Chemical Engineering 374

Fluid Mechanics

Exam 2 Review

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Spiritual Thought

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“Do we really know what we have?”

- Gertrude Specht

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Exam Review, By Content/Lectures

- OEPs
- Classes 13-21
- Chapter 5.6 Mechanical Energy
- Chapter 7.1-7.5 Dimensional Analysis
- Chapter 8.1-8.8 Pipe Flows
  - Laminar
  - Turbulent
  - Minor Losses
  - Single Pipelines
  - Pipe networks
  - Flow measurement

Open Ended Problems

- Think about what you are trying to figure out
  - What things do you know?
  - What things would be helpful to know?
  - Is there equations or laws that will help?
- Make simplifying but legitimate assumptions
  - Don't assume problem solution
  - Don't get too detailed. If an assumption (steady state vs. transient) makes a problem drastically easier, say so and get to work. Worst case you explain in the check what went wrong!
  - JUSTIFY!!!!
    - "This adds 10% error, but allows a theoretical solution"
    - "Assuming 1D flow is inaccurate, but only introduces a small error"
    - "By assuming roughness is constant, we don't account for additional friction at the end of pipe life, but it allows to get an approximate view of what the pressure drop is like…"
- Check with reasonable methods
  - If you see an oddity in the answer, say so; you'll get credit for thinking through it!!!
  - "Correct" is subjective… process, assumptions, and thoughts about the answer are better.

Class 13—Mechanical Energy

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- Mechanical Energy Balance
  - Steady state $\left(\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z\right) = \frac{\dot{W}}{\dot{m}} - \frac{\dot{F}}{\dot{m}}$
  - Friction losses included but just given
  - ME changes due to shaft work and friction
  - What if have heat transfer?
  - Head form?
- KE correction factor α
  - Know this, but usually ignore, unless laminar
- Examples similar to Chp 8, where we compute the friction losses explicitly.

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Class 14—Dimensional Analysis

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- Dimensional Homogeneity
- Nondimensionalize equations: $t^*=t/t_{ref}$; $t=(t_{ref})(t^*)$
- Scaling
  - Terms in equations become product of [..](..), where [..] give size, (..) is O(1).
  - Can use this to find functional forms.
    - Heat equation characteristic time
    - Boundary layer thickness.
- Similarity—3 types (geometric, kinematic, dynamic)
  - Equivalent systems are similar
  - The Π groups need to be equal between model and full-scale

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Class 15—Dimensional Analysis

- Find dimensionless groups (Π's)
  - 3 methods: governing equations, force ratios, Π **method**
- Π method is general (but can be cumbersome)
  - n parameters
  - j dimensions
  - k=n-j Π's
- Fluids, usually have 3 dimensions (m, s, kg)
- Can usually guess the Π's directly
  - Use all parameters
  - Helps to pick j=n-k repeating parameters that appear in all Π's
  - Re is common in fluids with viscous effects (friction, drag, etc.)

Class 16—Laminar Pipe Flow

- Pipe discussion
- More on the Re (physical intuition: force ratio, timescale ratio)
  - Re < 2300 is laminar; Re > 4000 is turbulent (transition in between)
  - 2300 is the number to remember as the laminar/turbulent cutoff
  - Most flows are turbulent
- Hydraulic diameter (for noncircular pipes) $D_h=4A_c/P_w$
- Entrance region
  - Fully developed flow takes time/space
  - Wall stress/friction/pressure drop is higher in entrance region.
- Derive the velocity profile
  - Force balance → ODE (pressure, wall friction) (BC: v=0 at wall, dv/dx=0 at CL)
  - 2 integrations → parabolic profile
  - dp/dx is constant
  - $v_{avg} = ½\, v_{max}$ (for circular pipes!)
  - f defined, f = 64/Re

$$f \equiv \frac{4\tau_w}{\rho v^2/2}$$

Class 17—Turbulent Flow

- Darcy friction factor: $f = 8\tau_w/\rho v_{avg}^2$
  - $\Delta P_L/\rho = fLv^2/2D$
  - $f = 4 * f_{Fanning}$
- Velocity profile is flatter, with steep wall gradients → higher wall friction → higher friction than laminar.
  - Power law velocity profiles available
- Pipe roughness is important due to thin viscous wall layer.
- Params are $\Delta P/L$, ρ, μ, ε, D, v → 3 Π's → f(Re, ε/D)
  - Colbrook equation (implicit)
  - Haaland equation
  - Moody chart
  - Note simplifications: rough pipes at high Re → f is constant (read off Moody, or simple Colbrook)
  - Smooth pipes give a simpler Colbrook equation

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Class 18—Minor (fitting) Losses

- Losses from fittings, bends, valves, etc.
- In a pipe of a given size, use ΣK_L
- Table 8-4
- Based on the smaller of two diameters
- Use loss coefficient or equivalent lengh and add to pipe length.
- Don't forget expansions! K=1 (K=α).
- Contractions → vena contracta
- Valves increase friction to decrease flow rate for given pressure difference.

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Class 19—Single Pipelines

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$$\left(\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z\right) = -\frac{fLv^2}{2D} - \frac{\Sigma K_L v^2}{2} + \frac{\dot{W}_s}{\dot{m}}$$

- Note well the mechanical energy balance equation above with shaft (pump) work, pipe losses, and minor losses.
- Three problem types.
  - Find P, find ***V***, find D
  - Some require iteration. Know how to do this to solve the three types.
    - Find ***V*** : Guess f, v from E eqn., get Re, get f=f(Re, e/D), repeat.
    - Find D : Guess D, get Re, get f=f(Re,e/D), D from E. eqn, repeat.
  - System demand curve can turn a harder type into an easy type to "bypass" iteration.
- Examples given
- Economic pipe diameter (velocity), pipe size charts.

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Class 20—Pipe Networks

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- 2 Key parameters: ΔP, ***V***
- Series Flow
  - $\Delta P_{tot} = \Sigma \Delta P_i$
  - Constant ***V***
- Parallel Flow
  - $V_{tot} = \Sigma V_i$
  - $\Delta P_i = \Delta P_j = \Delta P_k$ For pipes between the same two nodes
- Type 1 (find ΔP) and 2 (find ***V***) problems considered
- A system demand curve can help conceptually (and computationally)
- Can also set up and solve system of nonlinear equations
- More complex networks are the sum of the parts
  - $\Sigma Q_i=0$ at "nodes" (pipe junctions)
  - $\Sigma \Delta P_i=0$ around loops.
  - Like Kirchoff's laws for current flow (but nonlinear)

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Class 21—Flow Measurement

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- Flow meter types discussed
- Emphasis on Bernoulli types
  - Pitot, Pitot-static
  - Orifice meters
  - Nozzles
  - Venturi meters
- Rotameters
- Discharge coefficient correlations provided
  - Require iteration, but good guesses given low variation of C_d
  - Book provides average values (their values agree better with correlations at lower Re)
    - 0.61, 0.96, 0.98 for orifice, nozzle, venturi, respectively

$$v = C_d\sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

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OEP (I)

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Buzz Lightyear, the heroic Space Ranger from Star Command, is on an urgent quest to find and capture the evil Emperor Zurg before he can utilize his world-destroying tembler bomb, thus ending life in the Universe as we know it. He chased Zurg to a mystical and somewhat backward world known as Hyrule, but was there held up by some kid with a sword who was named after part of a chain. After convincing the youth that he was a friend, and promising that he had no idea why 3 triangles were so important, Buzz discovered that Zurg's hideout could be reached through a vent at the bottom of a deep chasm in the ocean. By following a strong current within this vent, Buzz is able to float quickly to the center of the planet. Just as Buzz is about to reach Zurg's hidden base, the tembler bomb activates, and will destroy the planet within 2 minutes. Buzz has 4 paths to choose from in order to reach Zurg's hideout at the planet's watery center in time to diffuse the bomb, as indicated by the drawing below. Which path provides the best chance of reaching Zurg's base in time, and why? (i.e. prove it...)

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OEP (II)

- What is the problem asking for?

The shortest time to get to Zurg's base. i.e. $t = L/v \Rightarrow$ shortest pathlength to velocity ratio

- What assumptions are you making in order to solve the problem?

ρ = constant, (liquid), No shaft work (No pumps or turbines)

Picture is to scale...

$D_1 = 0.25$ cm $D_2 = 0.75$

$D_3 = 0.5$ $D_4 = 1$ cm (from part d, 1 cm = 1000 m)

$K_1 = 1.5$ $K_2 = 2.35$ $K_3 = 3.5$ $K_4 = 9.2$

Fully turbulent flow, $f \sim$ constant

$P_{surface} = 1$ atm $g = 9.8\ m/s^2$

Buzz has plenty of O_2

SS, uniform properties, $D \sim$ constant in each pipe, etc.

OEP (III)

- Provide an answer based on your assumptions

4 pipes - $D_1 = 250m$, $D_2 = 750m$, $D_3 = 500m$, $D_4 = 1000m$
$L_1 = 6000m$, $L_2 = 2500m$, $L_3 = 3000m$, $L_4 = 6500m$

$$\Delta P_1 = \Delta P_2 = \Delta P_3 = \Delta P_4 = \Delta P_{total} \therefore \left(f\frac{L_1}{D_1} + K_1\right)\frac{\rho v_1^2}{2} = \left(f\frac{L_2}{D_2} + K_2\right)\frac{\rho v_2^2}{2} = \left(f\left(\frac{L_3}{D_3}\right) + K_3\right)\frac{\rho v_3^2}{2}$$
$$= f\left(\frac{L_4}{D_4} + K_4\right)\frac{\rho v_4^2}{2}$$

(plug in values)

$$782{,}88 v_1^2 = 11796 v_2^2 = 1758.22 v_3^2 = 4608.9 v_4^2$$

Now, since we don't have absolute values but we can do some comparisons... let's normalize these values using V_1

$$\frac{V_1}{V_1} = \sqrt{\frac{782.88}{782.88}} = 1 \quad \frac{V_2}{V_1} = \sqrt{\frac{1179.5}{782.88}} = 1.22 \quad \frac{V_3}{V_1} = 1.498 \quad \frac{V_4}{V_1} = 2.43$$

V_1 is fastest, V_4 is slowest

but, L is significant too!

L relative to $L_1 \Rightarrow \frac{L_1}{L_1} = 1$, $\frac{L_2}{L_1} = .42$, $\frac{L_3}{L_1} = .5$, $\frac{L_4}{L_1} = 1.08$

$$t_1 = \frac{L_1}{V_1}, \quad t_2 = \frac{L_2}{V_2}, \quad t_3 = \frac{L_3}{V_3}, \quad t_4 = \frac{L_4}{V_4}$$

$$t_1 = 1, \quad \underline{t_2 = 0.5}, \quad t_3 = 0.75, \quad t_4 = 2.63$$

Path 2 is fastest!

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- Assuming the drawing is 100,000 scale, how fast does the water in your chosen path need to be flowing in order for Buzz to make it with 30 seconds left to diffuse the bomb?

$$\Delta P = f\frac{L}{D}\frac{\rho v^2}{2} \quad t_2 = \frac{L_2}{V_2} \quad L_2 = 2{,}500\,m \quad K = 3.5$$

$$D_2 = 750\,m$$

$$V_2 = \frac{L_2}{t_2} = \frac{2500\,m}{90\,s} = \boxed{27.78\ m/s}$$

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Water flows from a reservoir, into a **sharp-edged pipe** (K_L=0.5), through a couple of **90° miter bends** (K_L=1.1) to a lower reservoir as shown in the figure. The difference in the levels of the reservoirs is **H=100 m**. The water flows through a pipe of length **L=200 m** and relative roughness **ε/D=0.002**. The velocity in the pipe is **v=10 m/s**, giving a very **high Re**. If the power generated by the turbine is **W_t=4,437,500 J/s**, **Find the required pipe diameter**.

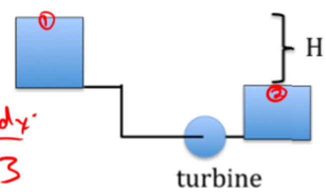
$$\dot m\left(\frac{\cancel{\Delta P}^{0}}{\rho}+\frac{\cancel{\Delta v^2}^{0}}{2}+\Delta g z\right)=\dot W_s-\dot F \qquad K_{L_{tot}}=2.7$$

$$\Delta g z=\frac{\dot W_s}{\dot m}-\left(f\frac{L}{D}+K_L\right)\frac{v^2}{2}$$

Using Moody: $f=.023$

solve for D →

$$\boxed{=0.616\,m}$$



We have seen how turbulence consists of large eddies breaking into smaller eddies until the scales are small enough that viscosity dissipates the kinetic energy (mechanical energy) into internal energy (friction). A most remarkable property of turbulence was found by Kolmogorov using dimensional analysis. He argued that the small scales are determined only by the **kinematic viscosity ν (m²/s)** and the rate that energy is fed to the small scales by the large scales—the **dissipation rate ε (m²/s³)**.

(a) Using some combination of these two parameters, **form a length scale η (m)**, and a **time scale τ (s)** for the small scales (the Kolmogorov (or small eddy) scales).

$$\eta \Rightarrow \varepsilon=\frac{m^2}{s^3} \quad \nu^3=\frac{m^6}{s^3} \quad \frac{\nu^3}{\varepsilon}=\frac{\frac{m^6}{\cancel{s^3}}}{\frac{\cancel{m^2}}{\cancel{s^3}}}=m^4 \quad \frac{\nu^{3/4}}{\varepsilon^{1/4}}=m \quad \therefore \boxed{\eta=\sqrt[4]{\frac{\nu^3}{\varepsilon}}}$$

$$\tau \Rightarrow \frac{\nu}{\varepsilon}=\frac{\frac{\cancel{m^2}}{s}}{\frac{\cancel{m^2}}{s^{3}}}=s^2 \quad \sqrt{\frac{\nu}{\varepsilon}}=s \quad \therefore \tau=\sqrt{\frac{\nu}{\varepsilon}}$$

(b) At high Re, the rate of energy transfer at large scales (big eddies breaking into small ones, etc.) does not depend on viscosity. **Write the dissipation rate ε as a combination of the large length scale L and the large scale velocity v.**

$$\varepsilon = \frac{m^2}{s^3} = \frac{v^3}{L} = \frac{m^3}{m \cdot s^3} = \frac{m^2}{s^3} \Rightarrow \boxed{\varepsilon = \frac{v^3}{L}}$$

(c) Now, find η in a 10 cm diameter pipe (that is, L=D=10 cm) at Re = 5,000?

$$Re = \frac{\rho v D}{\mu} = \frac{vD}{\nu} = 5000 \Rightarrow \frac{v}{\nu} = 500 \quad \nu^{3/4} = \frac{v^{3/4}}{500^{3/4}}$$

$$\eta = \frac{\nu^{3/4}}{\varepsilon^{1/4}} \qquad \varepsilon^{1/4} = \frac{v^{3/4}}{L^{1/4}} \qquad \frac{\nu^{3/4}}{L^{1/4}} = \frac{\frac{\cancel{v^{3/4}}}{500^{3/4}}}{\frac{\cancel{v^{3/4}}}{L^{1/4}}} = \frac{L^{1/4}}{500^{3/4}}$$

$$\boxed{\eta = 0.017}$$

- An orifice with a 1.8in-diameter opening is used to measure the mass flow rate of water at 60 ˚F (ρ = 62.38 lbm/ft^3 and μ = 7.536E-4 lbm/ft-s) through a horizontal 4 in-dimeter pipe. A mercury manometer is used to measure the pressure difference across the orifice. If the differential height of the manometer is 7 in, determine the volume flow rate of water through the pipe, the average velocity and the head loss caused by the orifice meter.

$$\beta = \frac{d}{D} = \frac{1.8}{4} = 0.45 \quad A_o = \frac{\pi d^3}{4} = \pi\left(\frac{1.8}{12\,ft}\right)^2/4 = 0.0177\,ft^2$$

Guess $C_d = 0.61$

$$\Delta P = P_1 - P_2 = (\rho_{Hg} - \rho_f) g h$$

$$\dot{V} = A_o C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = 0.2393\,ft^3/s \qquad v = 2.742\,ft/s$$

$$h_L = \frac{\Delta P}{\rho g} - \frac{\Delta v^2}{2g} = 5.2\,ft\ H_2O \qquad Re = \frac{\rho v D}{\mu} = 7.56\times10^4$$

$C_d = 0.6042 \rightarrow$ Iterate

$$\boxed{\dot{V} = 0.237\,ft^3/s,\ v = 2.72\,ft/s,\ h_L = 5.26\,ft\ H_2O}$$

In the pipe network shown in the figure, the total flow rate is 15.
Find the flow rate and pressure drop through pipe c.
The system demand curves for each pipe INDIVIDUALLY are shown in the figure.
Show your work for full/partial credit.