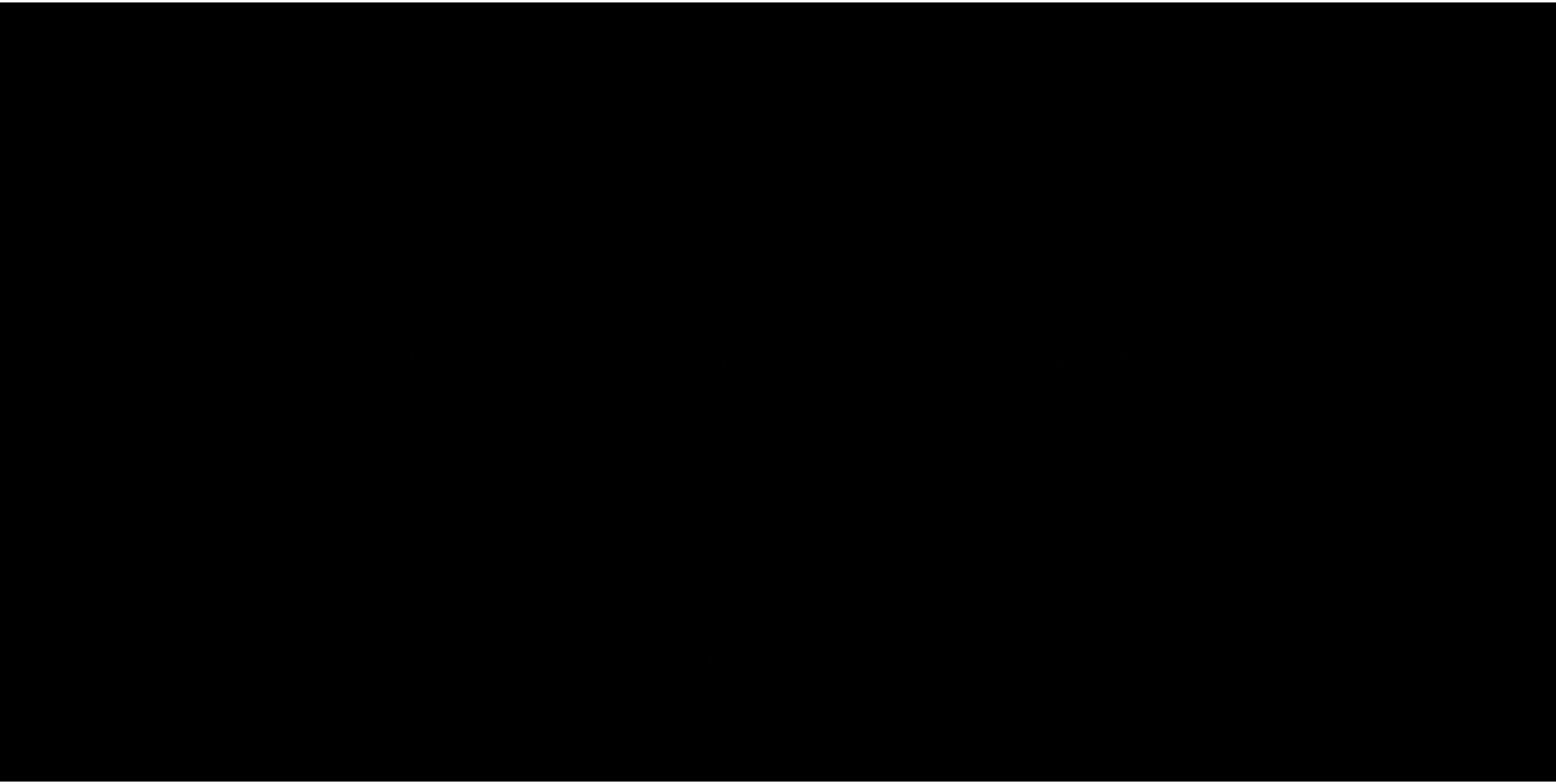
Chemical Engineering 374

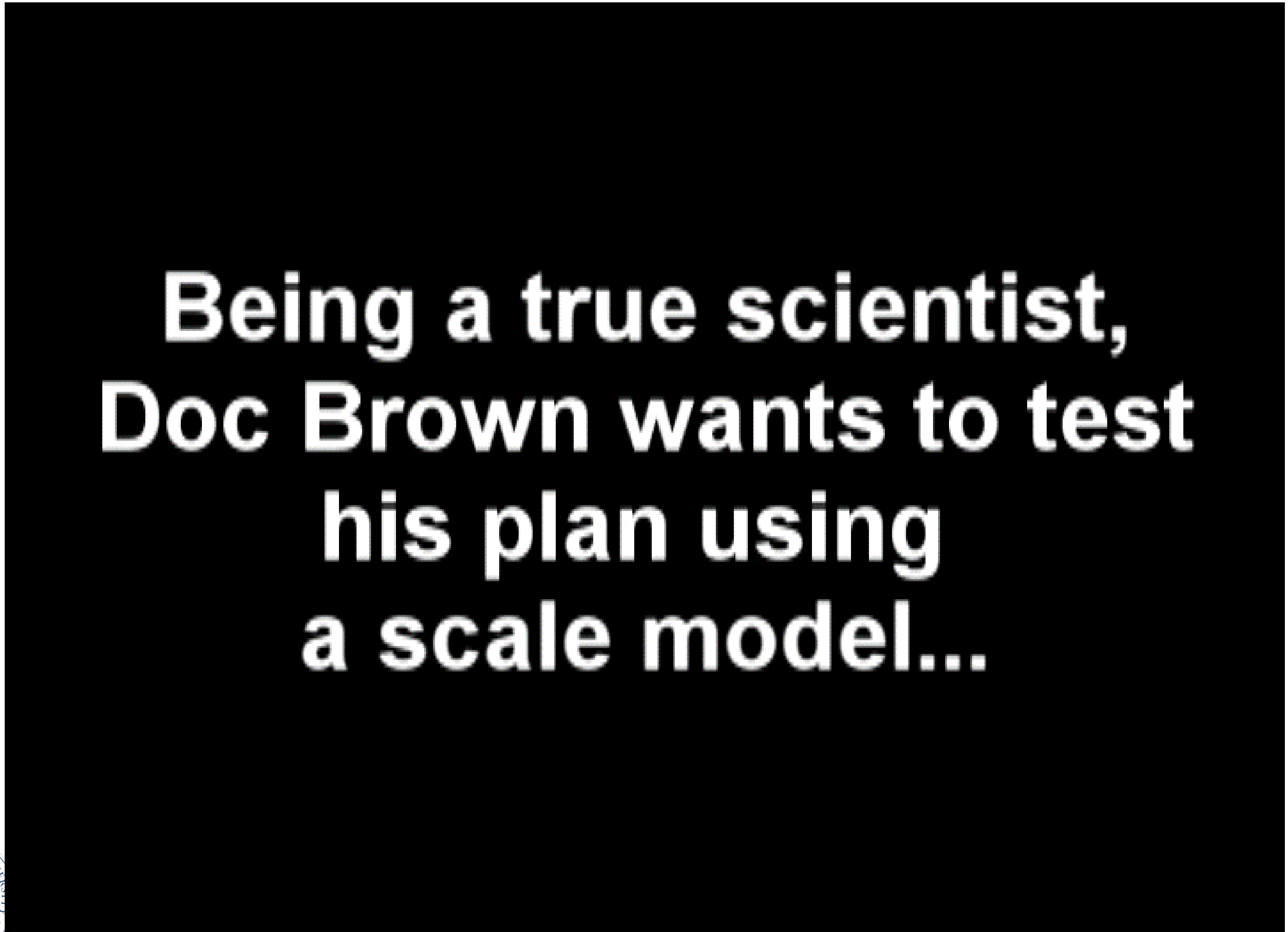
Fluid Mechanics

Laminar Flow in Pipes

Spiritual Thought



OEP 5 Clip


Being a true scientist,
Doc Brown wants to test
his plan using
a scale model...

OEP 5

Open Ended Problem #5

Exploding Trains and 88 mph

GROUP WORK OKAY, Due 10/16/24 at beginning of class

Great Scott!!! Reaching 88 mph is crucial to time travel, as the Brilliant Doc Brown has discovered. However, back in 1885, the only means he had of accelerating his DeLorian time machine was to use a steam engine locomotive. In order to make sure that this would work (since if it didn't they'd be in for a long free fall into a deep ravine), he decided to build a scale model, primarily to ensure that the drag forces on the train wouldn't preclude reaching 88 mph. How fast should the model train be moving in order to ensure similarity between the model and the actual attempt at reaching 88 mph?

OEP 5 (continued)

Open Ended Problem #5

Exploding Trains and 88 mph

GROUP WORK OKAY, Due 10/16/24 at beginning of class

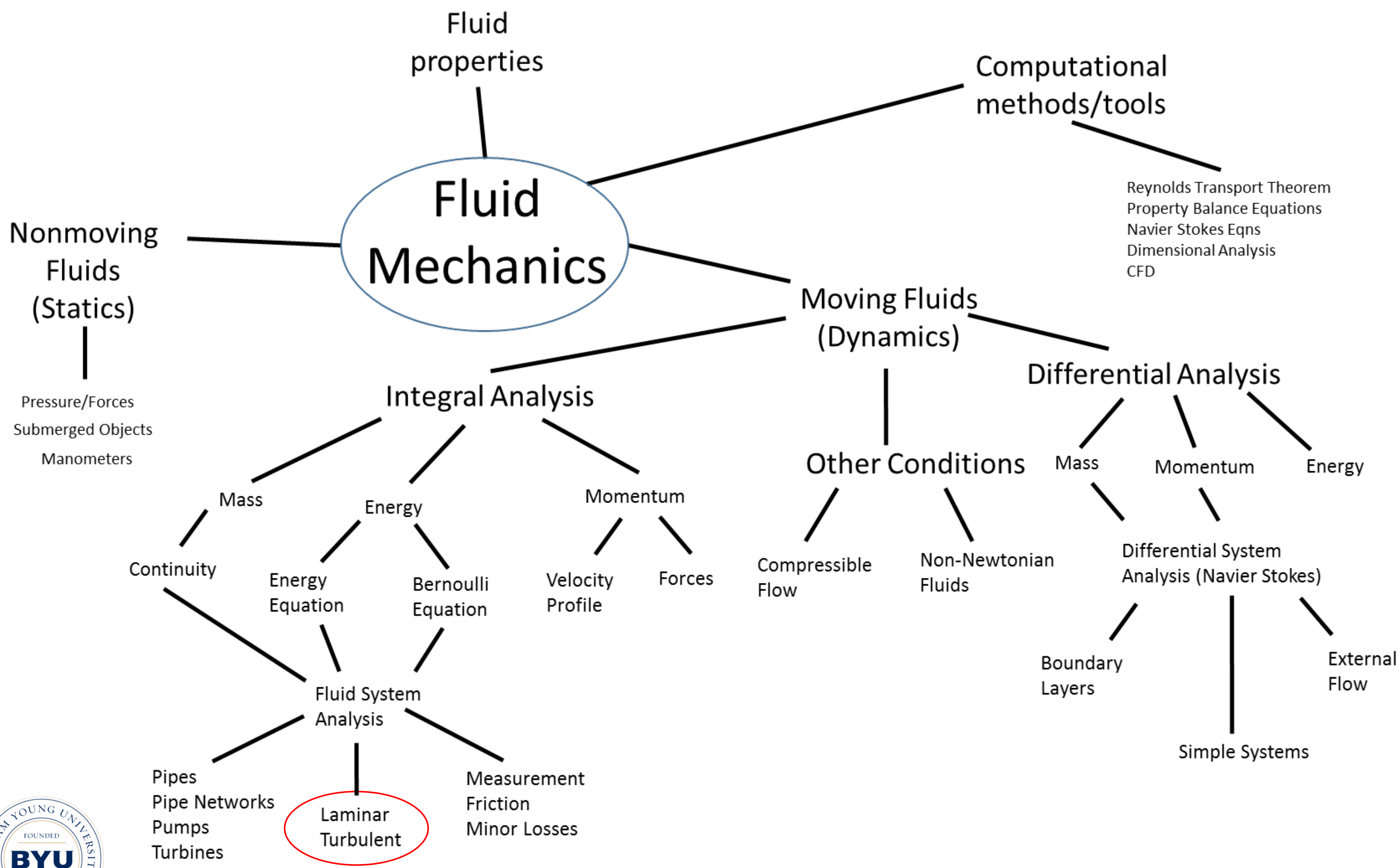
7. Verify your answer... Does it look reasonable? Anything odd about the calculation?

a) It turns out that at 72 mph, the drag forces (5718.2N) balance out the force of the steam engine, meaning no further acceleration is possible. To fix this problem, Doc Brown created exploding logs to create higher temperature, pressure, and thus acceleration in the steam engine. How much acceleration did the log need to provide in order to overcome the drag forces and bring the DeLorian to 88 mph (assume drag forces at 88 mph = 8542.1N)?

b) Could this burst of acceleration really have broken the rod that Clara was holding on to? Could Doc Brown have maintained his hold?

c) It is 3 miles from the location of the DeLorian (at 0 mph initially) to the ravine. Is this enough distance to enable reaching 88 mph, based on your analysis?

Fluids Roadmap

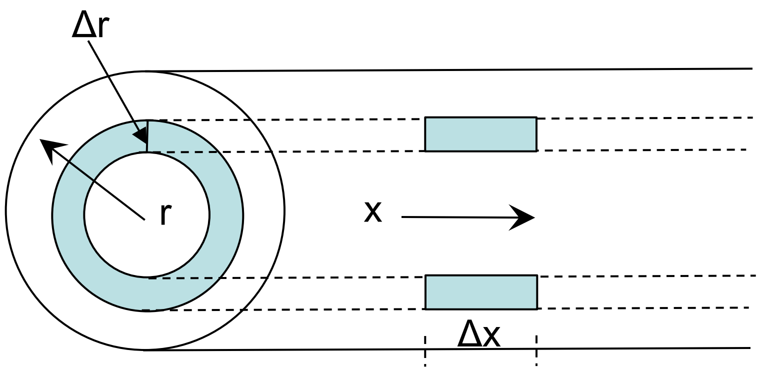


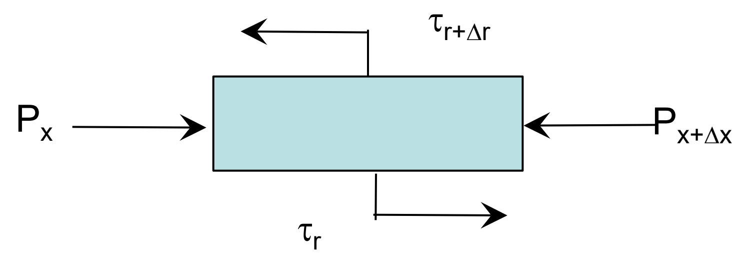
Laminar vs. Turbulent

- Laminar Flow – Smooth, Streamlines, ordered motion
- Turbulent Flow – Chaotic, Random, Disordered

- Reynolds number defines laminar vs. turbulent

Laminar Pipe Flow





Force Balance: Pressure, stress

$$\overbrace{(P_x - P_{x+\Delta x})(2\pi r \Delta r)} + \overbrace{(2\pi \Delta x r)}^{\text{Moving Fluid Shear Stress}}\tau_r - \underbrace{(2\pi \Delta x)(r + \Delta r)\tau_{r+\Delta r}}_{\text{Non-Moving Fluid (Resistive) Shear Stress}} = 0$$

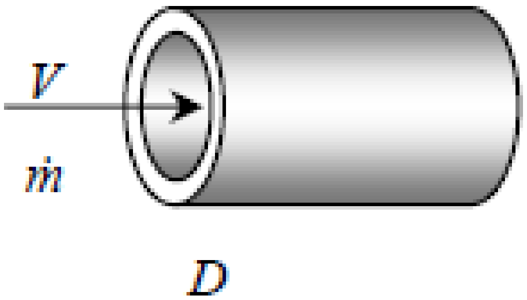
Divide $2\pi\Delta r\Delta x$

$$r\frac{P_x - P_{x+\Delta x}}{\Delta x} + \frac{r\tau_r - (r+\Delta r)\tau_{r+\Delta r}}{\Delta r} = 0$$

Limit Δx, $\Delta r \rightarrow 0$

$$-\frac{dP}{dx} = \frac{1}{r}\frac{d(r\tau)}{dr} = C$$

Laminar Friction Derivation



- $\frac{dP}{dx} = \frac{1}{r}\frac{d(\tau r)}{dr}$ == constant
- Solve for τ
- $\tau = -\frac{dP}{dx} \cdot \frac{r}{2}$, evaluate at wall:
- $\tau_w = -\frac{dP}{dx} \cdot \frac{R}{2} = -\frac{\Delta P}{L} \cdot \frac{D}{4}, \quad \therefore 4\tau_w = \frac{-\Delta P D}{L}$
- Insert shear stress expression, $\tau = -\mu\frac{du}{dr}$:
- $-\mu\frac{du}{dr} = -\frac{dP}{dx} \cdot \frac{r}{2}$, solve for u, (bounds: r to R & 0 to u):
- $u(r) = \frac{-R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$, Laminar velocity profile,
- $u_{max}/2 = u_{avg}$

Laminar Pressure drop

- *Recall that we had two dimensionless pi groups for pipe flow: Re and* $\frac{\Delta PD}{L\rho v^2}$
- $4\tau_w = \frac{-\Delta PD}{L}$, from last slide, divide by ρv²
- $\frac{4\tau_w}{\rho v^2} = \frac{-\Delta PD}{L\rho v^2}$, multiply both sides by 2
- $\frac{8\tau_w}{\rho v^2} = \frac{-\Delta PD}{L\rho\left({v^2}/{2}\right)}$, then second pi group is $\mathrm{f} = \frac{8\tau_w}{\rho v^2}$
- Recall $\mathrm{u(r)} = \frac{-R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right)$, umax = u(0) = $\frac{-D^2\Delta P}{16\mu L}$
- Insert $4\tau_w = \frac{-\Delta PD}{L}$ and $\mathrm{f} = \frac{8\tau_w}{\rho v^2}$, get $\mathrm{f} = \frac{64}{Re}$

Memorize!!! KEY EQUATION! $\Delta P = f\,\frac{L}{d}\frac{\rho v^2}{2}$

Non-Newtonian Pipe Flow

- Most non-Newtonian flows are laminar.
- Key results: (remember, Q is just volumetric flow rate-Vdot)

  - Force balance:
  - Power law constitutive relation
  - Integrate with B.C. v=0 at r=R

$$\tau = -\frac{r}{2}\frac{dP}{dx}$$

$$\tau = K\left(-\frac{dv}{dr}\right)^n$$

$$v = \left(-\frac{1}{2K}\frac{dP}{dx}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$$

  - Q = Av_{avg}
    - Q is volumetric flow rate

$$Q = \frac{\pi n D^3}{8(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n} \qquad V_{avg} = \frac{nD}{2(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$$

  - Kinetic Energy Correction Factor: $\alpha = \dfrac{3(3n+1)^2)}{(5n+3)(2n+1)}$

  - Momentum Flux Correction Factor: $\beta = \dfrac{3n+1}{2n+1}$