Chemical Engineering 374

Fluid Mechanics

Scaling

Spiritual Thought

"When prayer works as it should, we express the feelings of our hearts in simple words. Heavenly Father typically answers by putting thoughts in our minds accompanied by feelings. He always hears the sincere prayer we offer when we pray with a commitment to obey Him, whatever His answer and whenever it comes."

-President Henry B. Eyring

https://www.youtube.com/watch?v=n2Y8JaIsf54

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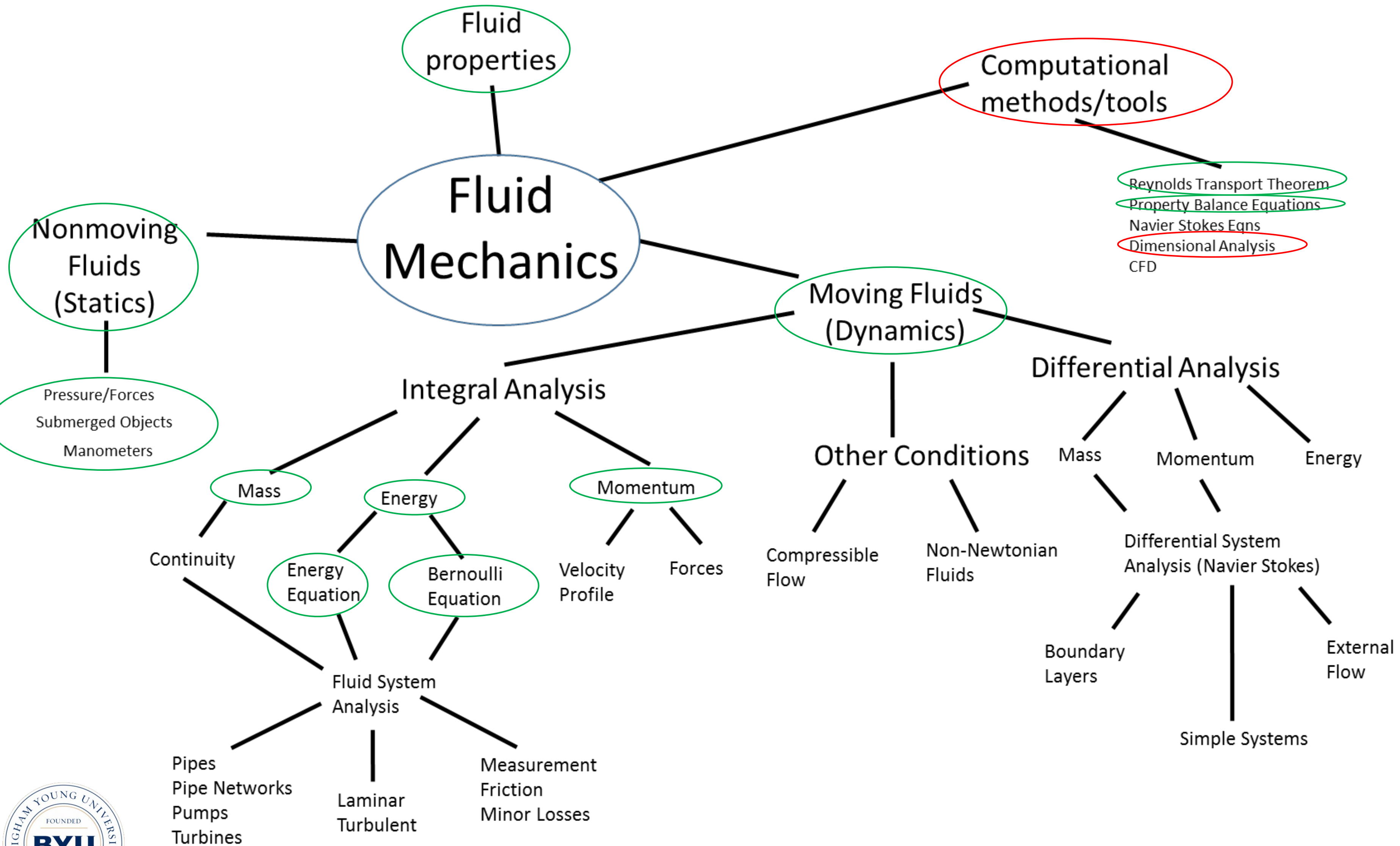
Fluids Roadmap



- Recall: Nature does not "know" about our units.
  - kilogram, pounds
  - meters, miles
  - seconds, hours
- Nature only knows about fundamental dimensions, and relative sizes with those dimensions.
  - When we assign values to those dimensions or combinations of those dimensions, we do so relative to our own contrived scales.
  - **Most problems have more natural scales than our units (1 m).**
  - What is the most natural length scale for air flow in a room?
    - Then measure length in "room" units → x/L_{room}.

How long to cook a roast?

- How long to cook?
  - Trial and error?
  - Approximate the geometry and properties and solve a heat transfer problem?
  - Solve a complex numerical solution?
- Cookbook Instructions call for **~20 min per lb**.
- Does this make sense? Why or why not?

www.ochef.com

"If you roast at 325° F (160° C), subtract 2 minutes or so per pound. If the roast is refrigerated just before going into the oven, add 2 or 3 minutes per pound. …

"…You thought this was going to be a simple answer, didn't you? "

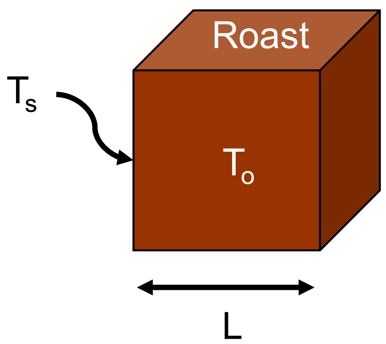
What does the heat equation say?

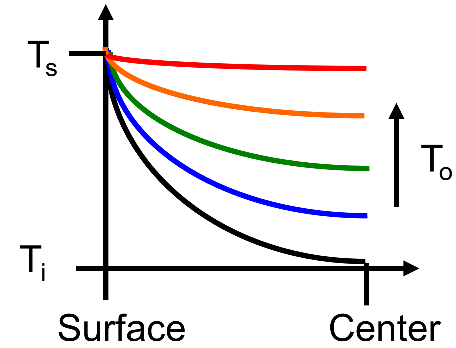
$$\frac{\partial T}{\partial t} = -\alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

$$\frac{\partial T}{\partial t} = -\alpha \left(\frac{\partial^2 T}{\partial x_i^2} \right)$$

$$\alpha = \frac{k}{\rho c_p}$$

(And 1 IC, 2 BCs)





- Unsteady heat equation
  - Assuming constant thermal conductivity
  - 3 dimensional, Cartesian coordinates
- Dimensional homogeneity: each term has the same **dimensions** and the same **units**.
  - k→ J/s*m*K;   ρ → kg/m^3; c_p→J/kg*K;   α→m^2/s
- We don't want to solve this equation, just examine it.
- Nondimensionalize, then scale the equation.

Nondimensionalize

$$t^* = \frac{t}{\tau} \rightarrow t = \tau t^*$$

$$T^* = \frac{T}{T_{ref}} \rightarrow T = T_{ref} T^*$$

$$x^* = \frac{x}{L} \rightarrow x = L x^*$$

$$\longrightarrow \frac{\partial T}{\partial t} = -\alpha \left(\frac{\partial^2 T}{\partial x_i^2} \right) \longrightarrow \left[\frac{T_{ref}}{\tau} \right] \left(\frac{\partial T^*}{\partial t^*} \right) = - \left[\frac{\alpha T_{ref}}{L^2} \right] \left(\frac{\partial T^*}{\partial x^{*2}} \right)$$

- Select reference quantities: T_{ref}, L, τ
- Make a direct substitution.
- Simplify: divide through by T_{ref}/τ

$$\frac{\partial T^*}{\partial t^*} = - \frac{\alpha \tau}{L^2} \left(\frac{\partial T^*}{\partial x^{*2}} \right)$$

- Now all terms are nondimensional.
- The group $\alpha\tau/L^2$ is called a dimensionless group and is the Fourier number: a ratio of the physical time to the characteristic diffusion time.

Scaling

- Now scale the equation:
  - Cengel and Boles call this "normalization"
- If the terms in (…) are O(1), then since the LHS is O(1), the term in […] must be O(1) also. Then what to choose for τ, L?
  - L is just the domain size.
  - Then $\tau = L^2/\alpha$
- These are the characteristic scales of the problem.
- Rather than measure length in units of "meters", nature prefers units of "roast size"

$$\left[\frac{T_{ref}}{\tau}\right]\left(\frac{\partial T^*}{\partial t^*}\right) = -\left[\frac{\alpha T_{ref}}{L^2}\right]\left(\frac{\partial T^*}{\partial x^{*2}}\right)$$

$$\left(\frac{\partial T^*}{\partial t^*}\right) = -\left[\frac{\alpha \tau}{L^2}\right]\left(\frac{\partial T^*}{\partial x^{*2}}\right)$$

"… in the process of scaling, one attempts to select intrinsic reference quantities so that each term in the dimensional equations transformes into the product of a constant factor which closely estimates the term's order of magnitude and a dimensionless factor of unit order of magnitude"

-- Lin and Segal 1988

- Now what does this say about cooking a roast?
- The timescale scales as L^2
- But mass scales as L^3

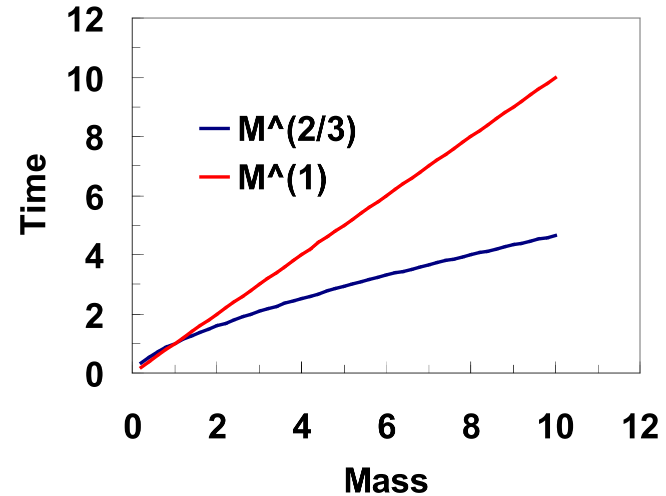
$$\tau \propto L^2$$

$$m \propto L^3$$

$$\downarrow$$

$$L \propto m^{1/3}$$

$$\rightarrow \tau \propto m^{2/3}$$



- So cooking time scales with $m^{2/3}$, not with m
- We have not had to solve anything, but we know something valuable about the problem.
  - If you know the time for one roast, you can extrapolate to another.
  - Relationships among parameters!

- Nondimensionalization also reduces the number of parameters by showing that they are not independent, but come in characteristic groups.
  - Conduction problem: vary the Fourier number alone, not alpha and L separately.
  - Book gives simple equation of motion, where initial position, velocity and gravity parameters are collapsed into one parameter, the Froude number.
  - The Fourier number was the ratio of timescales, the Froude number is the ratio of forces.
  - Many others: Friction factor, Drag coefficient, Knudsen number, Mach number, REYNOLDS number.