Chemical Engineering 374

Fluid Mechanics

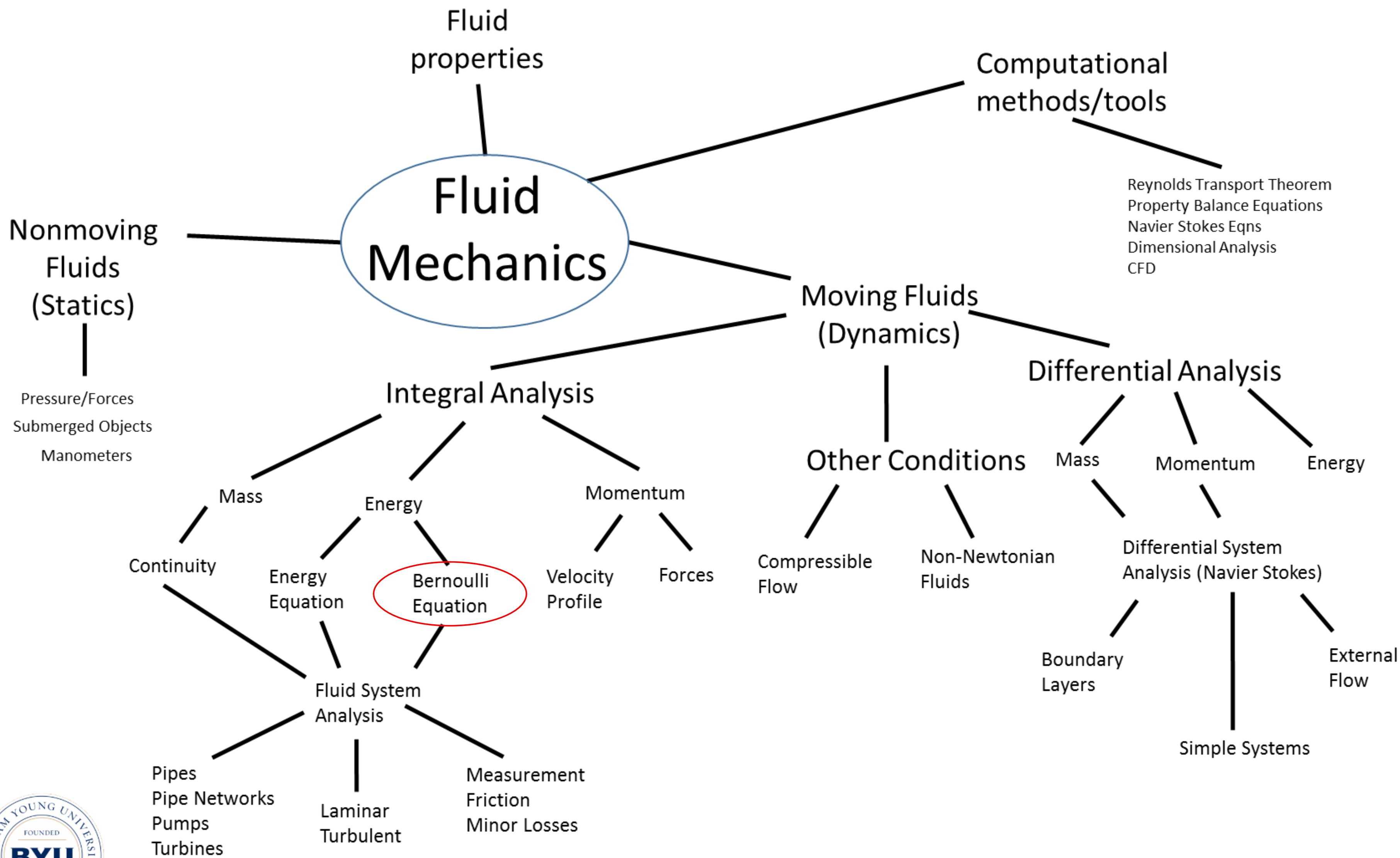
Bernoulli Equation

Spiritual Thought

Exam

- Take Home Exam
- Friday to Wednesday (turned in at start of class)
- 4 hour exam (only need 2) – ONE SITTING!
- Closed book, closed notes
- You can use 1 sheet (one side) of handwritten notes – stapled to back of exam
- Book info (like tables, units, properties) are provided.

Fluids Roadmap



"Generation" Accumulation Out In

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\rho\left(u + \frac{1}{2}v^2 + gz\right)V\right] + \left[\rho v A\left(u + \frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)\right]_{out} - [\,]_{in}$$

e_{mech} e_{mech}

Can rearrange to familiar **(Accumulation) = (In) – (Out) + ("Generation")**

Simplify

- Steady State
- Ws = 0
- Q = 0
- No friction (viscous effects)
  - This and no Q give const. u
- Incompressible → constant density

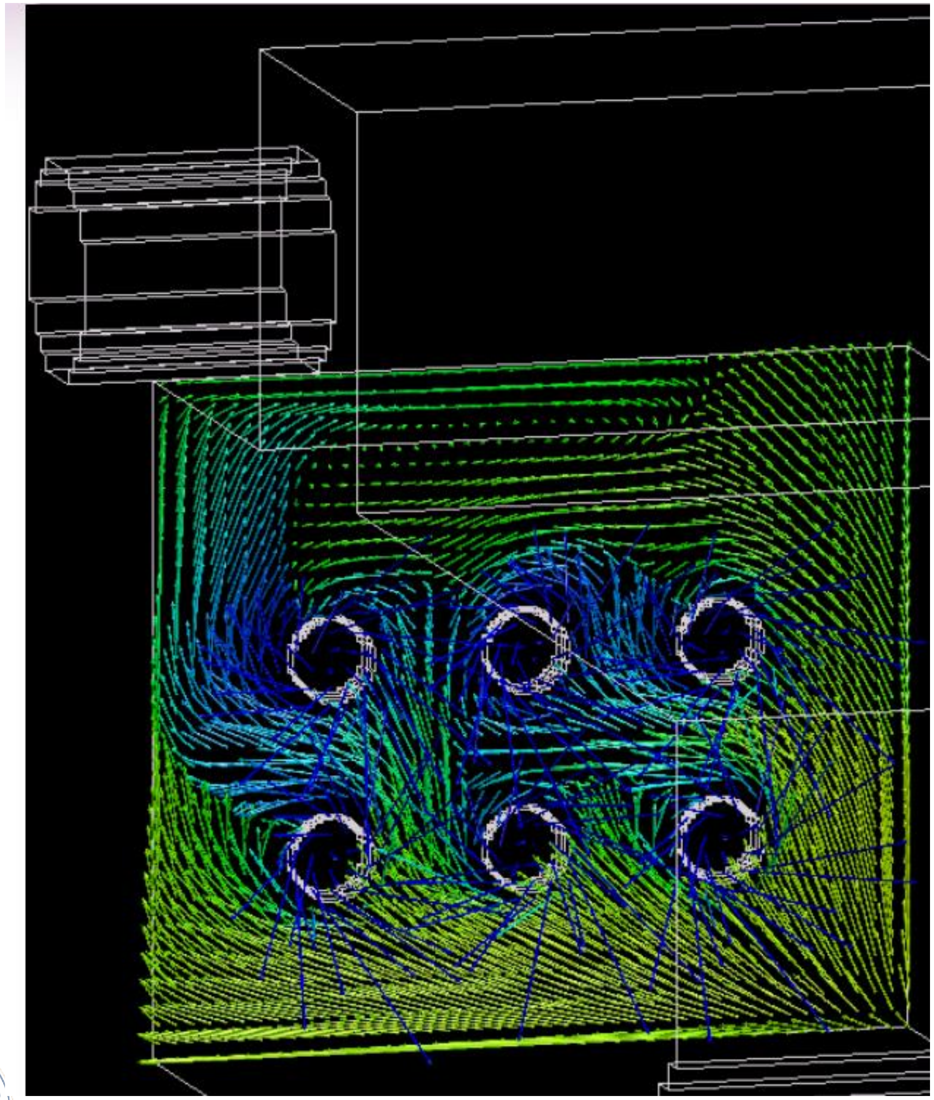
$$\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_{in} = \left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_{out}$$

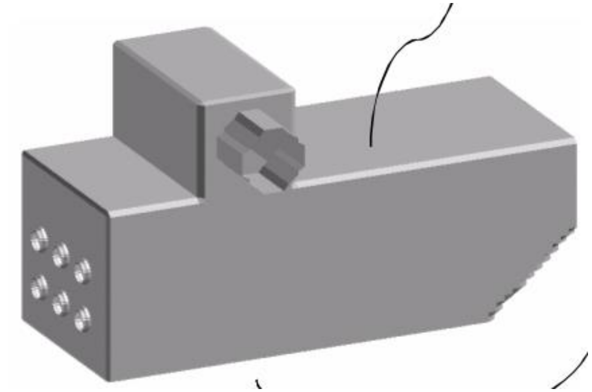
Or

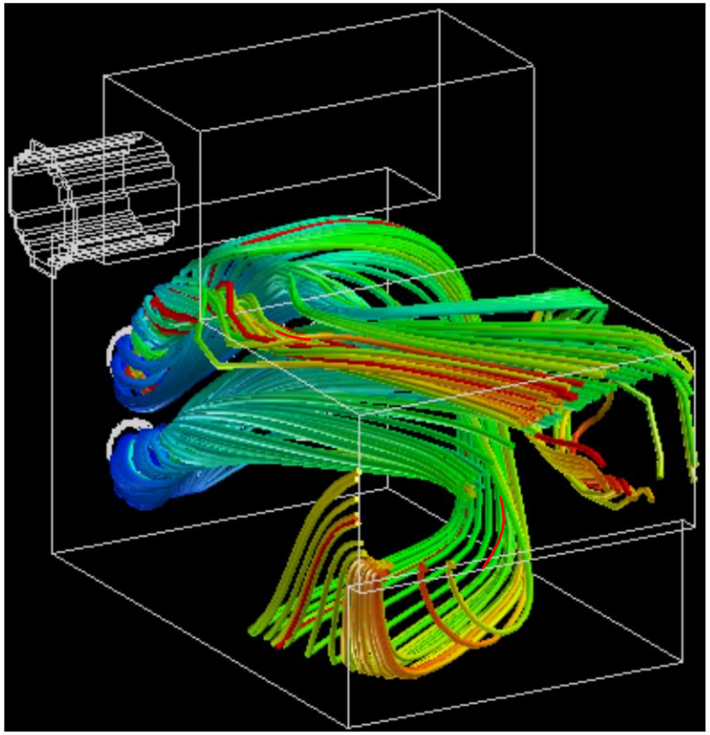
$$\Delta\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right) = 0$$

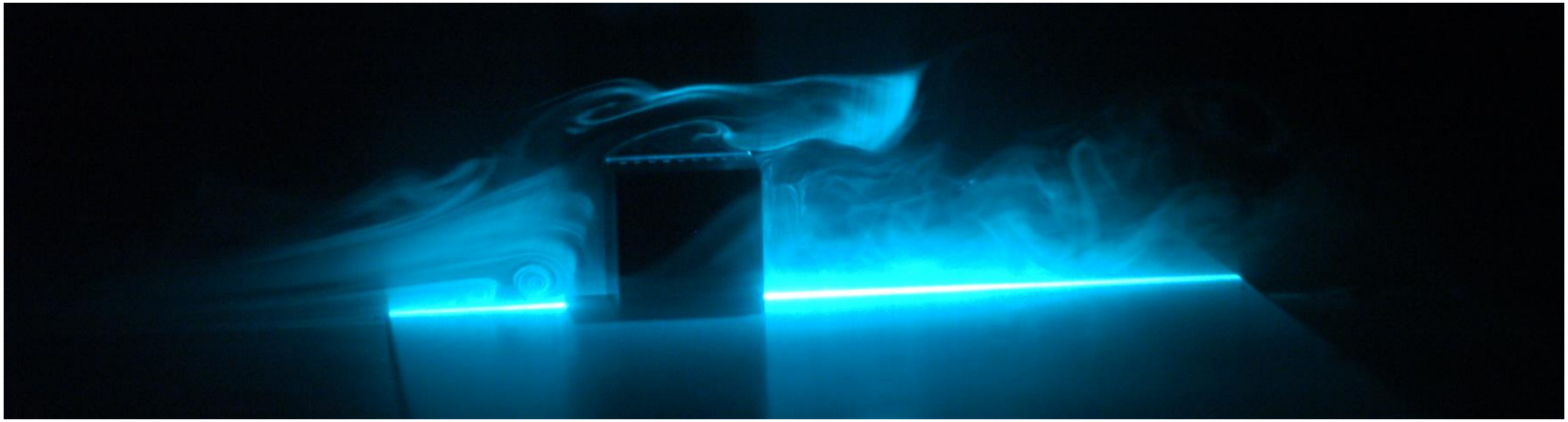
e_{mech} is conserved

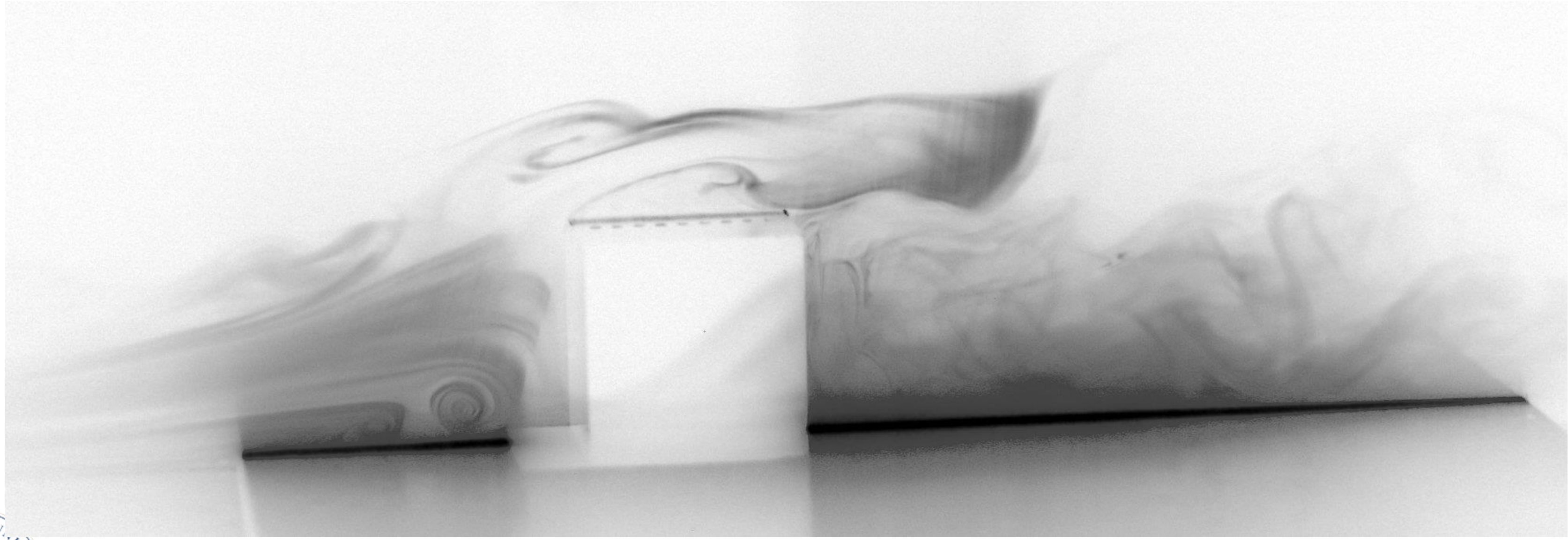
Streamlines

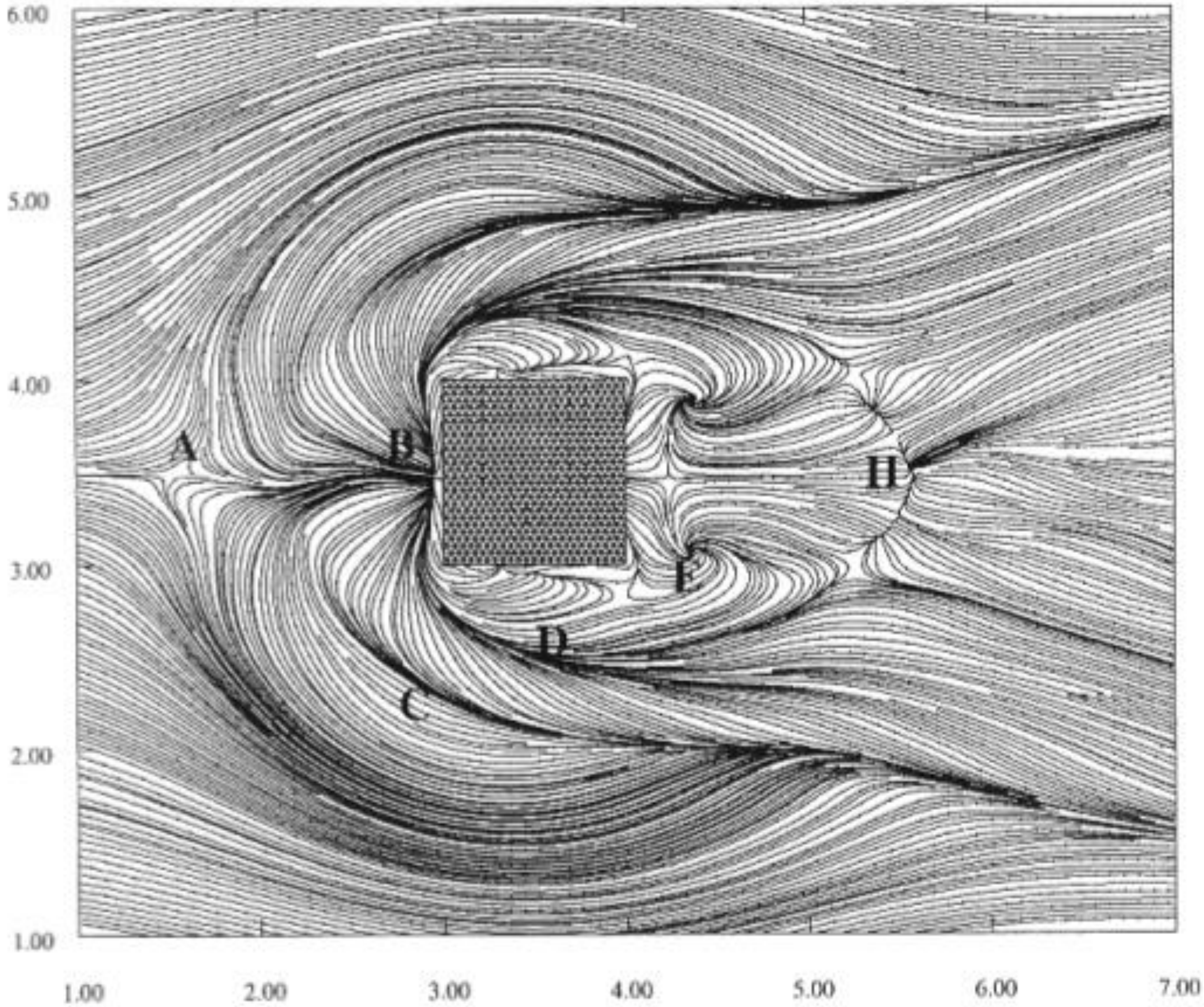








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E
H


Flow over aerofoils
H Babinsky
Cambridge University
Department of Engineering

- For streamlines, mechanical energy on a streamline is constant.
- Can derive the Bernoulli equation by making the same set of assumptions and “dot” the momentum equation (force balance equation) with displacement along a streamline.
- Cengel and Boles give a simpler derivation in terms of Newton’s Second Law (force balance), again along a streamline.
- Other forms of Bernoulli’s equation exist
  - Unsteady
  - Compressible
  - As usual, back up in the derivation when making assumptions.

Bernoulli Equation and Pressure

$$\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right) = C$$

Units

$$\frac{P}{\rho} (=) \frac{N}{m^2 \cdot kg/m^3} (=) \frac{kg \cdot m}{s^2 \cdot m^2 \cdot kg/m^3} (=) \frac{m^2}{s^2}$$

$$(=) \frac{J}{kg} (=) \frac{kg \cdot m^2}{kg \cdot s^2}$$

B.E. units are energy per unit mass

But since the mechanical energy is constant, can multiply through by density to give units of pressure.

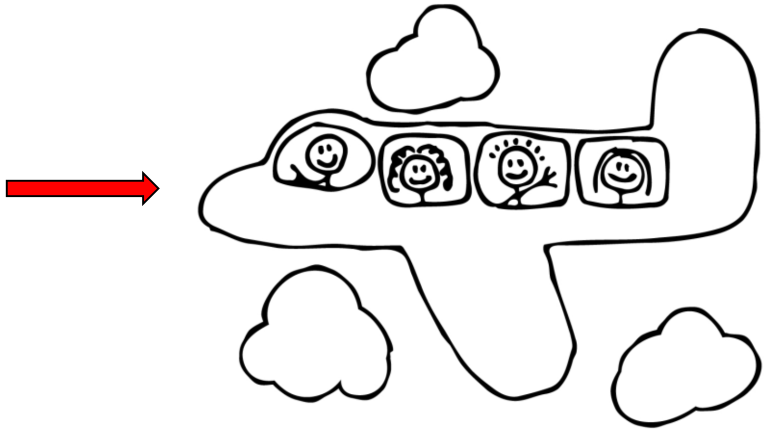
$$\left(P + \frac{1}{2}\rho v^2 + \rho g z\right) = C$$

Static | **Dynamic** | **Hydrostatic**

Total pressure

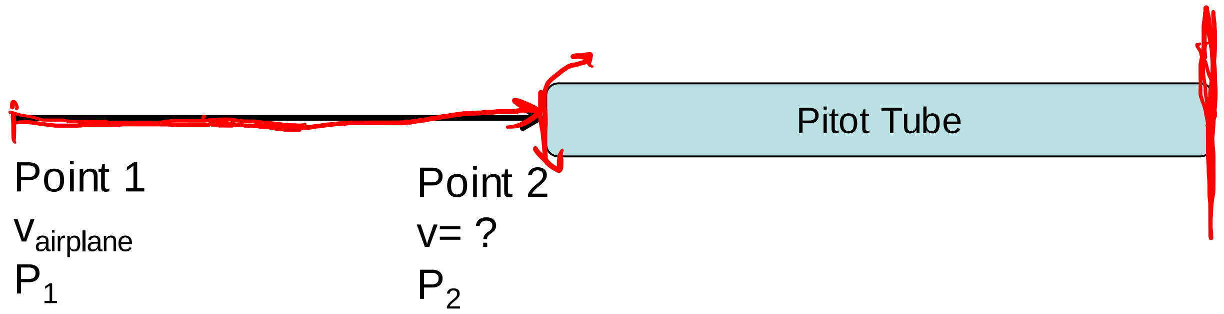
Application

- You are an airplane.
- Measure your velocity.
- How?
- Apply Bernoulli Equation $\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)_1=\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)_2$
- You have a bunch of variables
  - One is unknown.
  - The rest are either: *known*, *measured*, *controlled*
  - You have a constraint, what is it?



Pitot Tube

Pitot Tube



- Note the correlation between points and the device.
- Note the streamline.
- Note the control over v_2
- What is the principle: how does it work?

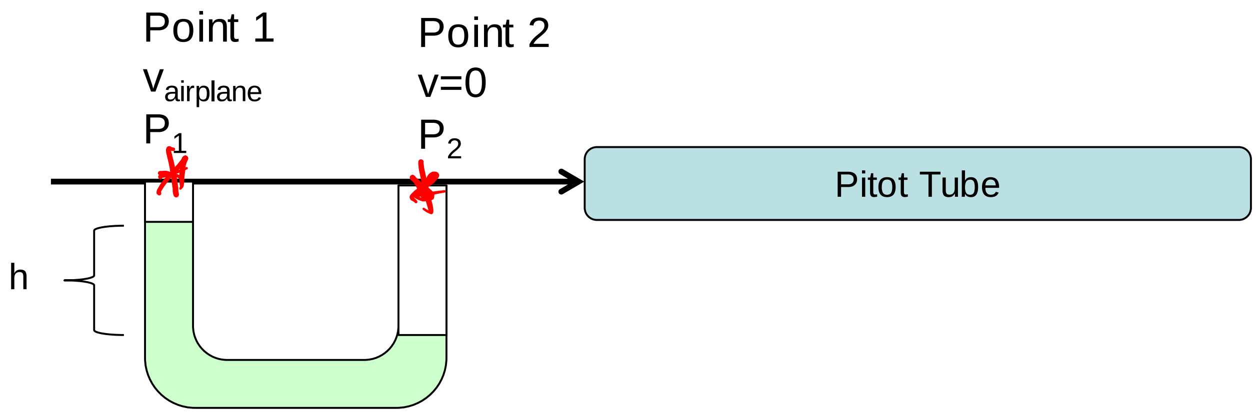
Velocity Measurement

- Velocity measurement
- Total pressure is constant along a streamline
- Measure pressure at two points on the same streamline
  - Where the velocity is desired
  - At a point where the velocity has stagnated
- $P_{stagnation} = P_{static} + P_{dynamic}$
- Stagnation pressure is the pressure to bring the fluid to zero velocity without friction.

$$\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_1 = \left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_2$$

$$\left(P + \frac{1}{2}\rho v^2\right)_1 = \left(P + \frac{1}{2}\rho v^2\right)_2 \Rightarrow v = \sqrt{\frac{2}{\rho}(P_2 - P_1)}$$

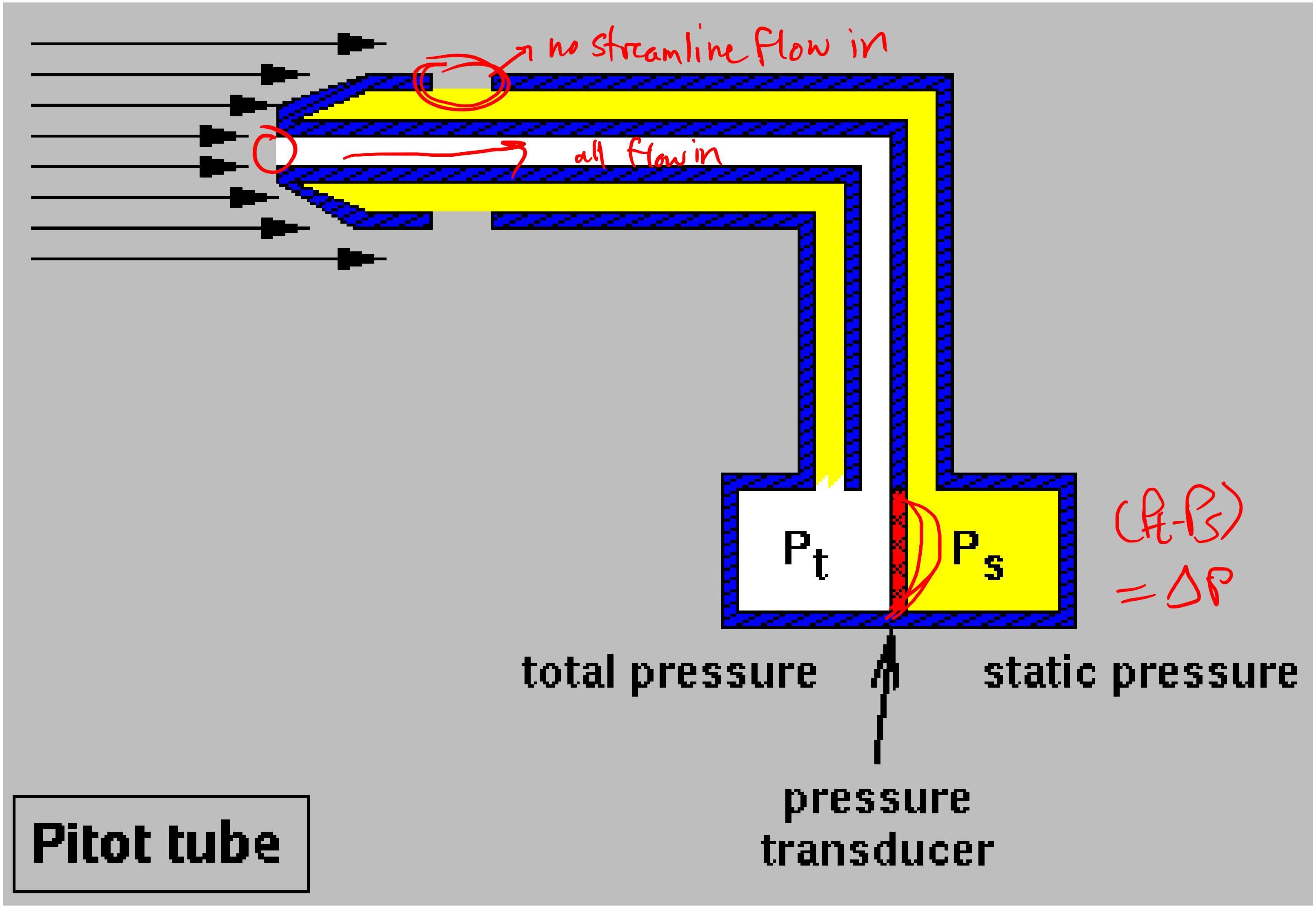
How to measure P_2-P_1



$$P_2 - P_1 = \rho g h$$

- Use a manometer,
- Or a pressure transducer, etc.
- Note, the real device is not laid out like this, but is analysed like this.

Pitot Tube



no streamline flow in
all flow in
P_t
P_s
(P_t-P_s)
$=\Delta P$
total pressure
static pressure
pressure transducer
Pitot tube

Velocity Measurement

- Problem solving with the Bernoulli equation amounts to:
  - Splitting configuration into points, evaluating P,v,z at one point and two of P,v,z at the other, and solving for the unknown with B.E.
  - Countless examples, all boil down to this.
  - Often involve multiple applications → two B.E. in two unknowns.

- Real flows are not ideal, and have friction losses.
- Friction results in a variation in internal energy (u).
- Rather than include Δu, include a friction loss term *F*
- For constant height and velocity, friction causes pressure drop.
  - Bigger fans, pumps, turbines needed for the same flow!
  - Minimize the pressure drop (friction).

$$\Delta\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)=0 \quad \longrightarrow \quad \Delta\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)=-F$$

lost energy