Chemical Engineering 374

Fluid Mechanics

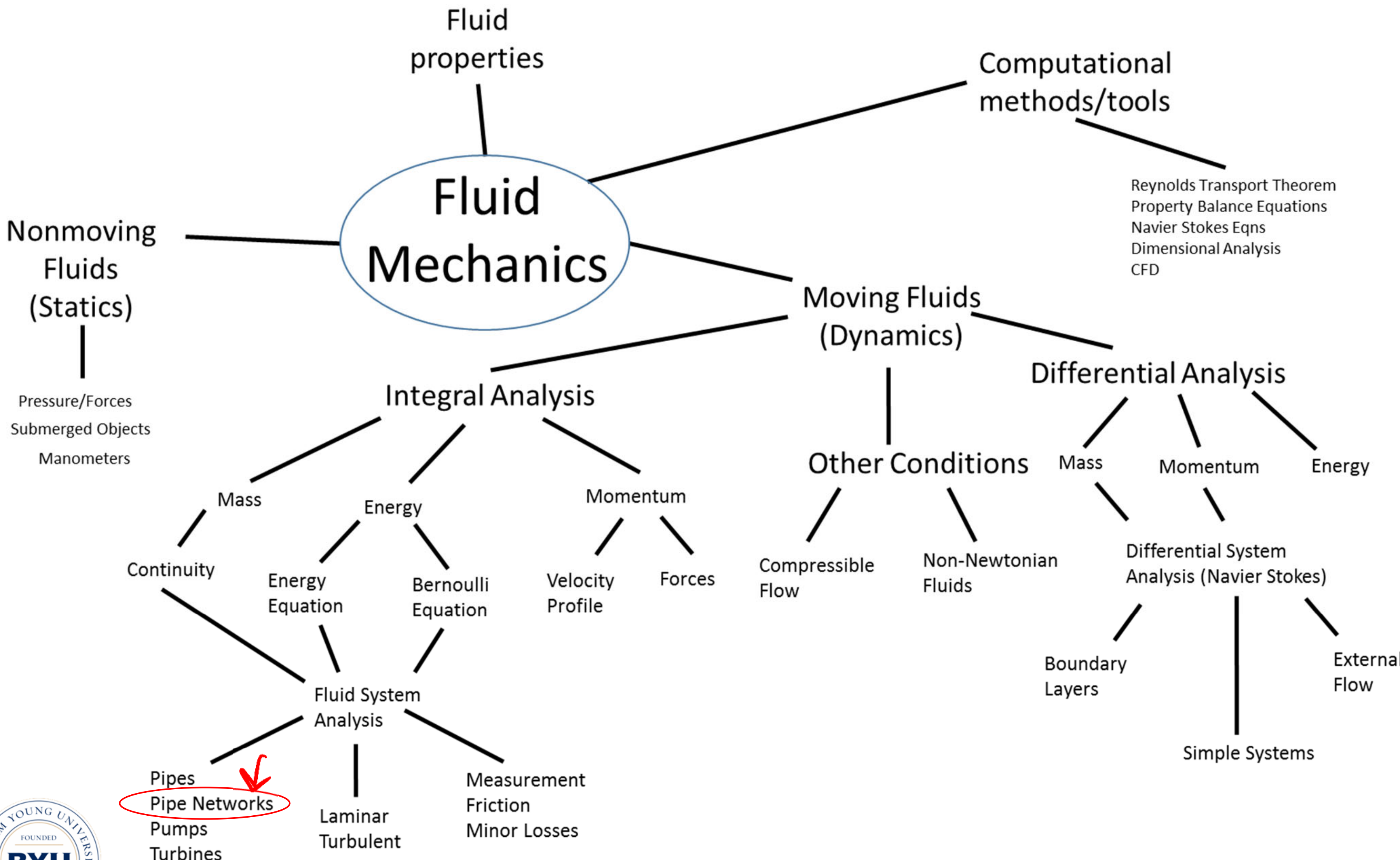
Pipe Networks

Spiritual Thought

Alma 26:22

22 Yea, he that repenteth and exerciseth faith, and bringeth forth good works, and prayeth continually without ceasing—unto such it is given to know the mysteries of God; yea, unto such it shall be given to reveal things which never have been revealed; yea, and it shall be given unto such to bring thousands of souls to repentance, even as it has been given unto us to bring these our brethren to repentance.

Fluids Roadmap



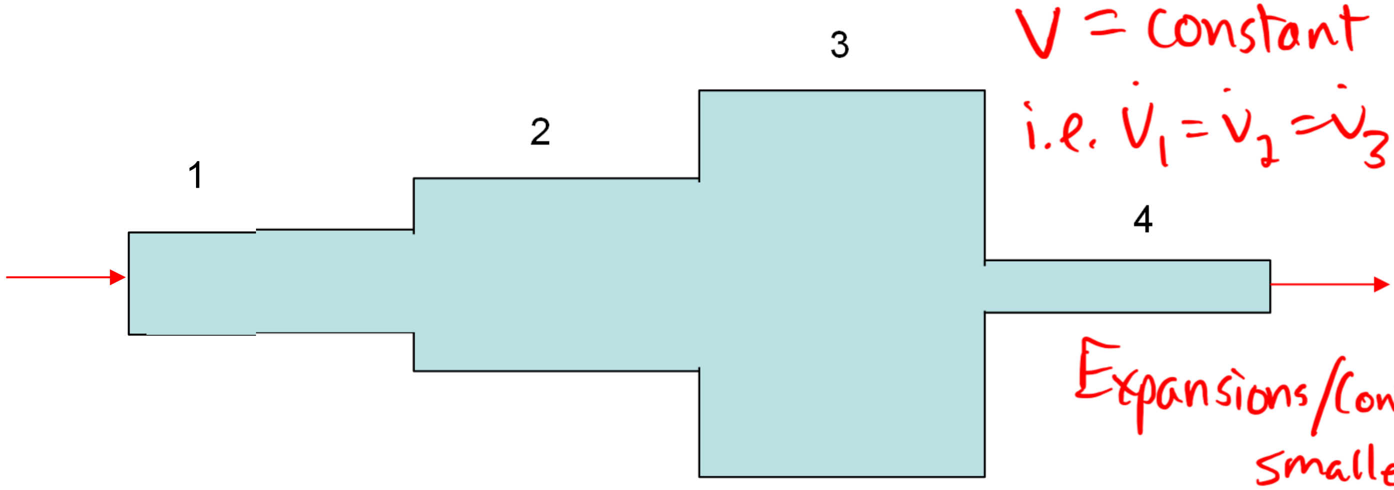
Key Points

- Pipe Networks composed of single pipes
- Pipes is series
  - Type I – find ΔP
  - Type II – find $\dot{V}$
  - Type III or IV – find D, Find L – Doesn’t work!!
- Pipes in parallel
  - Type I
  - Type II
  - Type III or IV – Find D, Find L – Doesn't work
- KNOW graphical solution method!!!!

Pipe Networks

- Most systems are not single pipes
  - Central heating in home
  - Nuclear reactor cooling systems
  - Water supply for cities
  - Oil/product pipes in refineries
  - Etc.
- Can be decomposed into series/parallel
- Similar analysis to electrical circuits

Pipes in Series (I)



$\dot{V}$ = constant by mass conservation

i.e. $\dot{V}_1 = \dot{V}_2 = \dot{V}_3 = \dot{V}_4$

$\Delta P_{tot} = \sum \Delta P_i$

Expansions/Contractions – evaluate using smaller pipe

$$\Delta P = \left(f\frac{L}{D} + K_L\right)\frac{\rho v^2}{2}$$

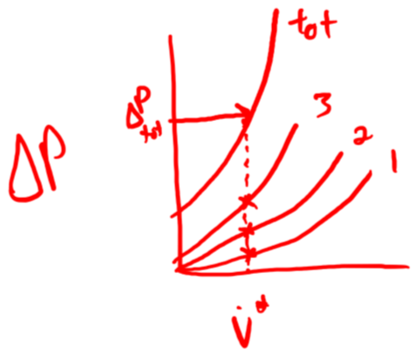
- Type 1 problem: Know L, D, $\dot{V}$, then find ΔP

Calc $Re_1 \rightarrow f_1 \rightarrow \Delta P_1 \rightarrow Re_2 \rightarrow f_2 \rightarrow \Delta P_2 \ldots$ $\Delta P_{tot} = \Delta P_1 + \Delta P_2 + \Delta P_3 + \Delta P_4$

Pipes in Series (II)

- Type 2 problem: Know ΔP, D, L, then find $\dot{V}$

- system demand curve → ΔP for any possible $\dot{V}$



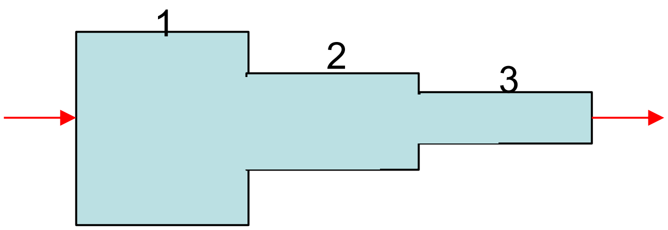
← sum ΔP up to find
total system curve
read off $\dot{V}$ from ΔP_{tot} vs $\dot{V}$ curve

- or solve $\Delta P_t = \sum_i \frac{L_i}{D_i} \frac{\rho v_i^2}{2} f_i$ using colbrook for f_i

→ solve the nonlinear equations

Type III or IV → not unique, can't solve!

Series Example



	L (m)	D (m)	ε (m)
1	100	0.05	0.00024
2	150	0.045	0.00012
3	80	0.04	0.0002

ΔP_t = 320,000 Pa; ignore K_L

Find $\dot{V}$?

$$\Delta P_1 + \Delta P_2 + \Delta P_3 = \Delta P_t$$

$$\Delta P_1 = f_1 \frac{L_1}{D_1} \frac{V^2 \rho}{2}$$

$$\Delta P_2 = \cdots\cdots$$

$$\Delta P_3 = \cdots\cdots$$

$$V_i = \frac{\dot{V}_i}{A_i}$$

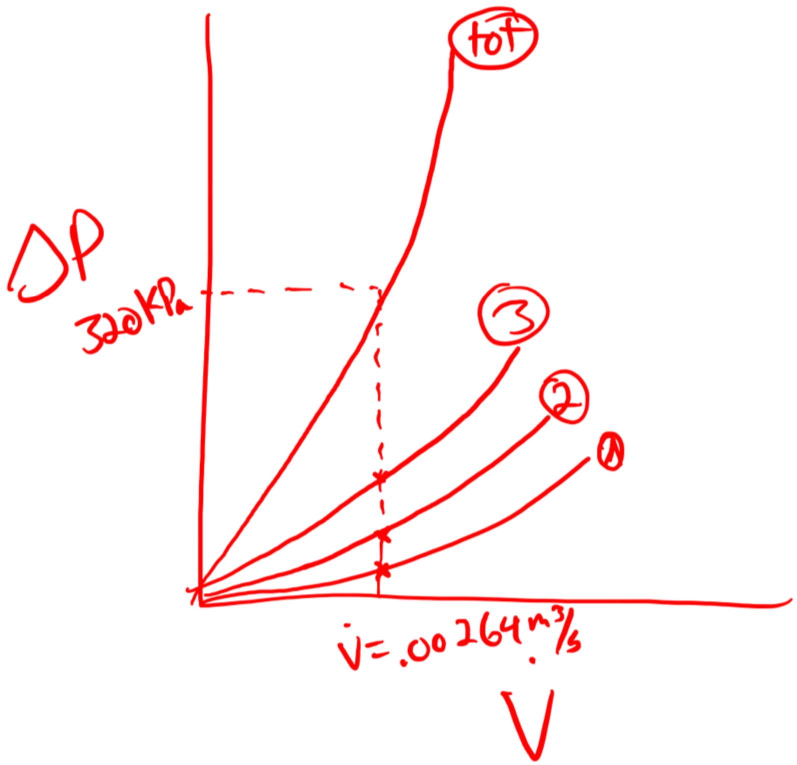
$$f_i = f_i(Re_i, \varepsilon_i/D_i)$$

$$Re_i = \frac{\rho V_i D_i}{\mu}$$

Solve system w/ MathCAD, python

Graphical Series

<u>OR</u> use graphical method:



MathCAD solution (Pipes in Series)

Mathcad - [Series_pipes.xmcd]

$Pt := 320000$ $\quad \rho := 998$ $\quad \mu := 1.002 \cdot 10^{-3}$

$e1 := 0.00024$ $\quad L1 := 100$ $\quad D1 := 0.05$

$e2 := 0.00012$ $\quad L2 := 150$ $\quad D2 := 0.045$

$e3 := 0.0002$ $\quad L3 := 80$ $\quad D3 := 0.04$

Guesses

$f1 := 0.01$

$f2 := 0.01$

$f3 := 0.01$

$Q := 0.1$

Given

$$Pt = f1 \cdot \frac{L1}{D1} \cdot \frac{\rho}{2} \cdot \left(\frac{4 \cdot Q}{\pi \cdot D1^2}\right)^2 + f2 \cdot \frac{L2}{D2} \cdot \frac{\rho}{2} \cdot \left(\frac{4 \cdot Q}{\pi \cdot D2^2}\right)^2 + f3 \cdot \frac{L3}{D3} \cdot \frac{\rho}{2} \cdot \left(\frac{4 \cdot Q}{\pi \cdot D3^2}\right)^2$$

$$\frac{1}{\sqrt{f1}} + 2 \cdot \log\left[\frac{e1}{D1 \cdot 3.7} + \frac{2.51}{\left(\frac{\rho \cdot D1}{\mu} \cdot \frac{4 \cdot Q}{\pi \cdot D1^2}\right) \cdot \sqrt{f1}}\right] = 0$$

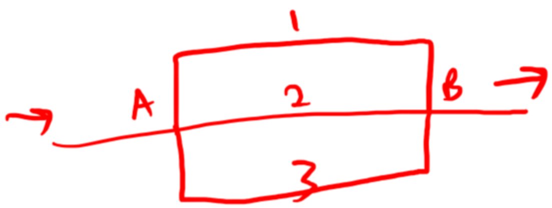
$$\frac{1}{\sqrt{f2}} + 2 \cdot \log\left[\frac{e2}{D2 \cdot 3.7} + \frac{2.51}{\left(\frac{\rho \cdot D2}{\mu} \cdot \frac{4 \cdot Q}{\pi \cdot D2^2}\right) \cdot \sqrt{f2}}\right] = 0$$

$$\frac{1}{\sqrt{f3}} + 2 \cdot \log\left[\frac{e3}{D3 \cdot 3.7} + \frac{2.51}{\left(\frac{\rho \cdot D3}{\mu} \cdot \frac{4 \cdot Q}{\pi \cdot D3^2}\right) \cdot \sqrt{f3}}\right] = 0$$

$$\begin{pmatrix} Q \\ f1 \\ f2 \\ f3 \end{pmatrix} := \mathrm{Find}(Q, f1, f2, f3)$$

$$\begin{pmatrix} Q \\ f1 \\ f2 \\ f3 \end{pmatrix} = \begin{pmatrix} 2.64074 \times 10^{-3} \\ 0.03141 \\ 0.02716 \\ 0.03148 \end{pmatrix}$$

Pipes in Parallel (I)



$$\dot{V}_{tot} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3$$

ΔP is constant accross each (from A to B by any path)

Type I: $\dot{V}_1$ known, ΔP & $\dot{V}_{tot}$ unknown: Find ΔP

- calculate ΔP, using $\dot{V}_1$, Then w/ ΔP calculate $\dot{V}_2, \dot{V}_3$ (type II problem)

$$\dot{V}_1 + \dot{V}_2 + \dot{V}_3 = \underline{\dot{V}_{tot}}$$

Pipes in Parallel (II)

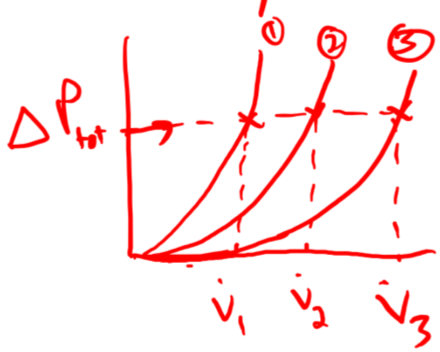
Type II : ΔP known, flow rates unknown

Calculate Type II problem for each pipe

$$\dot{V}_{tot} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3$$

or

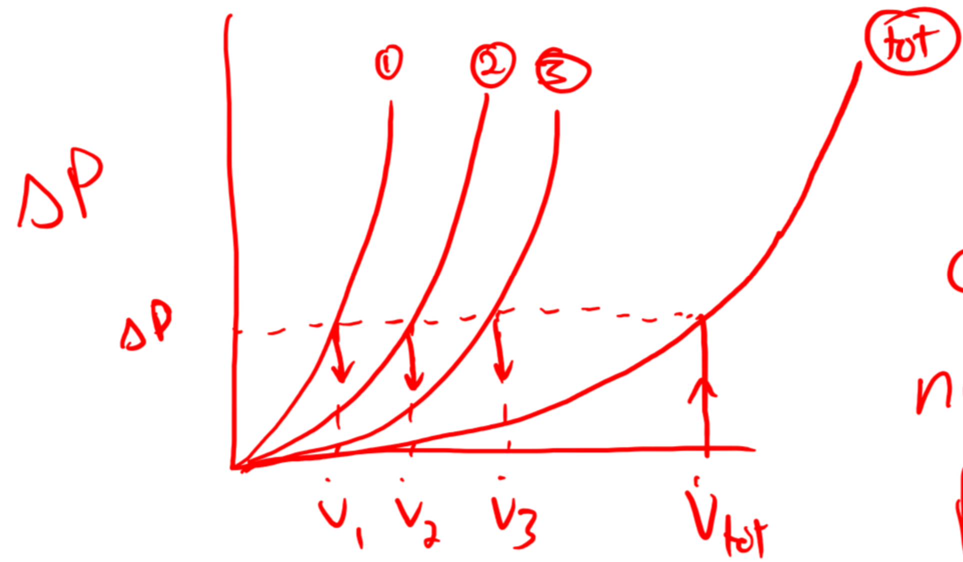
system demand curve



Pipes in Parallel (III)

- Type I: Total flow rate ($\dot{V}_{tot}$) known, ΔP unknown

  - system demand curve: sum to the right:

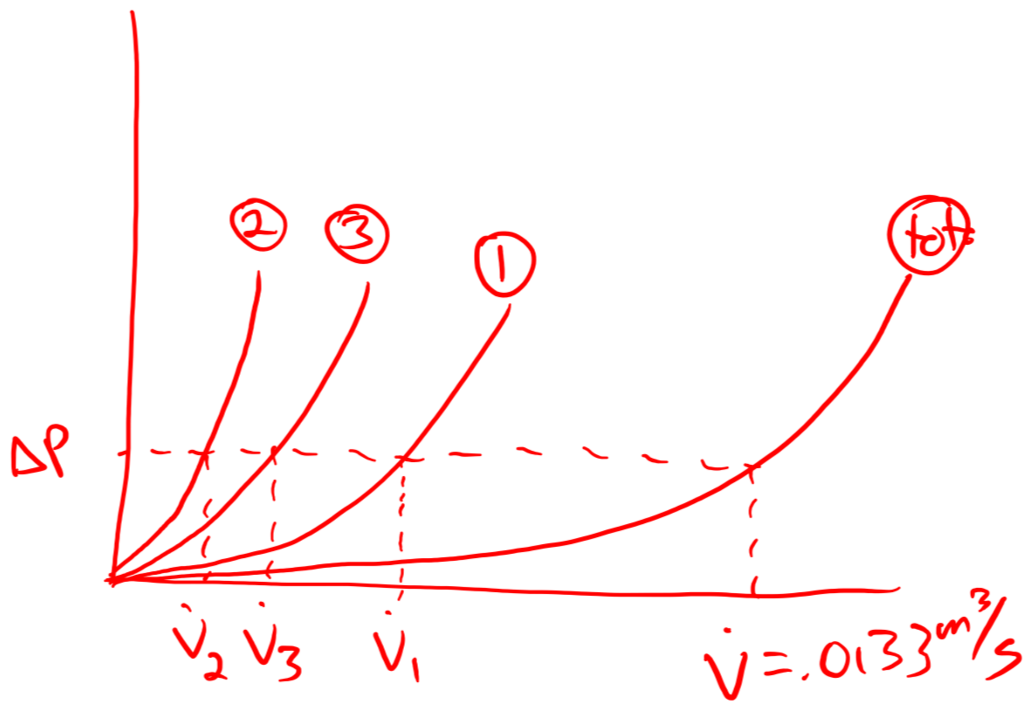


or solve multiple eq. numerically via Python or MathCAD

Parallel Example

	L (m)	D (m)	ε (m)
1	100	0.05	0.00024
2	150	0.045	0.00012
3	80	0.04	0.0002

$\dot{V}$ = .0133 m^3/s Pa; Find ΔP



MathCAD solution (Parallel Pipes)

Mathcad - [Parallel_pipes.xmcd]

$Qt := 0.01333 \quad \rho := 998 \quad \mu := 1.002\cdot 10^{-3}$

$e1 := 0.00024 \quad L1 := 100 \quad D1 := 0.05$

$e2 := 0.00012 \quad L2 := 150 \quad D2 := 0.045$

$e3 := 0.0002 \quad L3 := 80 \quad D3 := 0.04$

Guesses

$f1 := 0.01 \quad Q1 := 0.004$

$f2 := 0.01 \quad Q2 := 0.004$

$f3 := 0.01 \quad Q3 := Qt - Q1 - Q2$

Given

$$f1\cdot\frac{L1}{D1}\cdot\frac{\rho}{2}\cdot\left(\frac{4\cdot Q1}{\pi\cdot D1^2}\right)^2 = f2\cdot\frac{L2}{D2}\cdot\frac{\rho}{2}\cdot\left(\frac{4\cdot Q2}{\pi\cdot D2^2}\right)^2$$

$$f1\cdot\frac{L1}{D1}\cdot\frac{\rho}{2}\cdot\left(\frac{4\cdot Q1}{\pi\cdot D1^2}\right)^2 = f3\cdot\frac{L3}{D3}\cdot\frac{\rho}{2}\cdot\left(\frac{4\cdot Q3}{\pi\cdot D3^2}\right)^2$$

$$\frac{1}{\sqrt{f1}} + 2\cdot\log\left[\frac{e1}{D1\cdot 3.7} + \frac{2.51}{\left(\frac{\rho\cdot D1}{\mu}\cdot\frac{4\cdot Q1}{\pi\cdot D1^2}\right)\cdot\sqrt{f1}}\right] = 0$$

$$\frac{1}{\sqrt{f2}} + 2\cdot\log\left[\frac{e2}{D2\cdot 3.7} + \frac{2.51}{\left(\frac{\rho\cdot D2}{\mu}\cdot\frac{4\cdot Q2}{\pi\cdot D2^2}\right)\cdot\sqrt{f2}}\right] = 0$$

$$\frac{1}{\sqrt{f3}} + 2\cdot\log\left[\frac{e3}{D3\cdot 3.7} + \frac{2.51}{\left(\frac{\rho\cdot D3}{\mu}\cdot\frac{4\cdot Q3}{\pi\cdot D3^2}\right)\cdot\sqrt{f3}}\right] = 0$$

$$Qt = Q1 + Q2 + Q3$$

$$\begin{pmatrix} f1 \\ f2 \\ f3 \\ Q1 \\ Q2 \\ Q3 \end{pmatrix} := \mathrm{Find}(f1, f2, f3, Q1, Q2, Q3) \qquad \begin{pmatrix} f1 \\ f2 \\ f3 \\ Q1 \\ Q2 \\ Q3 \end{pmatrix} = \begin{pmatrix} 0.030663 \\ 0.026613 \\ 0.03118 \\ 5.775203\times 10^{-3} \\ 3.889447\times 10^{-3} \\ 3.66535\times 10^{-3} \end{pmatrix}$$

$$\Delta P := f1\cdot\frac{L1}{D1}\cdot\frac{\rho}{2}\cdot\left(\frac{4\cdot Q1}{\pi\cdot D1^2}\right)^2 \qquad \Delta P = 2.647\times 10^5$$