Chemical Engineering 374

Fluid Mechanics

Boundary Layers

Spiritual Thought

"Even though it appears that all may be lost, our beloved Father in Heaven will reach out… He will fight your battles. He will come to your aid."

Bishop Dean M. Davies
BYU Devotional, 11/4/2015

OEP 8 Clip

OEP 8

Open Ended Problem #8

The Lord of the Rings

GROUP WORK Okay, Due 11/13/24 at beginning of class

The Lord of the Rings

Gandalf, after successfully stopping the Balrog of Morgoth from crossing the Bridge of Kazad Dun, is entangled in the Balrog's fiery whip. He drops his staff and his sword, Glamdring, into the chasm, clinging to the broken ledge long enough to warn the fellowship to flee. Immediately afterward, Gandalf falls into his fate of battling fire and shadow. Could Gandalf have actually caught up to his sword, Glamdring? Could he have caught up to the Balrog?

7. Verify your answer... Does it look reasonable? Anything odd about the calculation?

a) Gandalf possibly used magic to catch up to the sword. How much "magical" thrust would Gandalf have needed to generate in order to catch Glamdring (if applicable)?

b) What acceleration is produced by this thrust? Could he have caught Glamdring before encountering the Balrog?

c) Let's suppose an updraft exists in the chasm. What velocity of the upward air is needed to reduce the terminal velocity of Gandalf by half?

d) How would your answers in 7 a & b change in the face of such an air flow?
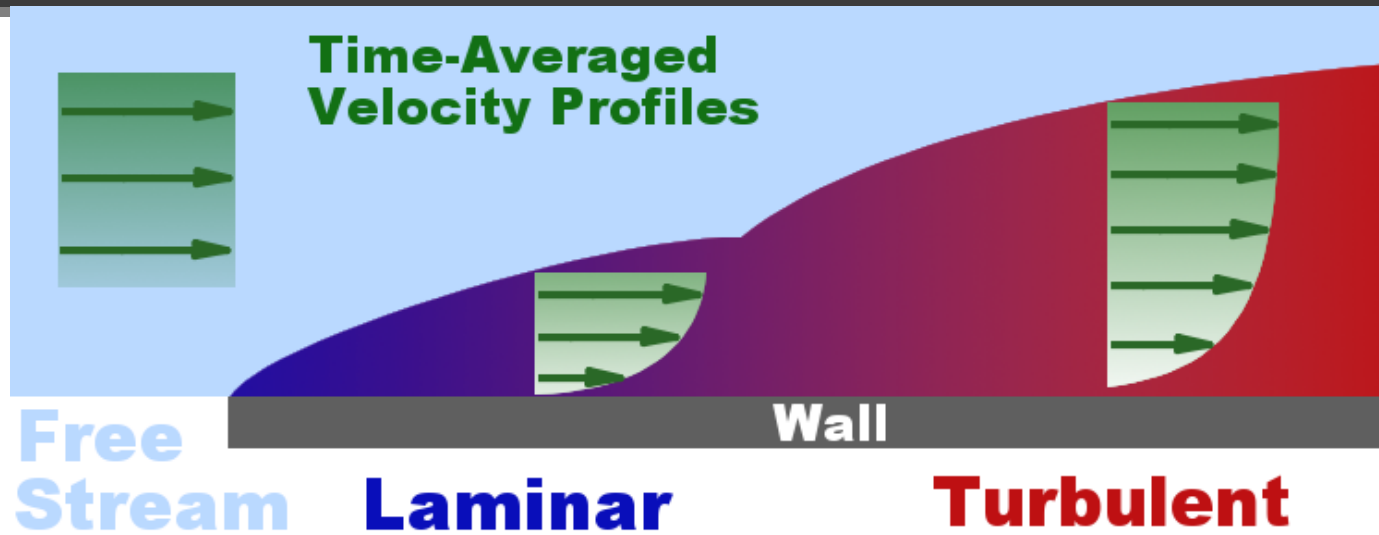
Fluids Roadmap
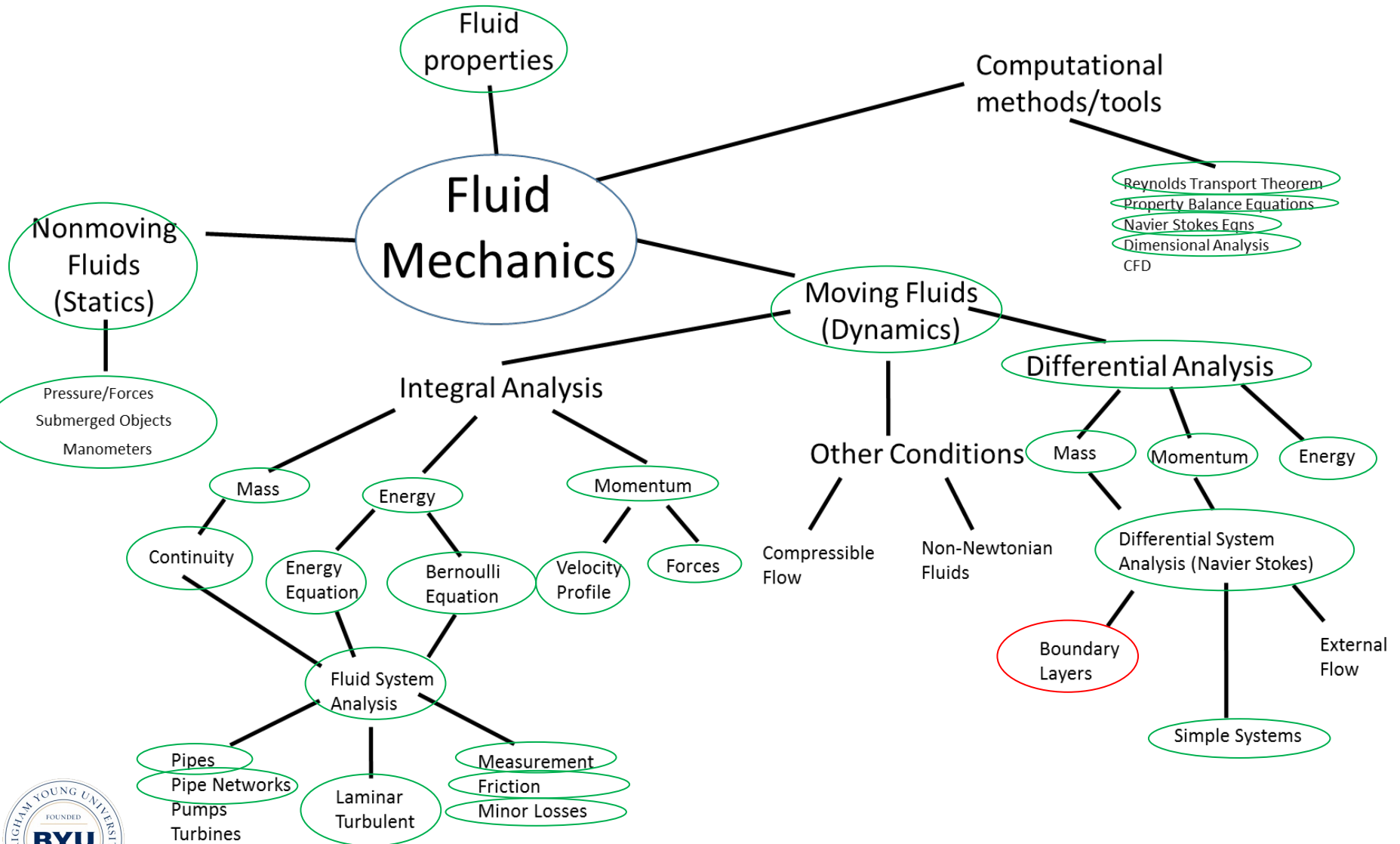
Why do we care about B.L.?

- Mass & Momentum – Navier Stokes
  - Only 3 ways to solve Navier Stokes Eq.
    1. Simplify the problem until it's very basic
    2. Boundary Layer Approximation
    3. CFD
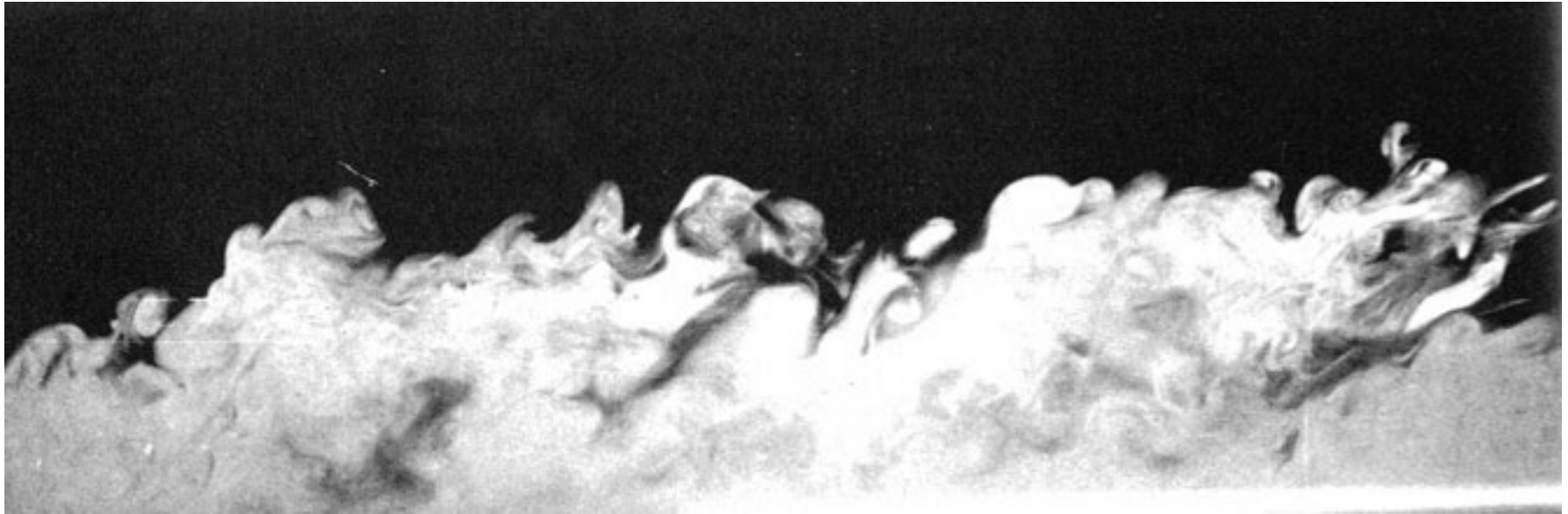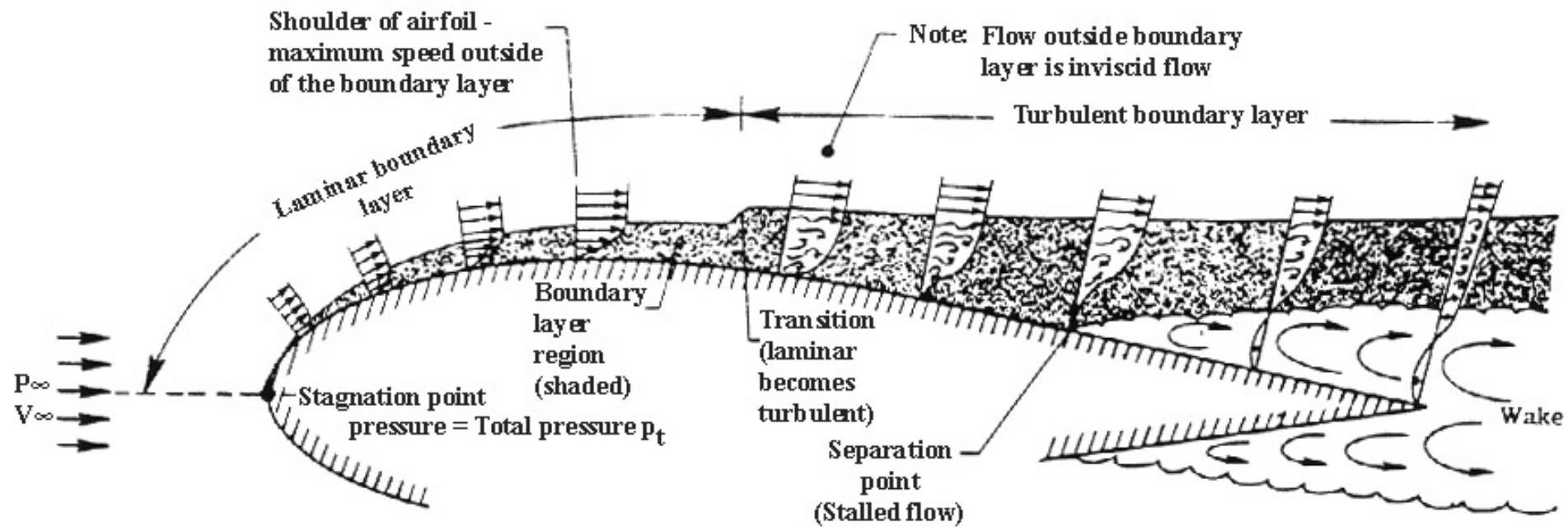- Airplane Wing – Need Drag, Lift

Boundary Layers

- Fluid flowing past wall, other fluid, etc.
  - Effect of viscous dynamics (no slip)
- What is velocity at top of boundary layer?
- What is velocity at bottom of B.L?

Turbulent B.L.

Characteristics of BL

- Re (flat plate) →$Re_x = \dfrac{\rho V x}{\mu}$

  – As Re increases, δ decreases

- Laminar layer – up to Re = 1×10^5

- Transition Flow – $1 \times 10^5 \leq Re \leq 3 \times 10^6$

- Turbulent Flow – $Re > 3 \times 10^6$

- $\tau = 0.332 \dfrac{\rho U}{\sqrt{Re_x}}$

- $C_F = \dfrac{FD}{A} \dfrac{2}{\rho v^2}; C_F = \dfrac{0.664}{Re_x}$ - Lam; $C_F = \dfrac{0.027}{Re_x^{1/7}}$ - Turb;

BL Solution Method

- Surfaces, jets, wakes, mixing layers
- Assume 2 different flow regions
  - Inviscid, bulk flow – Bernoulli or Euler Eqns.
  - Boundary Layer – Full N.S. solution
- Set boundary velocities equal
- Use N.S. Method 1
  - Non-dimensionalize and scale
  - Continuity Eqn
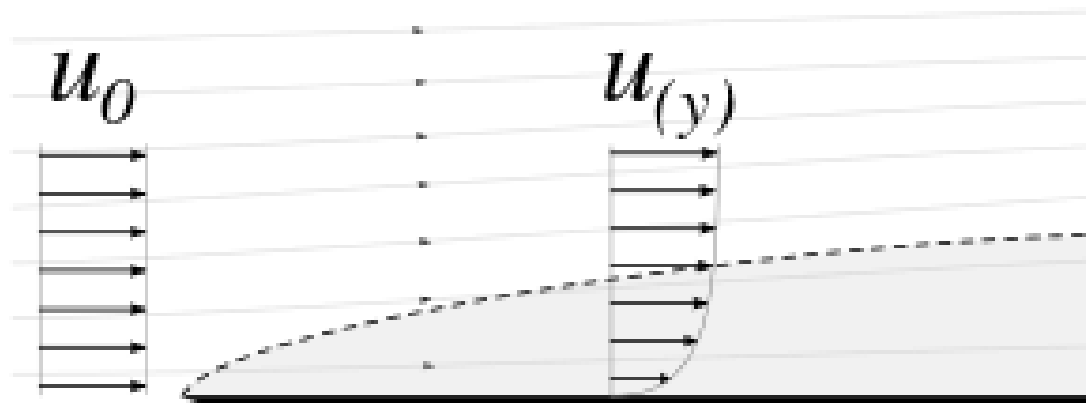 - Y component N.S.
  - X Component N.S.

B.L. Equation

- Assumptions:
  - 2D flow
  - No $\vec{g}$
  - S.S.
  - Thin (δ<<L)

- Start with dimensional Analysis

- $\vec{v} \cdot \nabla \vec{v} = -\dfrac{1}{\rho} \nabla P + \mu \nabla^2 \vec{v}$

- $\rightarrow \left(\overrightarrow{v^*} \cdot \nabla^* \overrightarrow{v^*} \right) = -\nabla' P^* + \dfrac{1}{Re} \left(\nabla^{*2} \overrightarrow{v^*} \right)$

  - As Re $\rightarrow \infty$ viscous forces go to 0??? NO!
    - Two different length scales, L and δ

Scaling/Continuity

- $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$

- Use dimensional Analysis:

- Scale u → U, x → L, y → δ, v → V$_{ref}$

- $\left(\dfrac{\partial u^*}{\partial x^*}\right) + \left[\dfrac{V_{ref}L}{\delta U}\right]\dfrac{\partial v^*}{\partial y^*} = 0$

- Since this is O(1), $V_{ref} = \dfrac{\delta U}{L}$, V_{ref} <<U

y-Momentum

- $u\dfrac{dv}{dx} + v\,\dfrac{dv}{dy} = -\dfrac{1}{\rho}\dfrac{dP}{dy} + \nu\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}\right)$

- Non-dimen. & scale it – like continuity eqn.

- $u^*\dfrac{dv^*}{dx^*} + v^*\,\dfrac{dv^*}{dy^*} = -\dfrac{L^2}{\delta^2}\dfrac{dP^*}{dy^*} + \dfrac{1}{Re}\left(\dfrac{\partial^2 v^*}{\partial x^{*2}}\right) + \dfrac{L^2}{\delta^2}\dfrac{1}{Re}\left(\dfrac{\partial^2 v^*}{\partial y^{*2}}\right)$

- Large, Re, small δ, $\dfrac{dP}{dy}$=0 (although P=P(x) still)

x-Momentum

- u$\frac{du}{dx} + v\frac{du}{dy} = -\frac{1}{\rho}\frac{dP}{dx} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$

- Non-dimen. & scale it

- u$^*\frac{du^*}{dx^*} + v^*\frac{du^*}{dy^*} = -\left(\frac{dP^*}{dx^*}\right) + \frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}}\right) + \frac{L^2}{\delta^2}\frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial y^{*2}}\right)$

  – Large Re, Small δ:

$$u\frac{du}{dx} + v\frac{dv}{dy} = -\frac{1}{\rho}\frac{dP}{dx} + \nu\frac{\partial^2 v}{\partial y^2}$$

Summary

- Dimensionalizing N. S. $\rightarrow \delta \ll L$

- Continuity($\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$) $\rightarrow v \ll U$

- y-mom $\rightarrow \frac{\partial P}{\partial y} = 0$

- X-mom $\rightarrow$ solution
  – Posted online