u + \frac{\partial \emptyset}{\partial y}v + \frac{\partial \emptyset}{\partial t}$ → $\frac{D\emptyset}{Dt} = \frac{\partial \emptyset}{\partial t} + \vec{v}\nabla\emptyset$

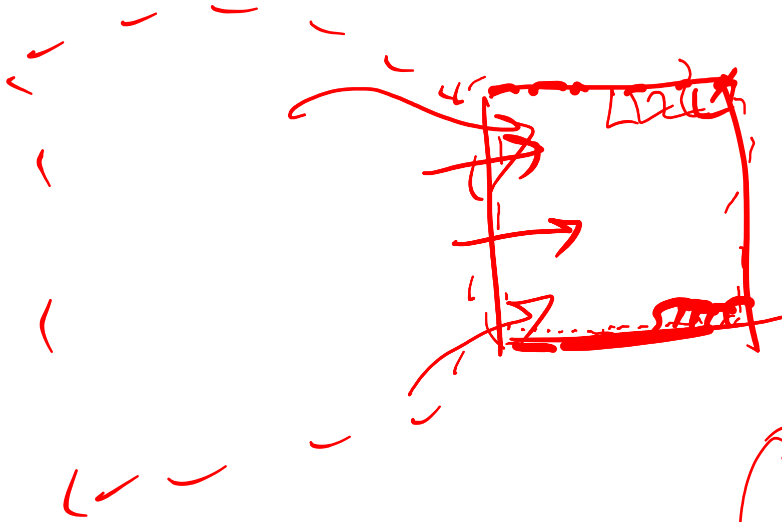
amount passing through C.V.

$\nabla\emptyset$

Lagrang

fixed point

Eulerian – Reynolds Transport Theorem



e.g.

$B = J = J/kg$

$b = B = m$ e.g. $b = 1$

- B is an extensive quality – mass (kg), energy (J)
- b is B/m → intensive property

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b \, dv + \int_{CS} \rho \vec{v} \cdot \vec{n} \, dA$$