Chemical Engineering 374

Fluid Mechanics

Turbulent Pipe Flows

Big whorls have little whorls
That feed on their velocity
And little whorls have lesser whorls
And so on to viscosity
--Lewis Richardson

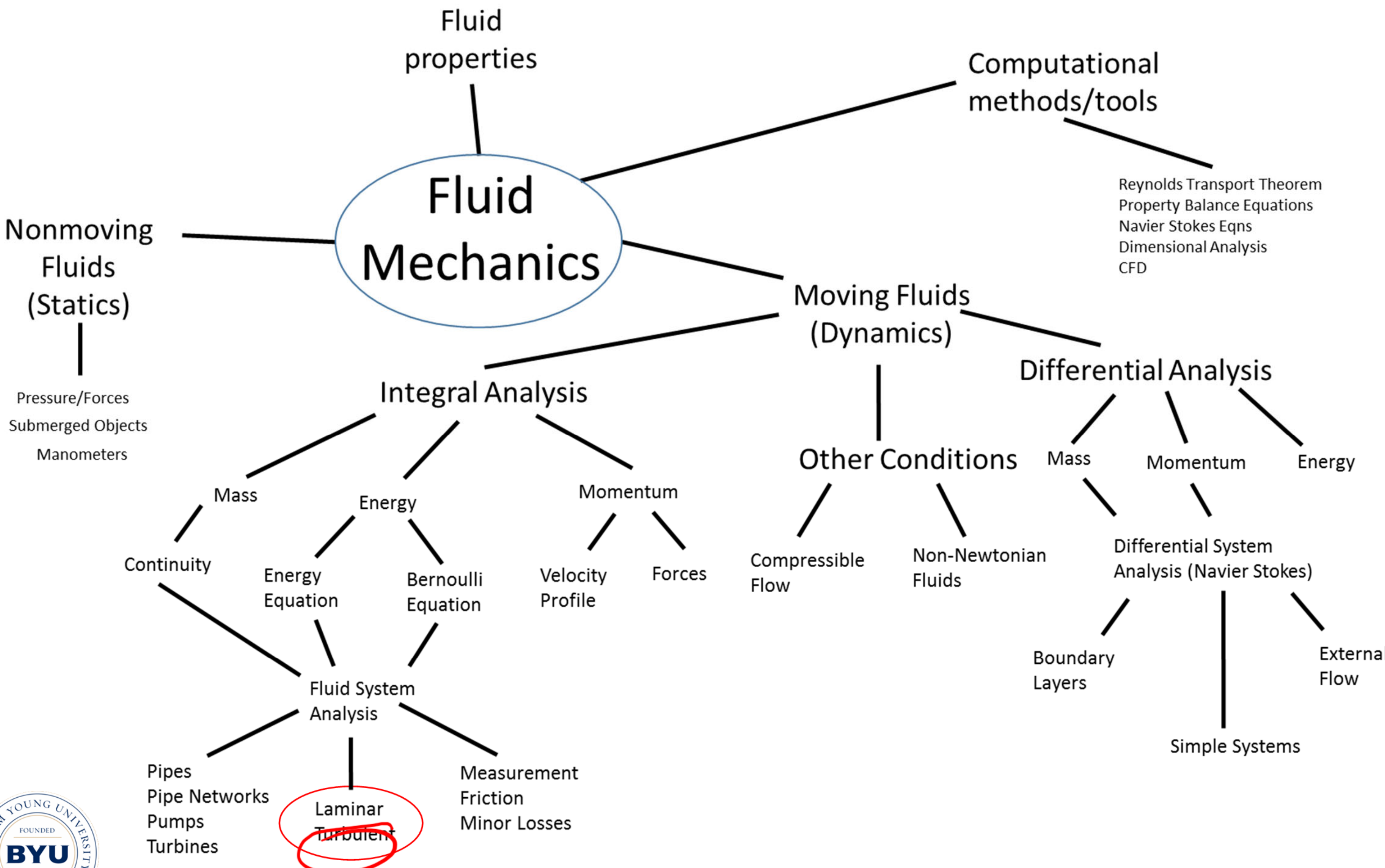
I am an old man now, and when I die and go to heaven there are
two matters on which I hope for enlightenment.
One is quantum electrodynamics, and the other is the
Turbulent motion of fluids.
About the former I am rather optimistic.
--Attr. to Horace Lamb

Spiritual Thought

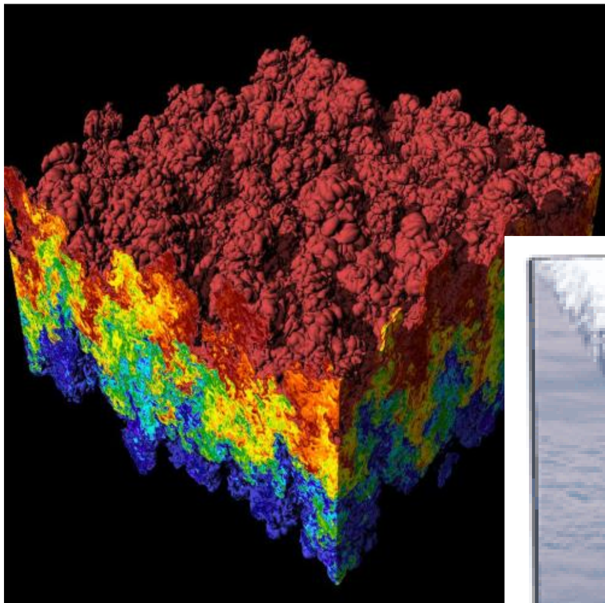
"Marriage is the foundry for social order, the fountain of virtue, and the foundation for eternal exaltation. Marriage has been divinely designated as an eternal and everlasting covenant. Marriage is sanctified when it is cherished and honored in holiness. That union is not merely between husband and wife; it embraces a partnership with God."

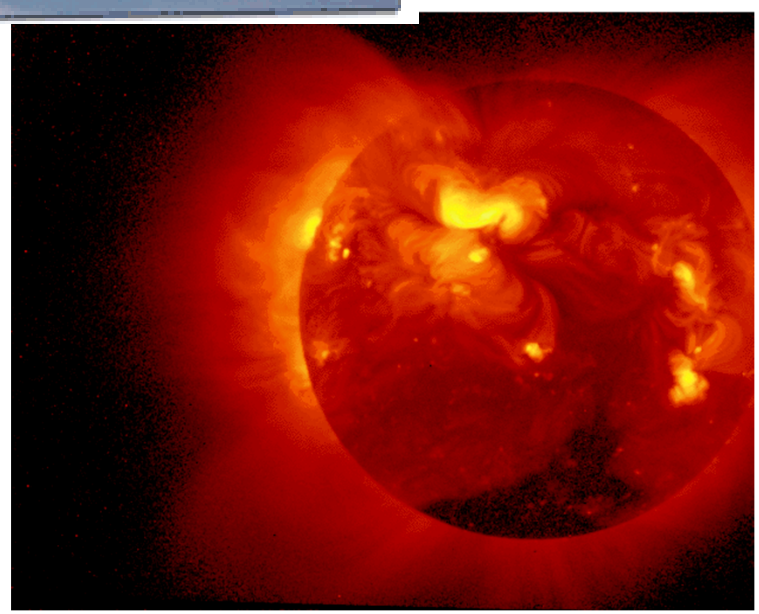
President Russell M. Nelson

Fluids Roadmap



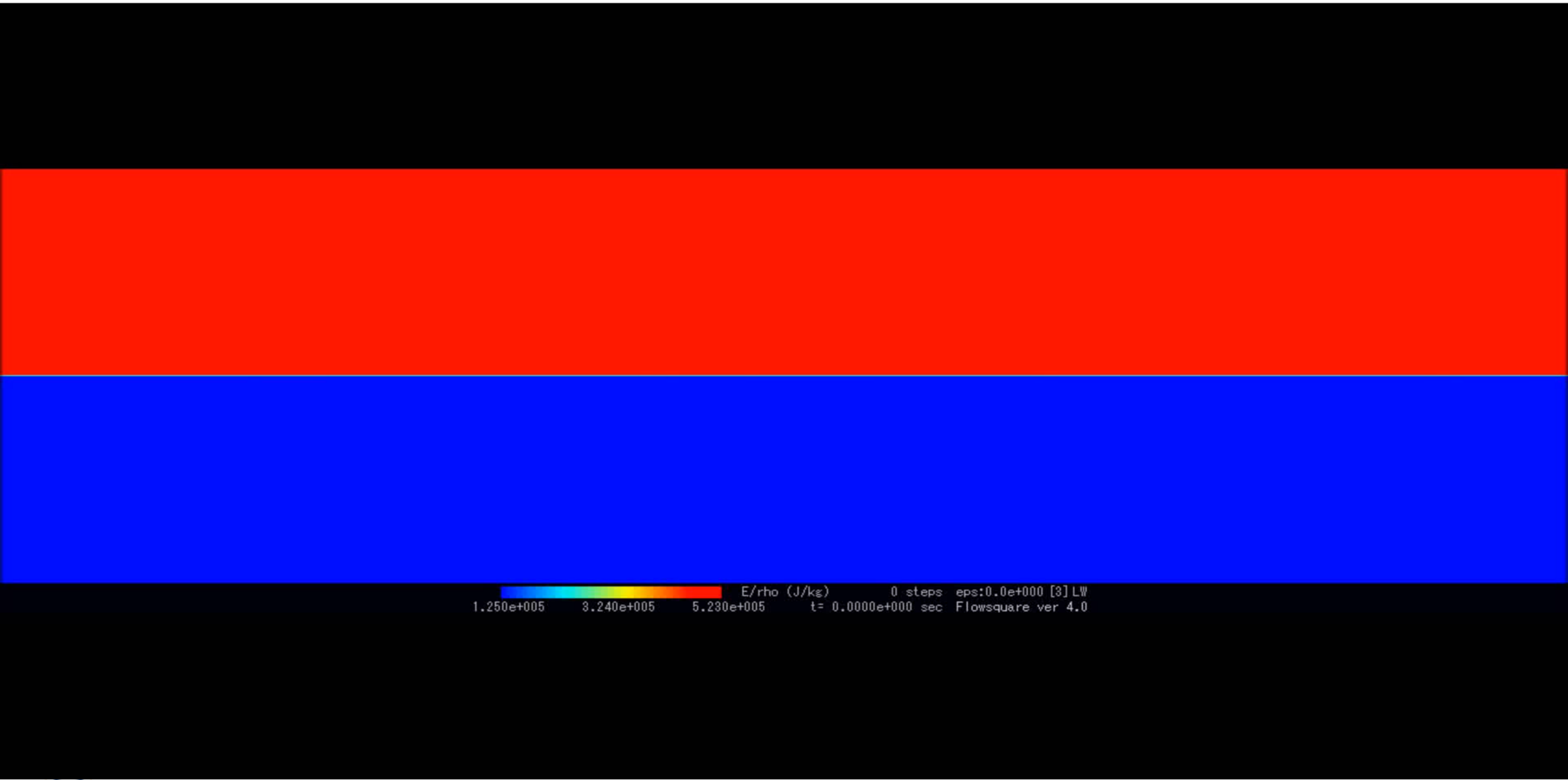
Turbulence examples





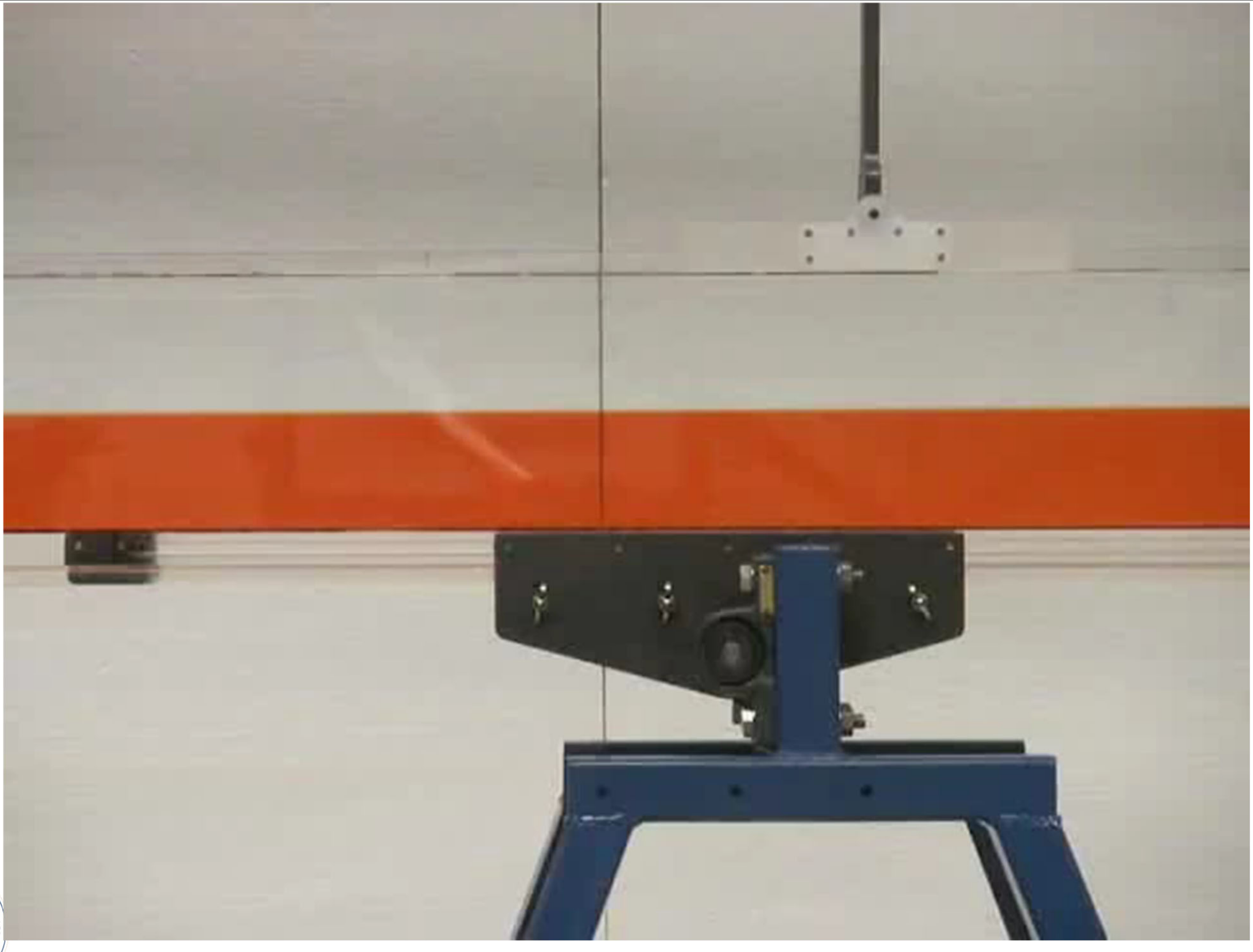
Turbulent Mixing Layer

Flow



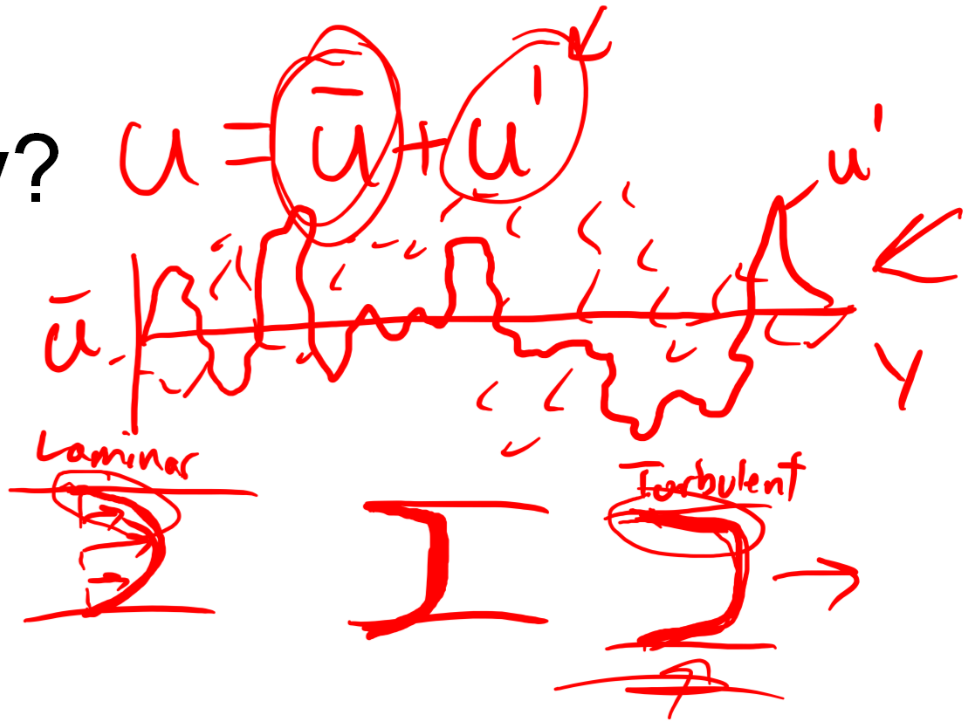
Flow

Instability Flow



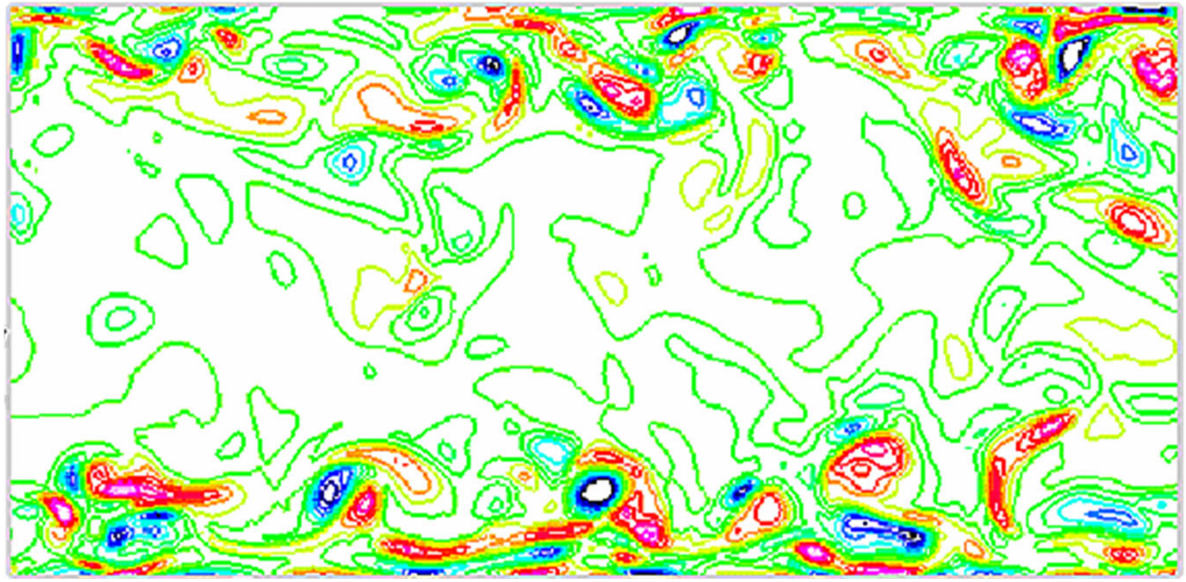
Turbulent Flow

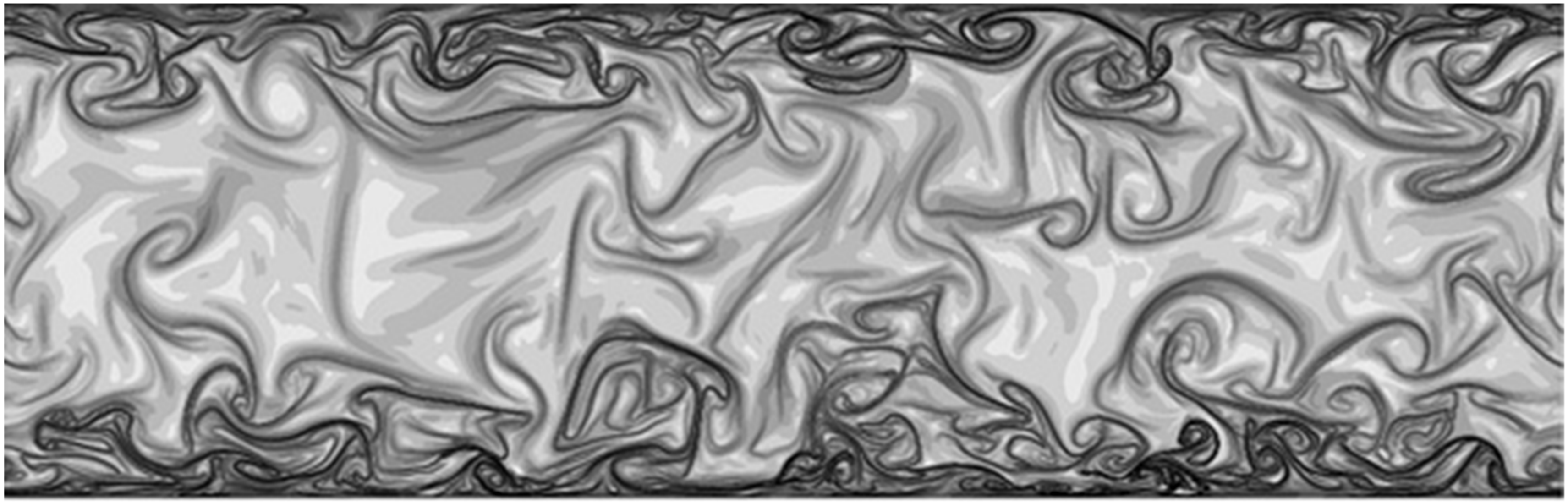
- Turbulent Velocity?
- Turbulent Profile?
- Properties:
  - Higher τ_w
  - Higher Friction
  - Higher Δp_{loss}
  - Shear Stress?



$\tau = -\mu \frac{du}{dy} \Rightarrow \tau = -\mu \frac{d\bar{u}}{dy}$

Channel Flow Simulations





Turbulent Eddies

- Average Shear Stress…
- Too small!
- Add new term for eddy motion
- Transport momentum:
  - $\dot{mom} = \dot{m} \cdot u' = (\rho A v')u' = \tau_t = \rho \overline{v'u'}$
    - Units of stress – F/A
    - Reynolds Stress
    - Don't know it… have to model it
    - $\tau_t = -\mu_t \frac{d\bar{u}}{dy}$

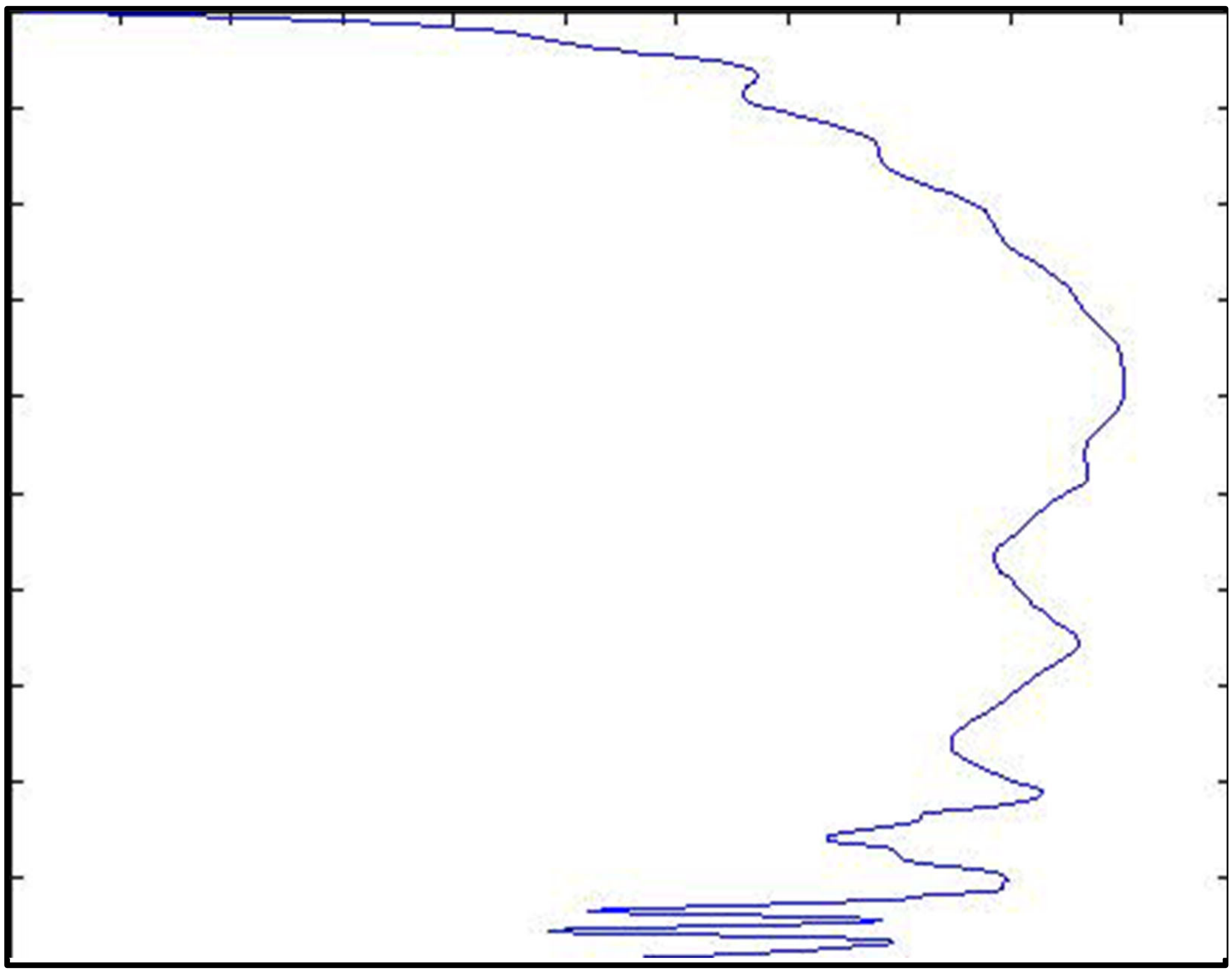
$\tau = \mu \frac{d\bar{u}}{dy}$

$\tau_{tot} = \tau + \tau_t$

Channel Flow Simulation

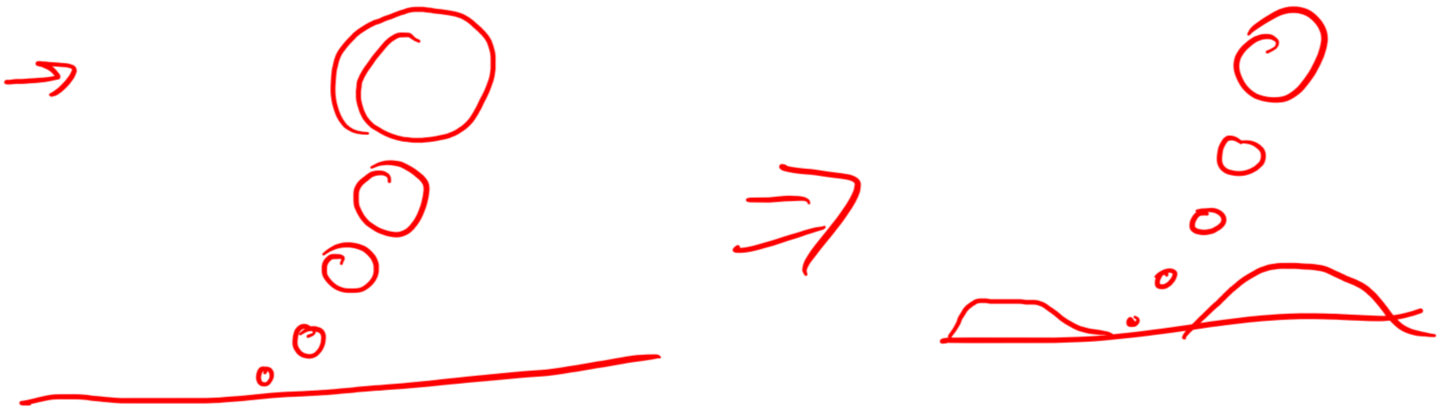
Wall

Flow



Wall

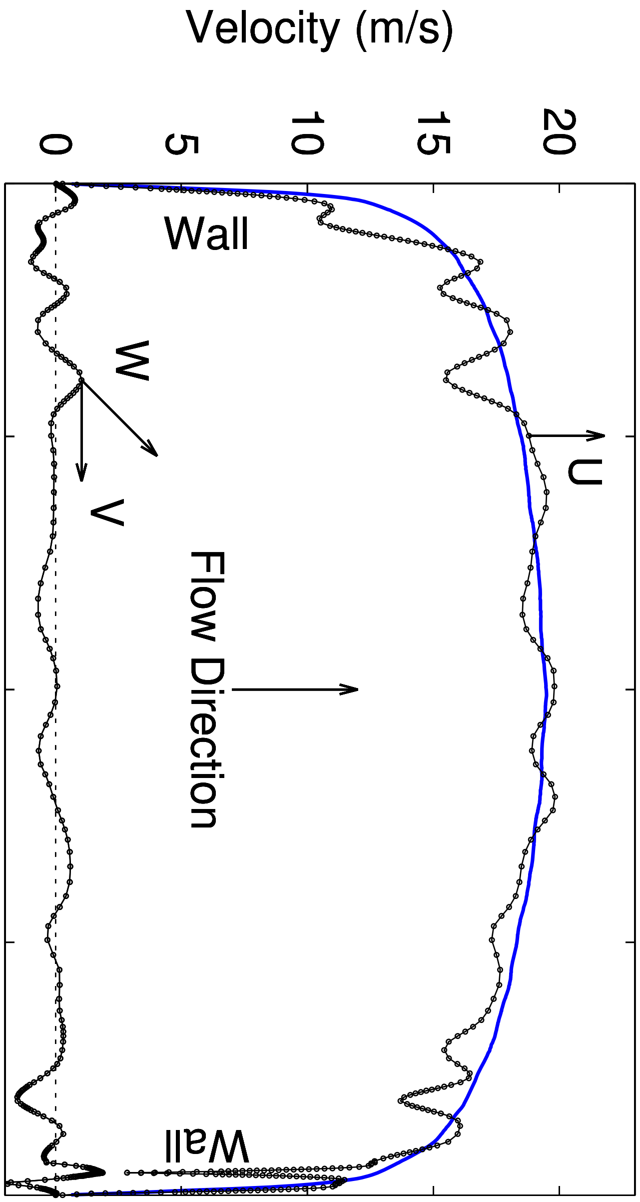
Roughness

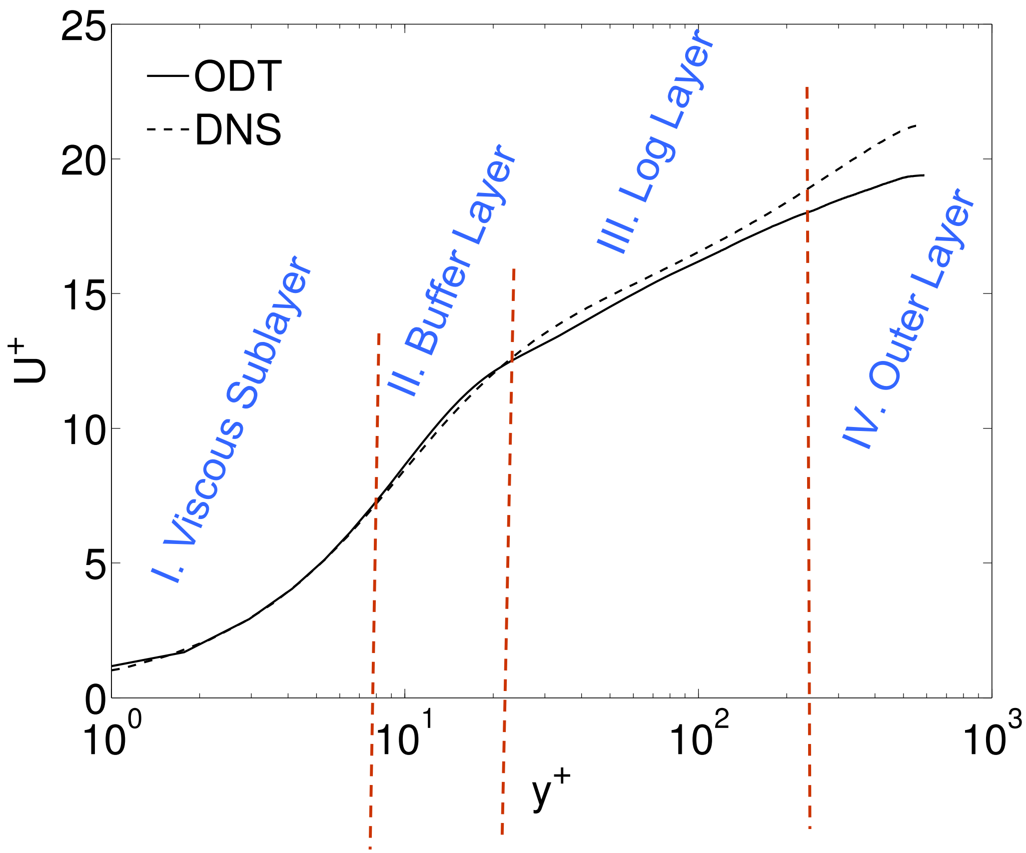


$$\varepsilon / D$$

$$f = f(Re, \varepsilon/D)$$

Instantaneous and Mean

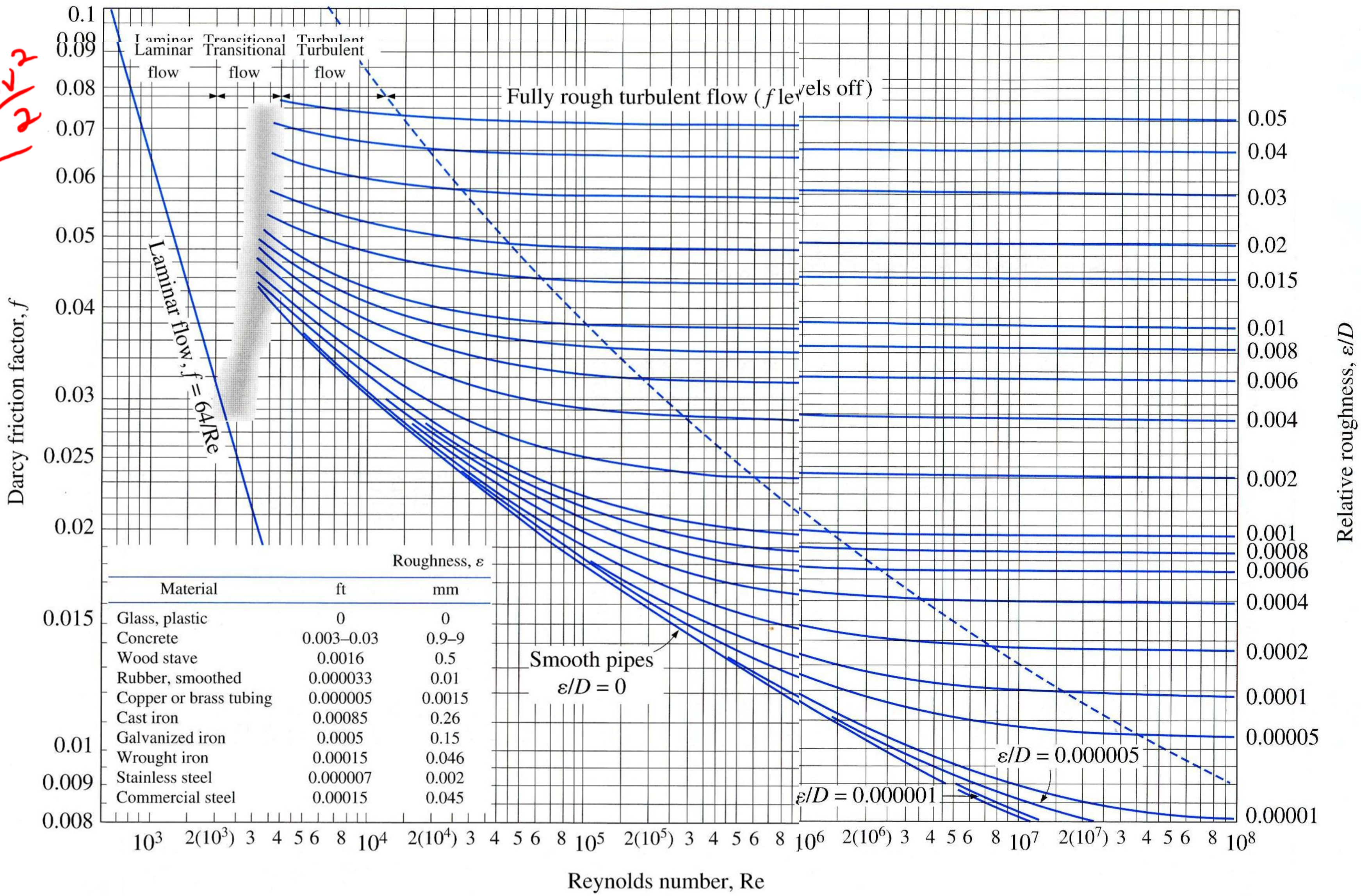




Shape of Regions I and III from dimensional analysis!

Moody Chart

$\Delta P = f \frac{L}{D}\frac{\rho V^2}{2}$



Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

FIGURE A–12

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $h_L = f\frac{L}{D}\frac{V^2}{2g}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$.

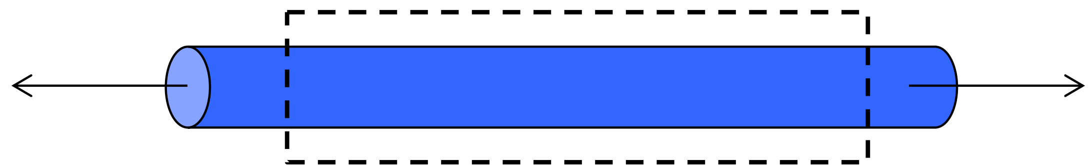
Example

$$\left(\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z\right) = \frac{\dot{W}_u}{\dot{m}} - \frac{\dot{F}}{\dot{m}}$$

$$\frac{\Delta P_L}{\rho} = -\frac{fLv^2}{2D}$$

$$\left(\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z\right) = \frac{\dot{W}_u}{\dot{m}} - \frac{fLv^2}{2D}$$

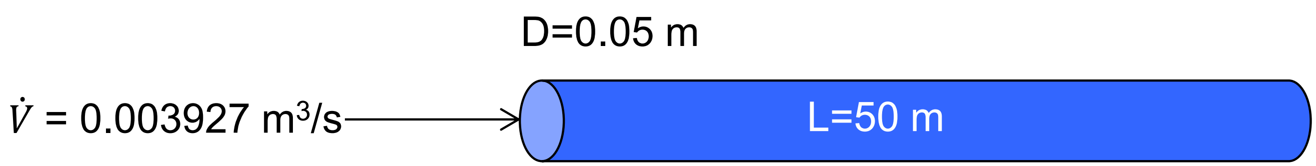
- Consider pipe flow.
- Question: How to simplify the above equation?
- Friction balances pressure drop.
- To find pressure drop for pipe/fluid, and velocity (flow rate):
  - Get f
    - f = f(Re)
    - Get Re = $\rho Dv/\mu$
  - Plug solve equation for ΔP_L



Colebrook $$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

Haaland $$\frac{1}{\sqrt{f}} = -1.8\log\left[\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]$$

Example



- Water flow in a smooth pipe:
- ρ = 1000 kg/m^3,
- μ = 0.001 kg/m*s → ν=1E-6 m^2/s

- Find ΔP_L, h_L, Power

- Question: What is happening physically?
- Question: What do I know?
- Question: What do I want?
- Question: What relationships do I have?

Colebrook $$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

Haaland $$\frac{1}{\sqrt{f}} = -1.8\log\left[\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]$$

- Solve Re → f → ΔP_L → h_L → Power
- Re = $\rho Dv/\mu$:
  - Need v:
  - v = $\dot{V}$ / A = $\dot{V}$ / ($\pi D^2/4$) = 2 m/s
  - Re = 100,000
- Colbrook → f = 0.0178
- $\Delta P = fL\rho v^2/2D$ = 35600 Pa
- $h_L = \Delta P/\rho g$ = 3.63 m
- Power = $\dot{V} \cdot \Delta P$ = 140 W

$$\frac{\Delta P_L}{\rho} = -\frac{fLv^2}{2D}$$

Turbulent Flow Problem Types

- 3 Main problem types:
  1. Find ΔP, given D, L, v

     easy → D, v → Re → f → ΔP

  2. Find $\dot{V}$ given D, L, ΔP

     guess v → Re → f → ΔP → repeat

  3. Find D given v, L, ΔP

     guess D → Re → f → D → repeat