

# Chemical Engineering 412

## *Introductory Nuclear Engineering*

### Lecture 8

### Radiation Interactions with Matter

### Exam Review



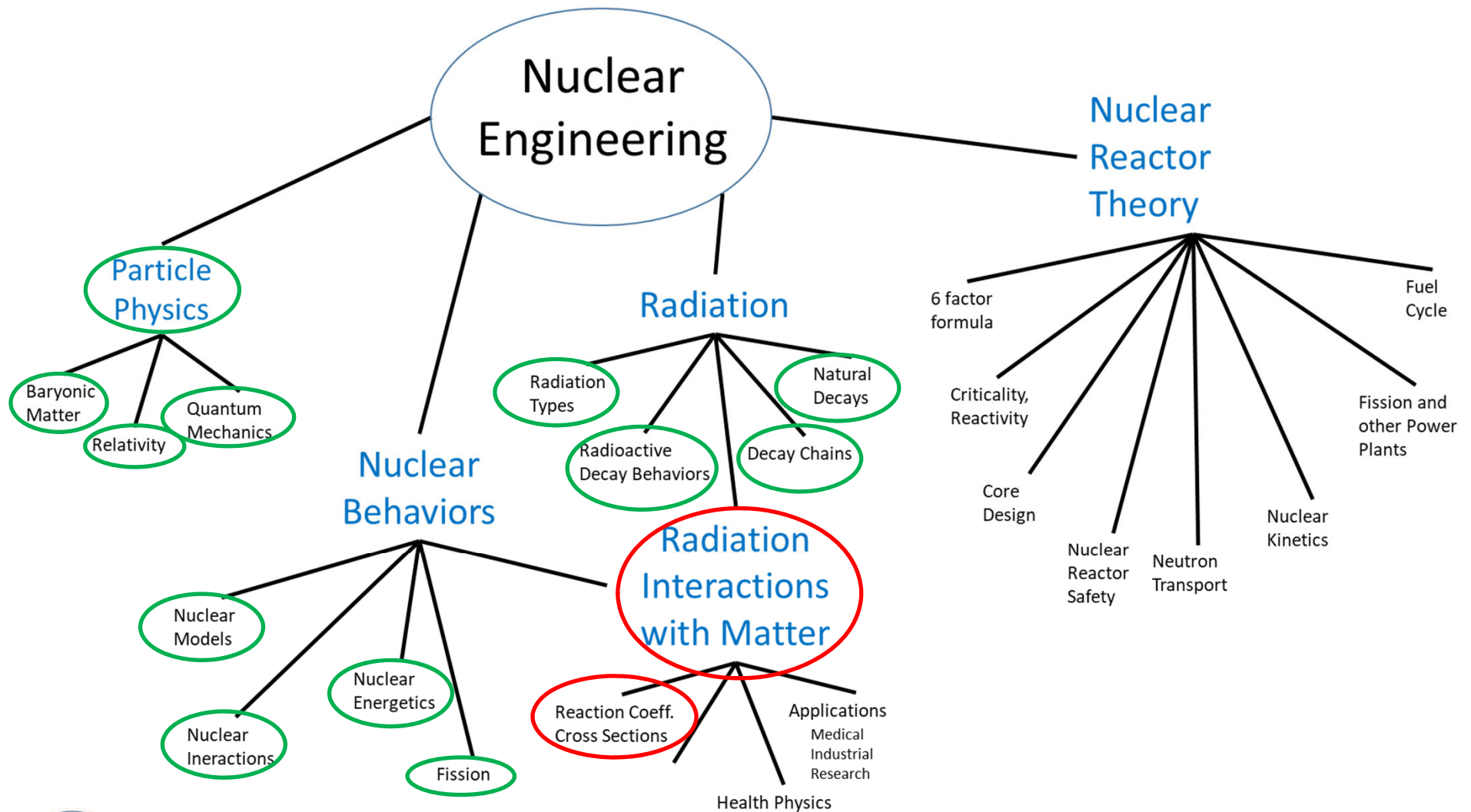
# Spiritual Thought

“Most people miss opportunity when it knocks because it comes to the door dressed in overalls and looks like work.”

Thomas S. Monson  
(quoting Thomas Edison)



# Roadmap



# Objectives

- Be able to calculate probabilities of interaction and radiation field intensities
- Understand both **linear interaction coefficients** and **cross-sections**
- Be able to calculate or find  $\mu$ ,  $\sigma$ , and  $\Sigma$
- Be able to read, understand and take values from cross section libraries: plots or tables
- **Know how to calculate reaction rates!!!**



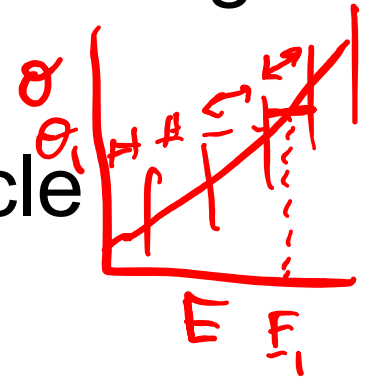
# Microscopic cross section

- The microscopic cross section
  - Independent of atomic density
  - Based strongly and complexly on particle kinetic energy
  - Play vital roles in nuclear engineering
- Behaviors are empirical!
  - (can be conceptually explained but not always quantitatively predicted by theoretical means)
- Typical unit is barns ( $1 \text{ barn} = 1 \times 10^{-24} \text{ cm}^2$ )
- 1 barn is approximate physical cross section of a uranium nucleus.



# Flux and Reaction Rate

- If  $\sigma$  is probability of one particle interacting with one nucleus
- And  $\Sigma$  is the probability of one particle interacting with many nuclei



- How do we evaluate many particles with many nuclei?
- FLUX- Essentially particle density per time
- Reaction Rate (number of reactions per volume per time)

$$\phi_t (\sigma_1 + \sigma_2) = \hat{R} \quad R_t = \sigma_1 N_1 \phi + \sigma_2 N_2 \phi$$

$$\hat{R}_i = \phi \sum_i = \phi N \sigma_i = \phi \sigma_i \frac{\rho N_a}{A}$$



# Example

- What is the power generation in a 1cm<sup>3</sup> section of U<sup>235</sup> fuel, assuming a thermal neutron flux of 1x10<sup>22</sup> neutrons/cm<sup>2</sup>-s?

From book:  $\sigma_f = 587 \text{ b}$   $N = \frac{\rho N_A}{M}$   $M = 235 \text{ g/mol}$   
 Appendix C  $\rho = 19.1 \text{ g/cm}^3$   $N = 4.81 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3}$

$$E_f \approx 200 \text{ MeV}$$

$$\Sigma_f = \sigma \cdot N =$$

$$\text{Power} = \Sigma_f \cdot \phi \cdot E_f = \underline{9.206 \text{ MW}}$$



# Mass Interaction Coefficient

- Photons – mass interaction coefficient
  - Interaction coefficient (macroscopic) divided by density
  - which depends only weakly on the properties of the medium (for photons)

$$\frac{\mu_i}{\rho} = \frac{\sigma_i N}{\rho} = \frac{N_a}{A} \sigma_i$$

- Homogeneous mixture properties can be determined from

$$\mu_i = \sum_j \mu_{i,j} = \sum_j N_j \sigma_{i,j} \quad \frac{\mu_i}{\rho} = \sum_j w_j \left( \frac{\mu_i}{\rho} \right)_j$$





# Cross sections for each interaction

$$\sigma_t = \sigma_e + \sigma_i + \sigma_\gamma + \sigma_f + \dots$$

total cross section

$$\sigma_a = \sigma_\gamma + \sigma_f + \sigma_\alpha + \sigma_p + \dots$$

absorption cross section

$$\sigma_s = \sigma_e + \sigma_i$$

scattering cross section

$$\sigma_t = \sigma_s + \sigma_a$$

total cross section

t = total

e = elastic scattering

i = inelastic scattering

$\gamma$  = radiative capture

f = fission

$\alpha$  = alpha (charged) particle

p = proton (charged) particle

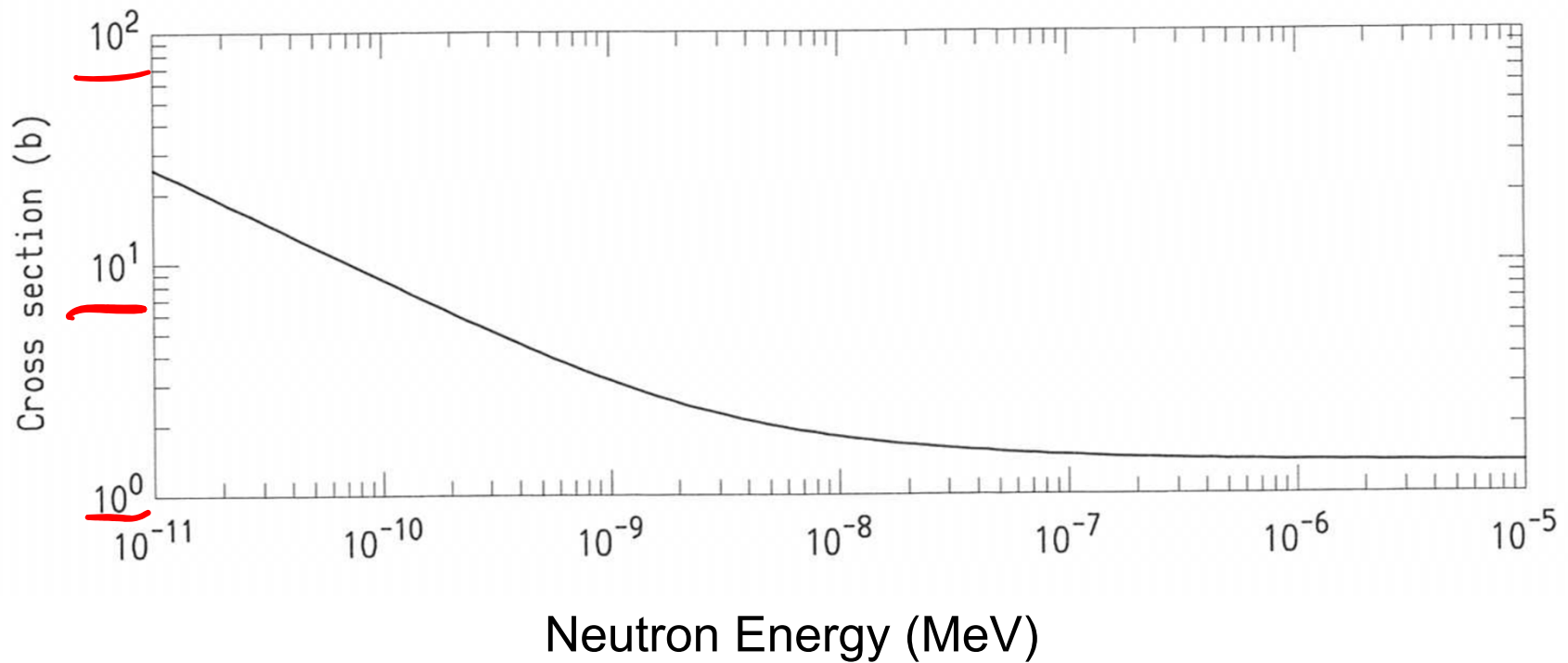


# Cross Section Trends

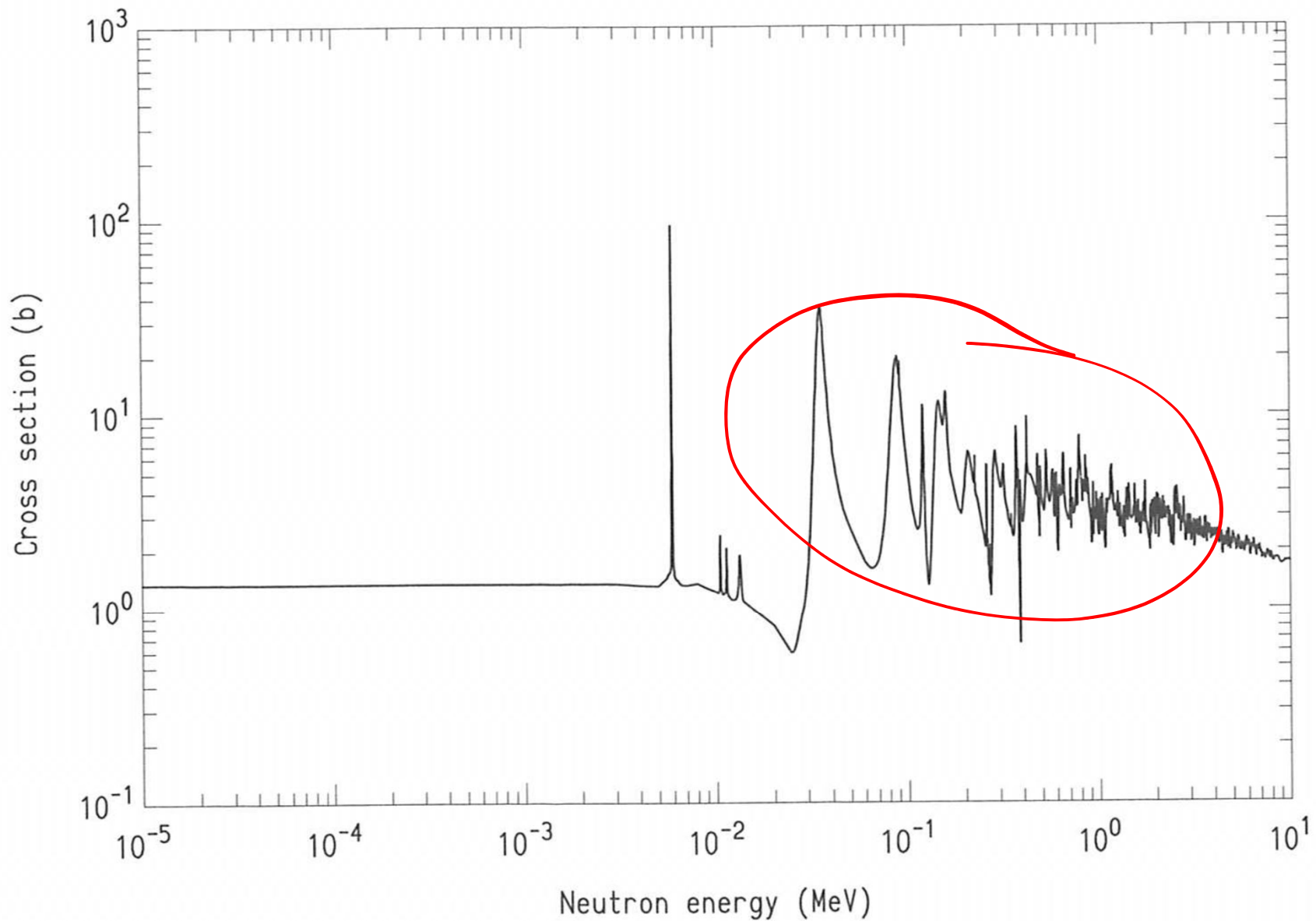
- Most Isotopes
  - Cross sections rise as neutron energy decreases.
  - Resonance regions with narrow and rapidly varying interactions that eventually are not resolvable
- Light isotopes ( $A < 25$ )
- Heavy isotopes ( $A > 150$ )
- Intermediate



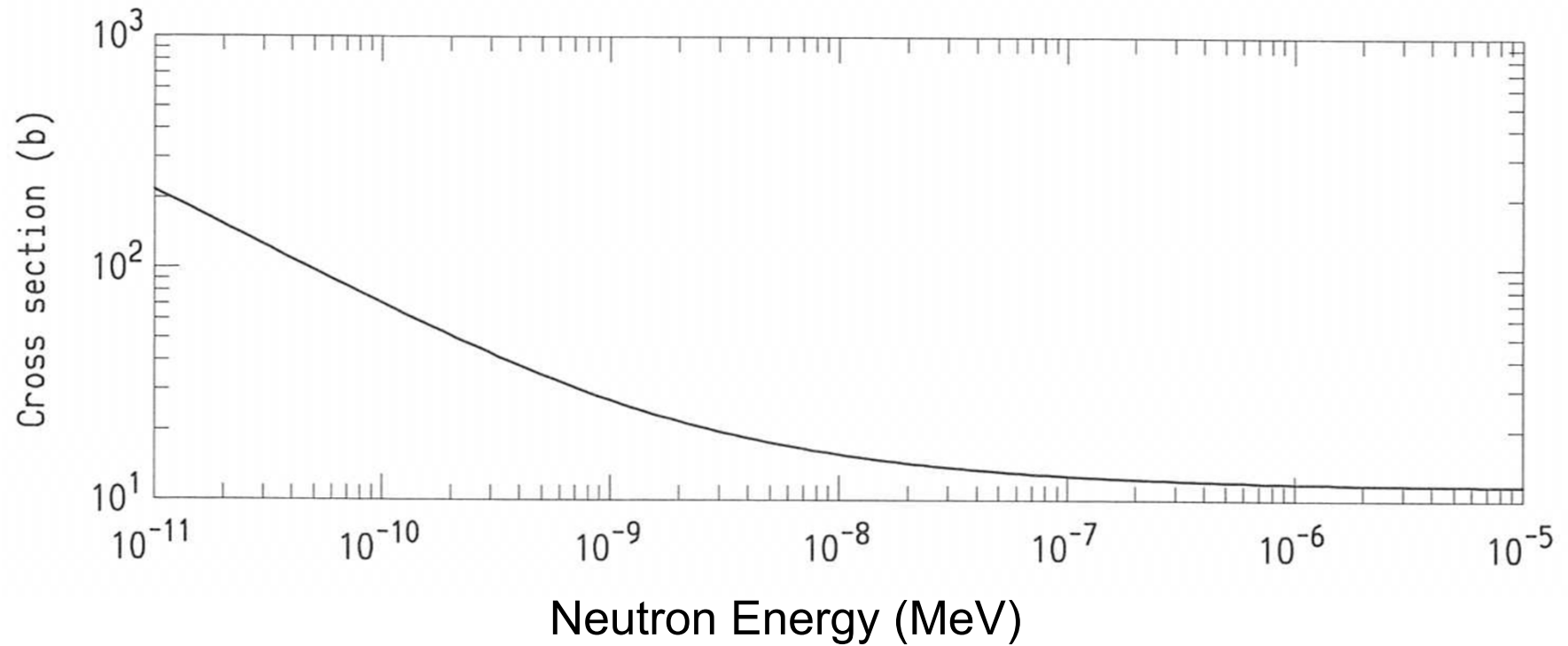
# Al Total Neutron Cross Section



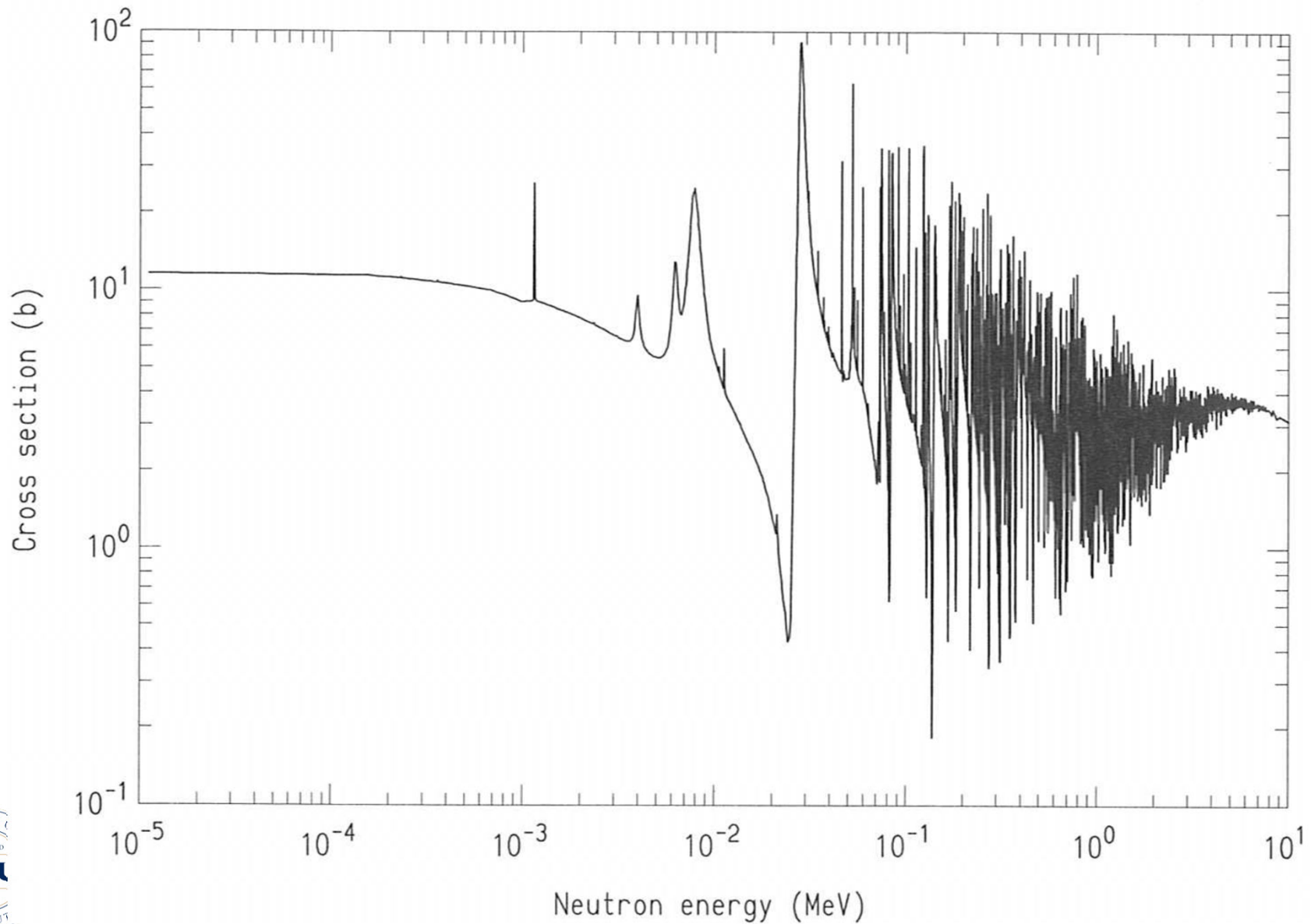
# Al Total Neutron Cross Section



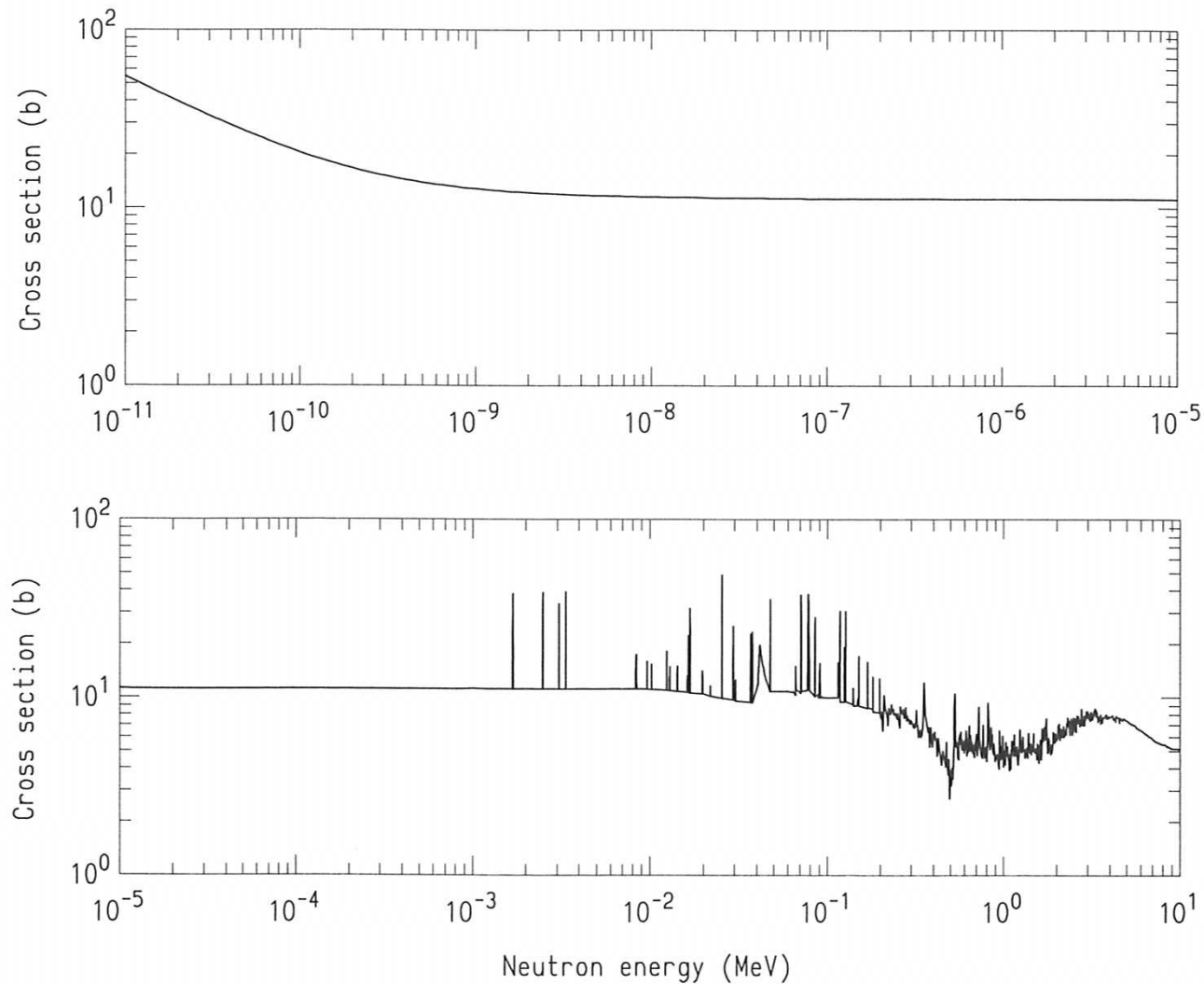
# Fe total neutron cross section



# Fe total neutron cross section



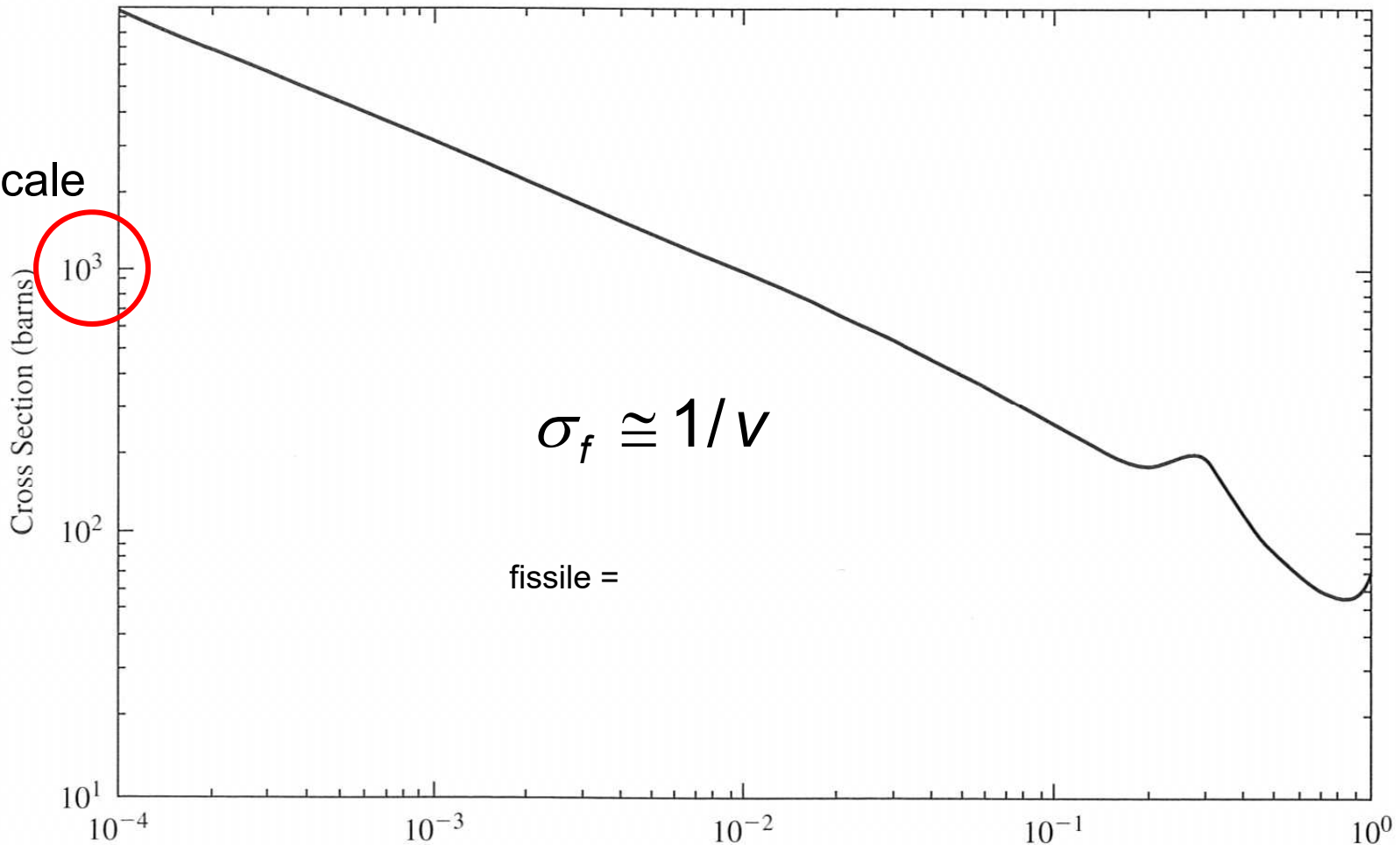
# Lead Total Neutron Cross Section



# Fission Cross Sections $^{235}\text{U}$

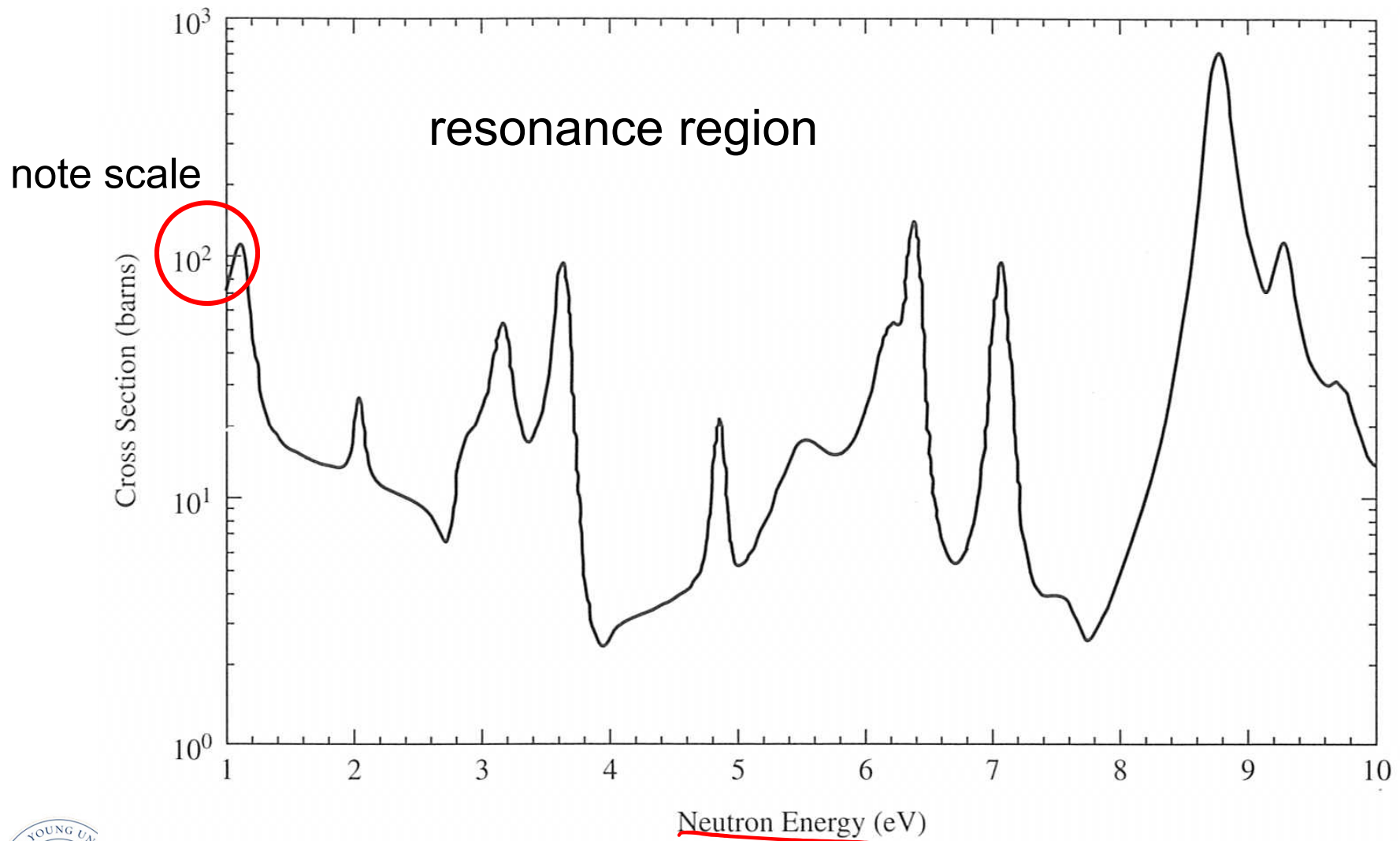
Uranium 235 Fission Cross Section MT = 18

note scale



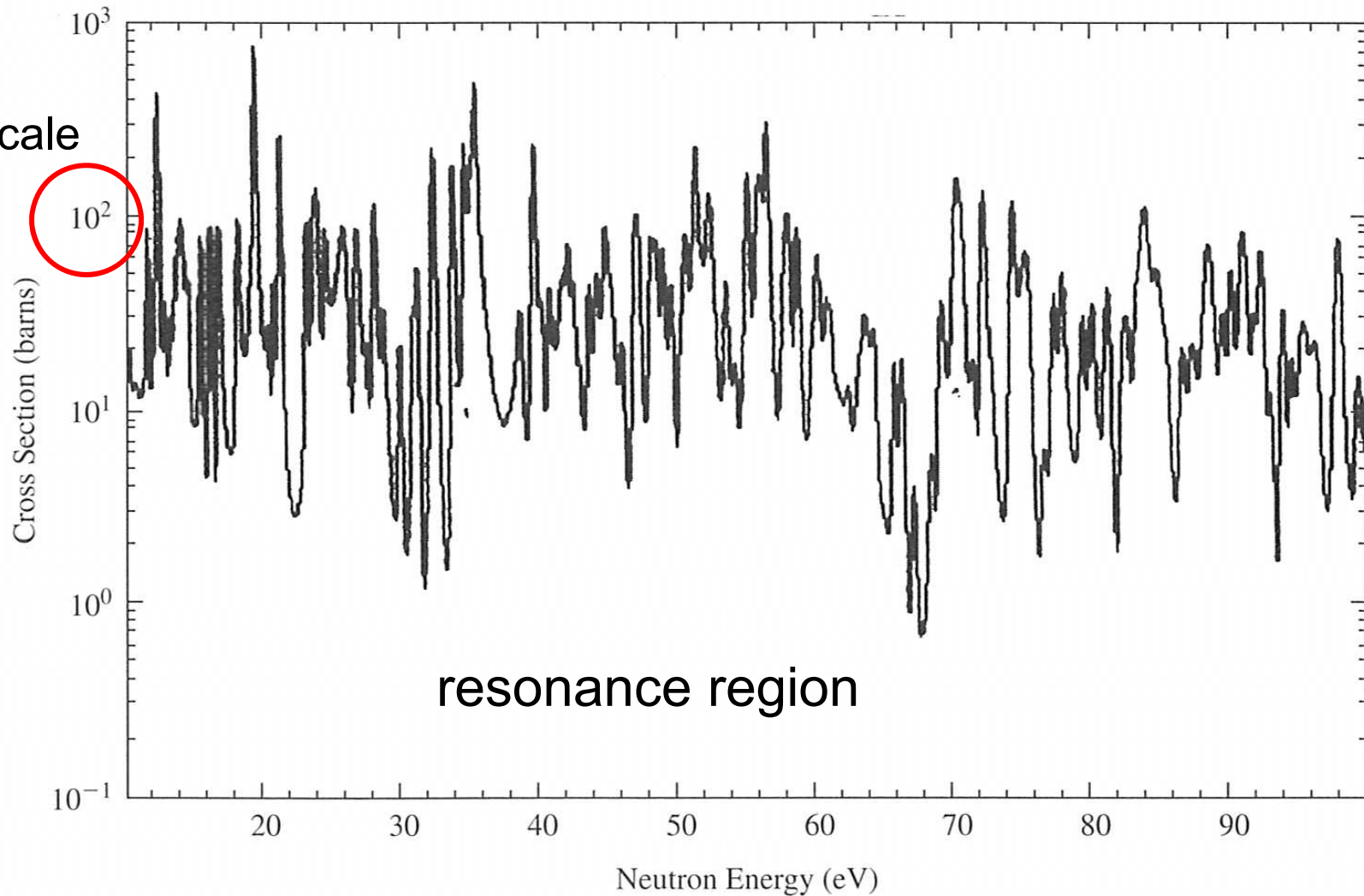


# Fission Cross Sections $^{235}\text{U}$

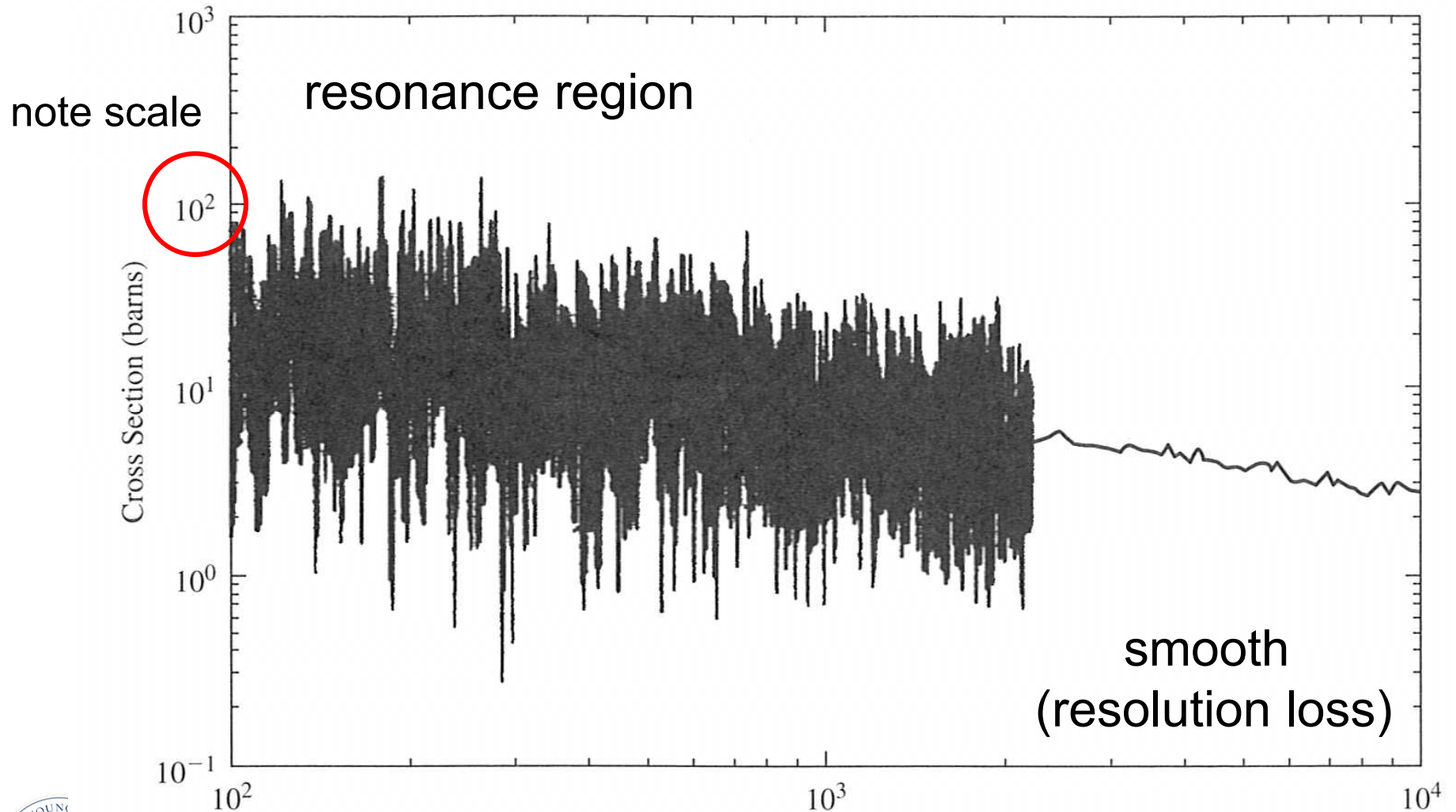


# Fission Cross Sections $^{235}\text{U}$

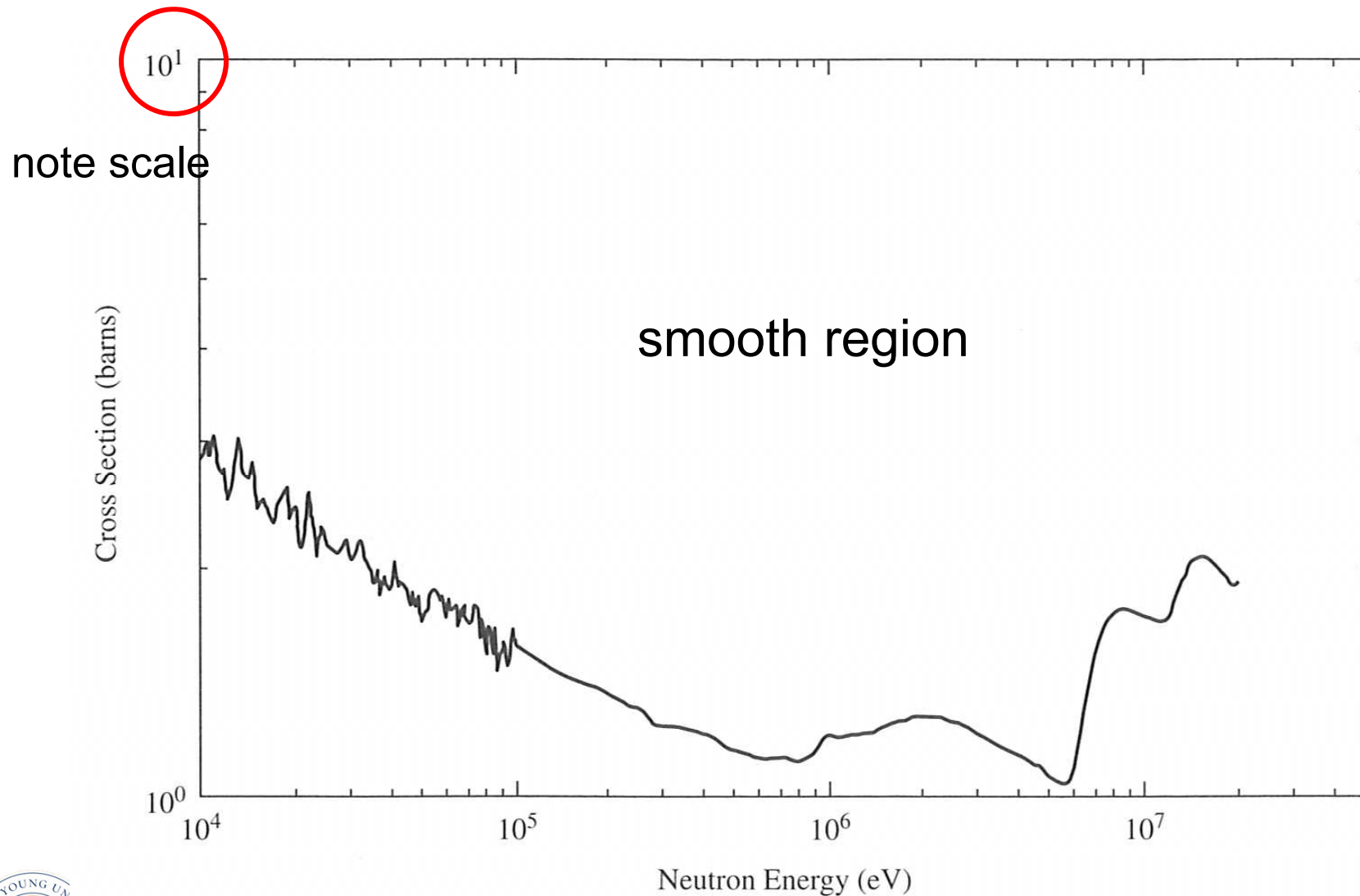
note scale



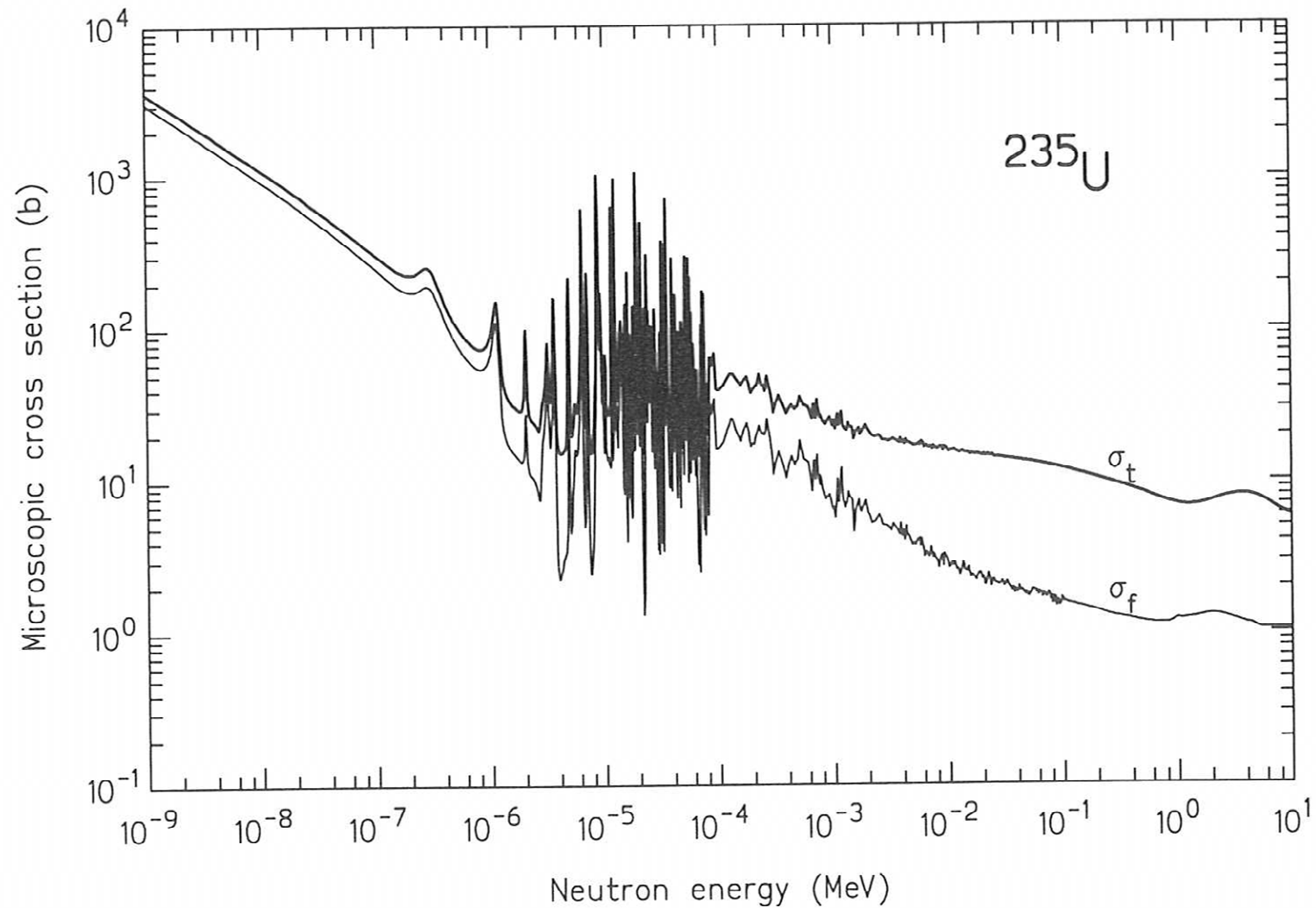
# Fission Cross Sections $^{235}\text{U}$



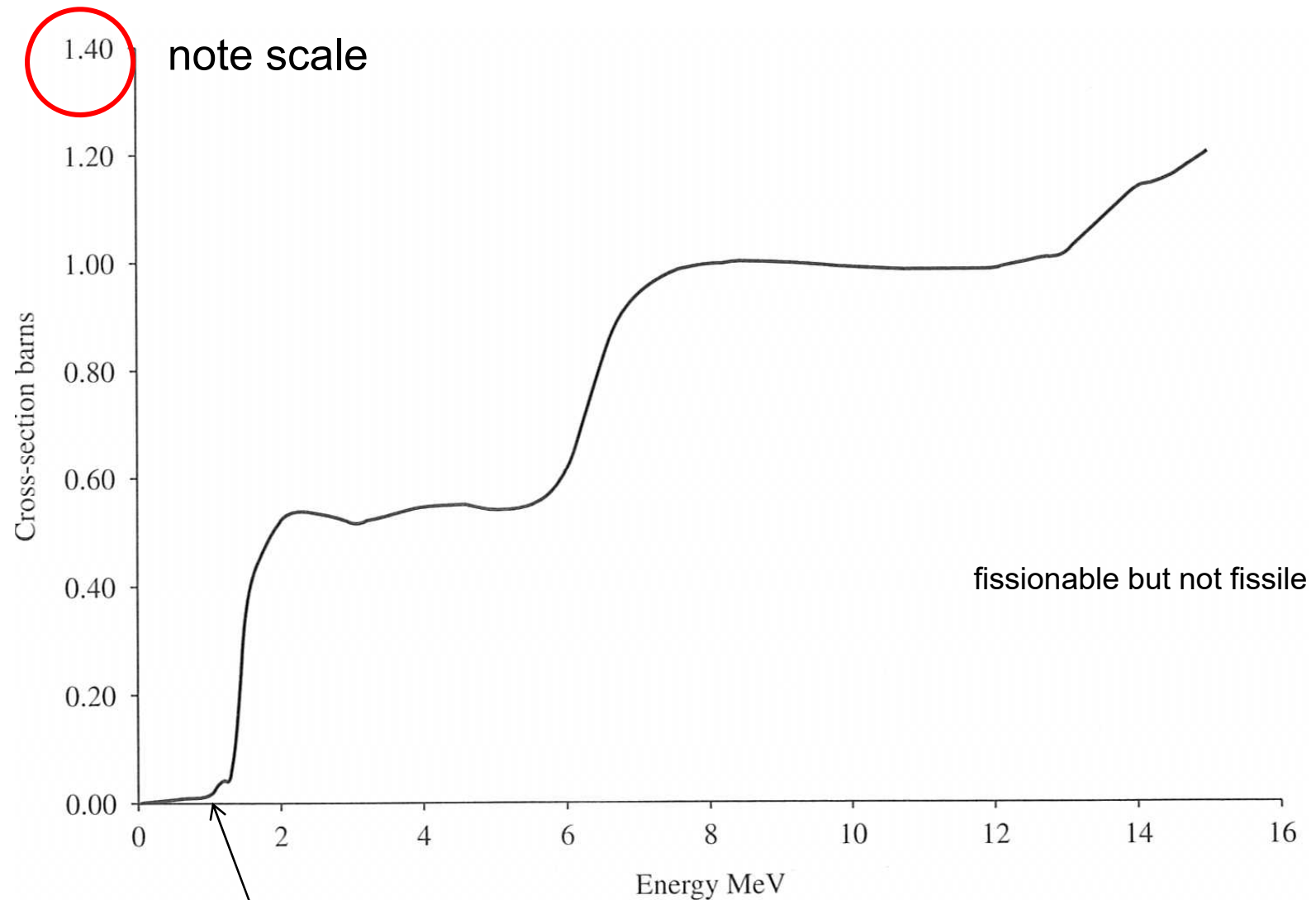
# Fission Cross Sections $^{235}\text{U}$



# Cross section over entire range

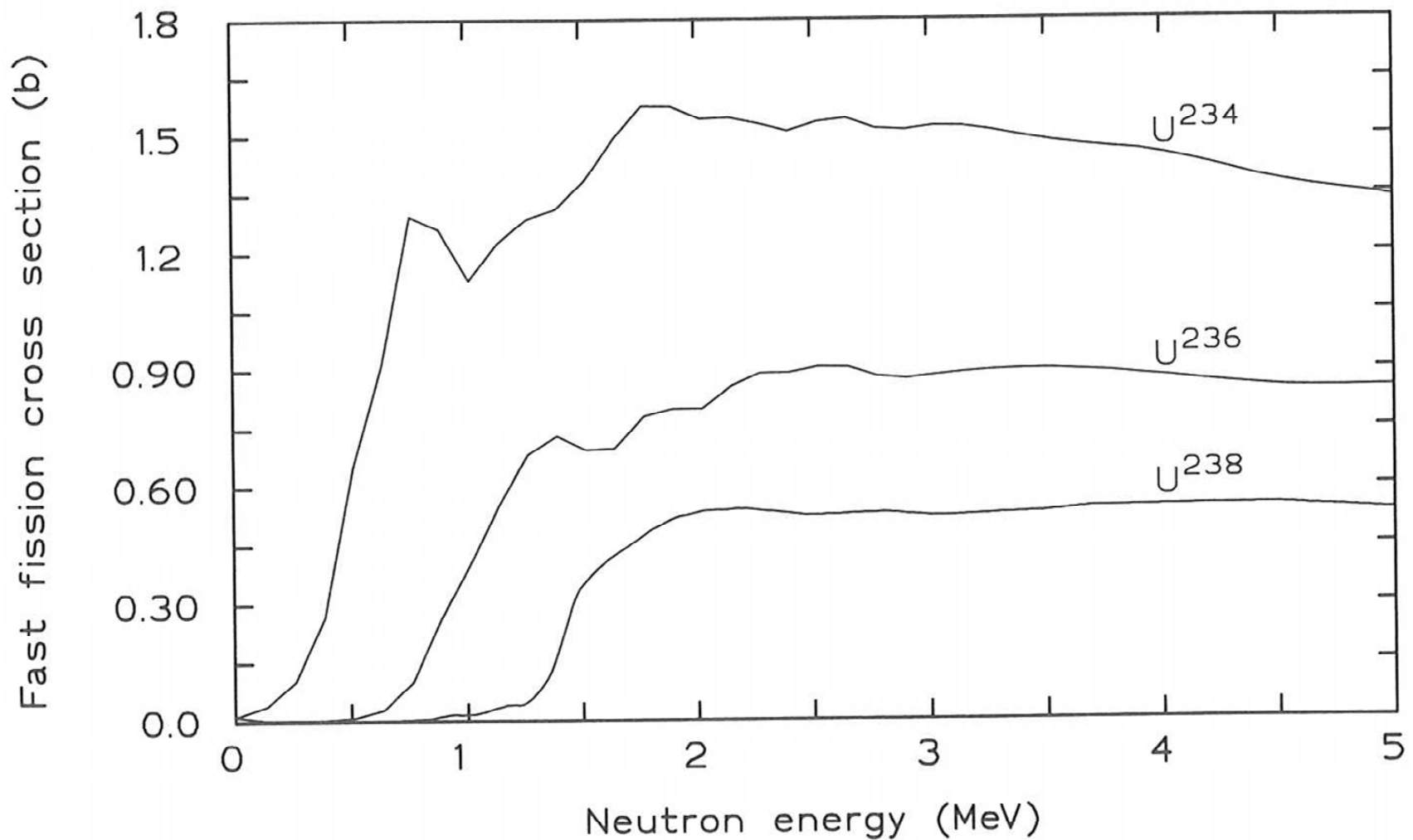


# Fission Cross Section of $^{238}\text{U}$



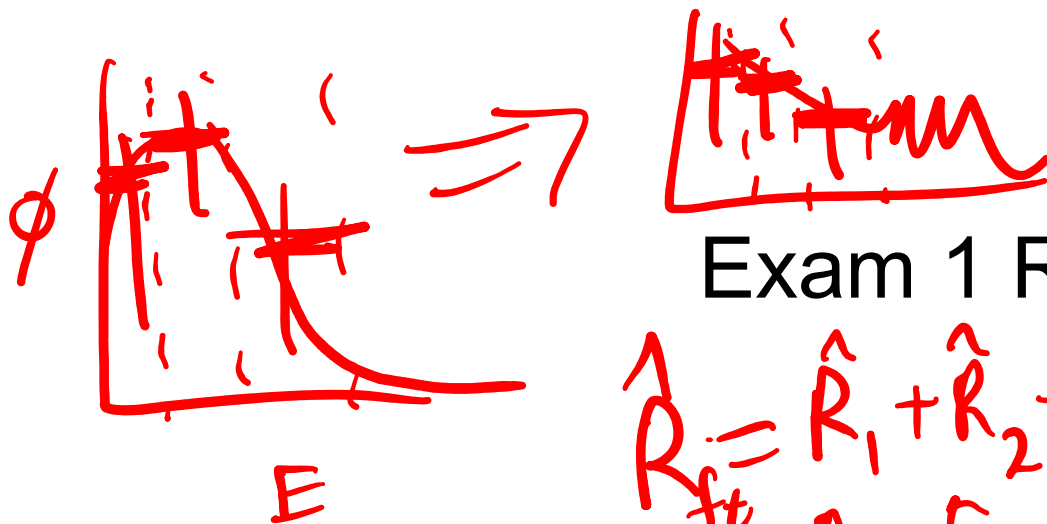
threshold energy (> resonance region energy)

# Fissionable Cross Sections



# Chemical Engineering 412

## *Introductory Nuclear Engineering*



Exam 1 Review

$$\hat{R}_{ft} = \hat{R}_1 + \hat{R}_2 + \hat{R}_3$$

$$= \rho_1 \theta_1 (N_1 + N_2 + N_3) + \rho_2 \theta_2 (N_1 + N_2 + N_3) \dots$$





# Chapter 1 - Fundamentals

- Nuclear units
- Elementary particles/particle physics
- Isotopic nomenclature
- Atomic weight/number density
- Chart of nuclides
- Mass energy equivalency



# Chapter 2 – Quantum Mechanics

- Special Relativity – time, length, mass changes
- Relativistic mass/momentum/energy relations
- Particle-wave duality
- Schrödinger's wave equation
- Heisenberg's uncertainty principle



# Chapter 3 – Nuclear Models

- Nuclear energy states
- Liquid Drop Model
- Nuclear mass equation
- Shell Model
- Nuclear stability
- Binding energy/mass excess
- Modern Nucleus concepts



# Chapter 4 – Nuclear Energetics

- Terminology
- Mass defect/BE
- Nuclear reactions
- Conserved quantities for various situations (not all the same!)
- **\*\*\*\*Q-Value\*\*\*\* (know how to calculate for ALL reactions)**
  - Know how to deal with charge
  - Know how to deal with excited nuclei
  - Know how to deal with electrons/binding energy of electrons



# Chapter 5 – Nuclear Decay

- Conservations
- Decay mechanisms – distinguishing features, Q values, energy/momentum balances
- \*\*\*Energy Diagrams\*\*\*
- Alpha/Beta particle energy distribution
- Decay Constant
- Half-Life
- Activity



# Chapter 5 – Nuclear Decay (cont)

- Parallel/Series Decay Routes
- Decay Chains
- Solutions to decay chain equations  
Secular Equilibrium
- Radionuclides in nature
- Carbon 14 dating
- Other isotopic dating methods
- Three component decays
- Isobars and most stable masses



# Chapter 6 – Binary Nuclear Reactions

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- Definitions
- Types of binary reactions
- Reaction Mechanisms
- Kinematics (scattering example)
- Threshold Energy
- Neutron Reactions
- Neutron Scattering/slowness
- Neutron Energy Spectrums

Lethargy



# Chapter 6 – Binary Nuclear Reactions (cont.)

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- Neutron capture vs. slowing
- Fission reactions
- Emitted/recoverable fission energy
- Critical energies for fission
- Fertile vs. fissile vs. fissionable
- Fission product distribution
- Prompt vs. delayed neutrons
- Fission steps/timeline





# Chapter 7 – Radiation Interactions with matter

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- Linear Interaction Coefficient (micro vs. macro)
- Cross section (micro vs. macro)
- Attenuation in Material
- Derivation of material interaction
- Buildup factor
- Mass Attenuation Coefficient
- Energy dependence of cross sections
- Cross section Trends



# Chapter 7 – Radiation Interactions with matter (cont.)

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- Cross Section of mixture
- Total intensity/flux
- Neutron flux
- Time/space/position dependence of flux
- Fluence
- Uncollided flux transmission
- Thermal vs. fast neutrons
- Photon Interactions – types, trends, energies, cross sections
- Charged Particle interactions
- Stopping Power (collision vs. radiative)
- Range



# Example 1

- The Radionuclide  $^{41}\text{Ar}$  decays by  $\beta^-$  emission to an excited level of  $^{41}\text{K}$  that is 1.293 MeV above the ground state. What is the maximum kinetic energy of the emitted  $\beta^-$  particle?
- What makes this the maximum energy?



# Solution

## Q Equation

$$Q_{\beta^-} = \{M(^{41}_{18}\text{Ar}) - [M(^{41}_{19}\text{K}) + E^*/c^2]\}c^2$$

$$[40.9645008 - 40.9618259] \cdot 931.5 - 1.293 \text{ MeV}$$

$$= 1.199 \text{ MeV}$$

B) Because an antineutrino is also released, which carries away some energy – this maximum is when the antineutrino has zero energy



# Example 2

Assume a fuel rod has a diameter of 1 cm and a length of 5 m.  
Assuming an enrichment of 5%  $^{235}\text{U}$  and a thermal flux of  $2 \times 10^{13}$  neutrons, what is the reaction rate in the fuel rod for:

a) scattering?  $1) R_s = \phi \cdot N \cdot \sigma_s \Rightarrow \text{Energy?} \rightarrow \text{thermal, } E = 0.025 \text{ eV}$

b) fission?  $R_f = \phi \cdot N \cdot \sigma_f \Rightarrow E = 0.025 \text{ eV}$   
 $\sigma_{s,m} = 2 \text{ parts! } ^{235}\text{U} \text{ \& } ^{238}\text{U}!$

$$2) \sigma_{s-235} = \overset{\text{total}}{700\text{b}} - \overset{\text{f}}{587\text{b}} - \overset{\text{r}}{103\text{b}} = 10\text{b} \quad \sigma_{f-235} = 587\text{b}$$

$$\sigma_{s-238} = 12.2\text{b} - 0.0000118\text{b} - 2.73\text{b} = \sim 9.47\text{b} \quad \sigma_{f-238} = 0.0000118\text{b}$$

$$3) N_{235} = \frac{\rho N_A x}{A} = \frac{(19.1\%/\text{cm}^3)(6.022 \times 10^{23} \frac{\text{atoms}}{\text{mol}})}{(235 \text{ g/mol})} (5\%) = 2.447 \times 10^{21} \frac{\text{atoms}}{\text{cm}^3}$$

$$N_{238} = \frac{\rho N_A x}{A} = \frac{(19.1\%/\text{cm}^3)(6.022 \times 10^{23} \frac{\text{atoms}}{\text{mol}})}{(238 \text{ g/mol})} (95\%) = 4.591 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3}$$

$$4) R_{s-235} + R_{s-238} = \phi N_{235} \sigma_{s-235} = (2 \times 10^{13} \frac{\text{neut}}{\text{cm}^2 \cdot \text{s}})(2.447 \times 10^{21} \frac{\text{atoms}}{\text{cm}^3})(10\text{b})(\frac{1 \text{ scatter}}{\text{atom}}) + \phi N_{238} \sigma_{s-238}$$

$$= (9.185 \times 10^{13} \frac{\text{scatters}}{\text{cm}^2 \cdot \text{s}})(\pi \frac{(1\text{cm})^2}{4})(5\text{m}) = \boxed{3.607 \times 10^{15} \frac{\text{scatters}}{\text{atom}}} \quad \boxed{6 = 1.1283 \times 10^{16} \frac{\text{fissions}}{\text{s}}}$$



## Example 3

- What is the probability of producing  $^{91}\text{Br}$  in a fission reaction?
- Use fission product mass distribution chart:
- $\sim 8.5\%$



# Example 4

- What is the amount of thermal neutrons that are absorbed in water per  $\text{cm}^3$  over 1 hour in a fission reactor if the thermal flux is  $2.2 \times 10^{16}$  neutrons/ $\text{cm}^2/\text{s}$ ? ( $\Sigma_a = 0.0197 \text{ cm}^{-1}$ )

- $1.56 \times 10^{18}$  absorptions per  $\text{cm}^3$

$$\phi \cdot t = \Phi$$

$$\hat{R}_a = \phi \cdot \Sigma_a \text{ or } \Phi \cdot \Sigma_a$$



# Example 5

