Chemical Engineering 374

Fluid Mechanics

Exam 3 Review

Spiritual Thought

After you have gotten the message, after you have paid the price to feel His love and hear the word of the Lord, go forward. Don’t fear, don’t vacillate, don’t quibble, don’t whine. You may, like Alma going to Ammonihah, have to find a route that leads an unusual way, but that is exactly what the Lord is doing here for the children of Israel. Nobody had ever crossed the Red Sea this way, but so what? There’s always a first time. With the spirit of revelation, dismiss your fears and wade in with both feet. In the words of Joseph Smith, “Brethren [and sisters], shall we not go on in so great a cause? Go forward and not backward. Courage, brethren; and on, on to the victory!”

–Elder Jeffery R Holland

Exam Review, By Content/Lectures

- Classes 25-32 (plus review)
- Environmental
- Chapter 9.1-9.2, 9.4-9.6 Differential Balances
- Chapter 10.6 Boundary Layers
- Chapter 11.1-11.6 External Flows: Drag
- Chapter 14.1-14.5 Pumps and Turbines
- HW 19-27

Class 26—Differential Balances

- Mass Balance
- Momentum Balance
- Derived from the RTT
- Continuity Equation
  - Used as a constraint for velocity
  - Book/handouts give expanded forms for Cartesian, Cylindrical, Spherical coordinates.
  - Incompressible (const ρ) means rate in = rate out.
  - If rate in = is not rate out then the density is changing (or there's accum)
- Momentum equation: just a differential force balance
  - (accum) = (in)-(out) + (body and surface forces)
  - Book/handouts give expanded forms in three coordinates
  - Again, use to find velocity field or pressure field.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \qquad \nabla \cdot \vec{v} = 0$$

Class 27—Navier-Stokes Equations

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 \vec{v} + \vec{g}$$

- Incompressible, Newtonian, vector
- The equations look scary, but we seriously simplify them
  - Know how to drop out terms:
- Solution procedure:
  - Is it steady? (usually, here), is it compressible (no, usually here const ρ), is it 1-D? (yes, usually here).
  - 1D → one momentum equation (one component of the vector expression).
  - 1D → maybe vary in y direction, but may have only u_x velocity (still 1-D)
    - Use only x-momentum, but varies in y-direction.
  - Solve for a velocity component OR…
  - Given a velocity field, can solve for the pressure field.
- Boundary conditions
  - Velocity (mass flow) given at some point (inflow/outflow)
  - Velocity zero at walls.
  - Symmetry (like at centerlines) → velocity gradients are zero.
  - Air/liquid interface → no shear → velocity gradient zero.

Big Picture Moment…

- Integral Balances
  - Energy, Momentum & Mass (last test)
- Differential Balances
  - Energy – next semester (heat transfer)
  - Mass & Momentum – Navier Stokes
    - Only 3 ways to solve Navier Stokes Eq.
      1. Simplify the problem until it’s very basic (last HW)
      2. Boundary Layer Approximation (today)
      3. CFD (First week of December)

Navier-Stokes: Method 1

- New and Difficult… PRACTICE!!
- Don't take shortcuts!!! Easy to get lost
- Methodology to maximize points (on tests)
  1. List and number all assumptions
  2. Write continuity equation
     - Reduce continuity equation using assumptions
     - Attach a number to the result
  3. Write Navier Stokes for non-main direction(s)
     - Eliminate terms using assumptions (number)
  4. Write Navier Stokes for main direction
     - Eliminate terms using assumptions (number)
  5. Solve Remaining Equation with BC's

Class 28—Boundary Layers

- Mostly conceptual here.
  - Euler equations (Navier-Stokes without viscosity) far from walls.
  - Boundary layer equations near walls (Simplified Navier-Stokes equations).
- Boundary Layer results.
  - Boundary layers have two length scales (length along the plate, and thickness).
  - Boundary layers are very thin compared to length. (So Euler equations solved ignoring BL thickness)
  - Thin, relatively parallel streamlines imply no pressure gradient normal to the plate. (Then the pressure gradient along the plate is that in the free stream (=0 for flat plate BL).
  - Navier-stokes (or x-mom) told us there are two length scales.
  - Continuity told us the scaling for v-velocity.
  - Y-momentum told us there is no pressure gradient normal to the plate.
  - X-momentum then lost a viscous term (du/dx), and dp/dx written in terms of dU/dx with Bern. Eq.
- Computed BL thickness
- Transition for laminar and turbulent Reynolds numbers (5E5 is the engineering cutoff).
- BL grows with distance, Re grows with distance → becomes turbulent.
- Shear stress decreases with distance.
- Turbulent shear > laminar shear.
- RECALL HOW WE USE THESE FACTS, remember the 2 plate direction prob.

Class 29—External Flow

- Lift, Drag, Drag Coefficient.
- Drag is emphasized and is the net force in flow direction.
  - Pressure forces (form drag)
    - Flow separation
  - Viscous forces (friction drag)
  - Streamlining reduces pressure drag, but increases friction drag, but usually, pressure drag dominates.
- $F = C_d \rho v^2 A/2$
  - A is the projected area, or the planview area
  - Use Table 11-1, 11-2 (given on exam if needed)
  - If v is unknown, guess laminar or turbulent → Cd → v → Re → confirm or adjust Cd. (Tables → only one try needed)
  - For small particles guess laminar, for big objects, guess turbulent.
- Terminal velocity follows from a force balance. Object falls under gravity until forces balance: buoyant, weight, drag

Class 30—Pumps

- Pump types: Positive displacement, dynamic.
- Dynamic = Centrifugal, Mixed Flow, Axial.
- bhp, efficiency. ρ*g*Q*H/bhp
- Pump performance curves (PPC): Head versus Flow
  - Shutoff head, free delivery flow rate, intermediate.
  - Individual pump whether alone, series or parallel operates on PPC.
  - System curve is system head versus flow.
  - Operating point where the SC and PPC intersect.
- Pump selection: size for required flow/head, maximize η, may have to overspecify pump (higher flow for given head)
- NPSH.
  - Specified for given pump.
  - Lookup P_{vap} for given flow. Flow parameters → P_2, v_2. Vary operation to ensure greater than NPSH. (Pressure drops prior to pump may lead to cavitation if $NPSH_{have} < NPSH_{need}$).

Class 31—Pump Scaling

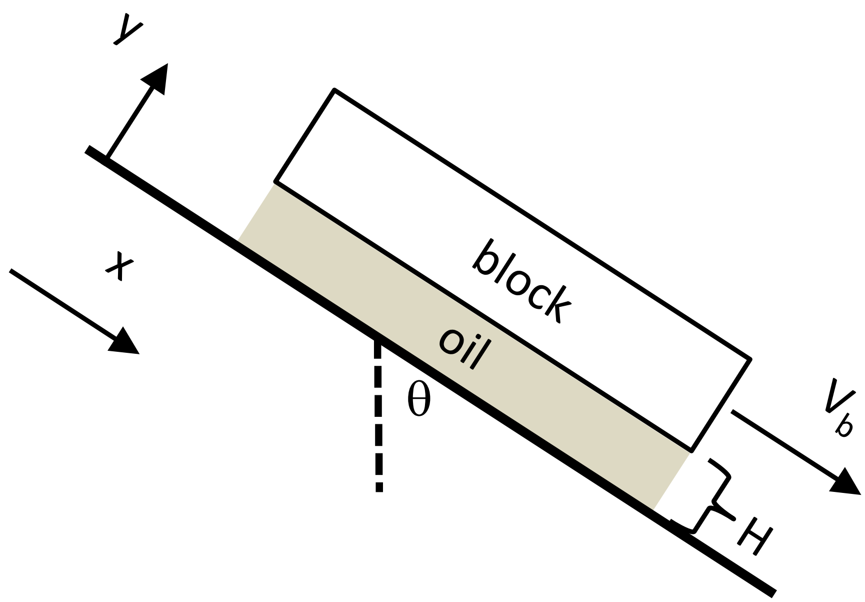
- Series and Parallel
- Series, increase head for given flow.
  - PPC is sum (vertically) of individual pumps PPC
  - Don't operate beyond Free Delivery Flow for smallest pump, or shutoff/bypass that pump.
  - Can act as a head loss above the free delivery flow.
  - Know how the series pumps work with respect to a system demand curve
- Parallel, increase flow for given head.
  - PPC is sum (horizontally) of individual pumps PPC.
  - Don't operate beynod the shutoff head for the smallest pump, or bypass that pump.
  - Backflow above the shutoff head.
  - Each pump operates on its own PPC, but combined system operates on the compined PPC.
  - But the head over each pump is equal → provides a link (horizontal line on PPC) between individual and combined PPC
  - Again, know how this works on a system curve
- Scaling:
  - C_H, C_P, $\eta = f(C_Q, Re, \varepsilon/D) \sim f(C_Q)$. gH, bhp, $\eta = f(D, Q, \omega, \rho, \mu, \varepsilon)$
  - To scale, as usual, match these groups. Often, keeping one or several parameters const →simplifies the problem.
- Choose pumps based on value of N_{sp}
  - if you were given a set of requirements, know how to compute Nsp and choose a pump or turbine.

Class 32—Turbines

- Types: Hydraulic, wind, steam, gas, etc.
- Positive displacement, Dynamic
- Dynamic
  - Impulse (Pelton Wheel)
  - Reaction (like inverse pump)
- Pelton Wheel Analysed using momentum balance
- Reaction turbines:
  - Francis (Radial/mixed flow)
  - Kaplan (Axial flow)
- $\eta = 1/\eta_{pump}$
- Wind turbine
  - Work available, $\eta = C_p < 0.59$ (more like 0.45)
- Scaling laws are like pumps: C_H, C_Q, C_p, η
- Take C_P as independent parameter
- Turbine selection, based on N_{sp}.

Practice Problem #1

- A block slides down an incline at a steady velocity Vb with a layer (thickness H) of an oil, (with density ρ and viscosity μ) between the block and the incline. The flow is laminar and the oil is exposed to the atmosphere around the edges of the block. Find the symbolic expression for the velocity profile of the oil. Show your work and state assumptions.



Assumptions:

1. SS
2. const ρ
3. Open Flow
4. Parallel Flow } 1D flow
5. Infinite Plate }
6. Fully Developed Flow

Continuity

$$\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{4} + \cancel{\frac{\partial w}{\partial z}}^{5} = 0$$

$$\frac{du}{dx} = 0 \Rightarrow \#6$$

BC1: @ $y=0, u=0$

BC2: @ $y=H, \frac{du}{dy}=0$

X-dir

$$\rho\left(\cancel{\frac{\partial u}{\partial t}}^{1} + u\cancel{\frac{du}{dx}}^{6} + \cancel{v}^{4}\frac{du}{dy} + \cancel{w}^{5}\frac{du}{dz}\right) = -\cancel{\frac{\partial p}{\partial x}}^{3} + \rho g_x + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{6} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}^{5}\right)$$

$$\int \mu \frac{\partial^2 u}{\partial y^2} = \int -\rho g_x$$

$$\int \frac{\partial^2 u}{\partial y^2} = \int -\frac{\rho g_x}{\mu} \Rightarrow \frac{\partial u}{\partial y} = -\frac{\rho g_x}{\mu} y + C_1$$

$$\text{BC2: } \frac{du}{dy} = 0 = -\frac{\rho g_x H}{\mu} + C_1 \qquad \boxed{C_1 = \frac{\rho g_x H}{\mu}}$$

$$\int du = \int \frac{\rho g_x}{\mu}(y-H)\,dy \qquad u(y) = \frac{\rho g_x}{2\mu} y^2 - \frac{\rho g_x H}{2\mu} y + C_2$$

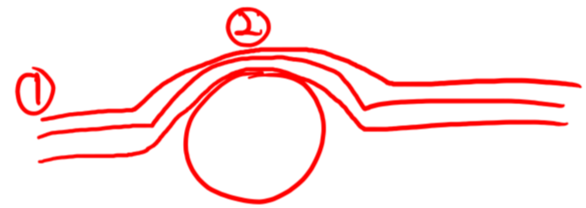
$$@\, y=0,\ u=0 \Rightarrow 0 = 0 - 0 + C_2 \qquad \underline{C_2 = 0}$$

$$\therefore \boxed{u(y) = \frac{\rho g_x}{2\mu} y^2 - \frac{\rho g_x H}{2\mu} y}$$

Practice Problem #2

- A golf ball is moving through air (P=1 atm) at 50 m/s. As the air above the ball and outside the boundary layer curves around the ball, its speed relative to the ball increases by 20%. What is the GAGE air pressure (Pa) at the top surface of the ball?

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} = 0$$



$P_1 = 0\,\text{psig}$ $\quad P_2 = ?$ $\quad V_2 = 1.2V_1$ $\quad \therefore \dfrac{P_2}{\rho} = \dfrac{V_1^2 - V_2^2}{2} = \dfrac{V_1^2(1^2 - 1.2^2)\rho}{2}$

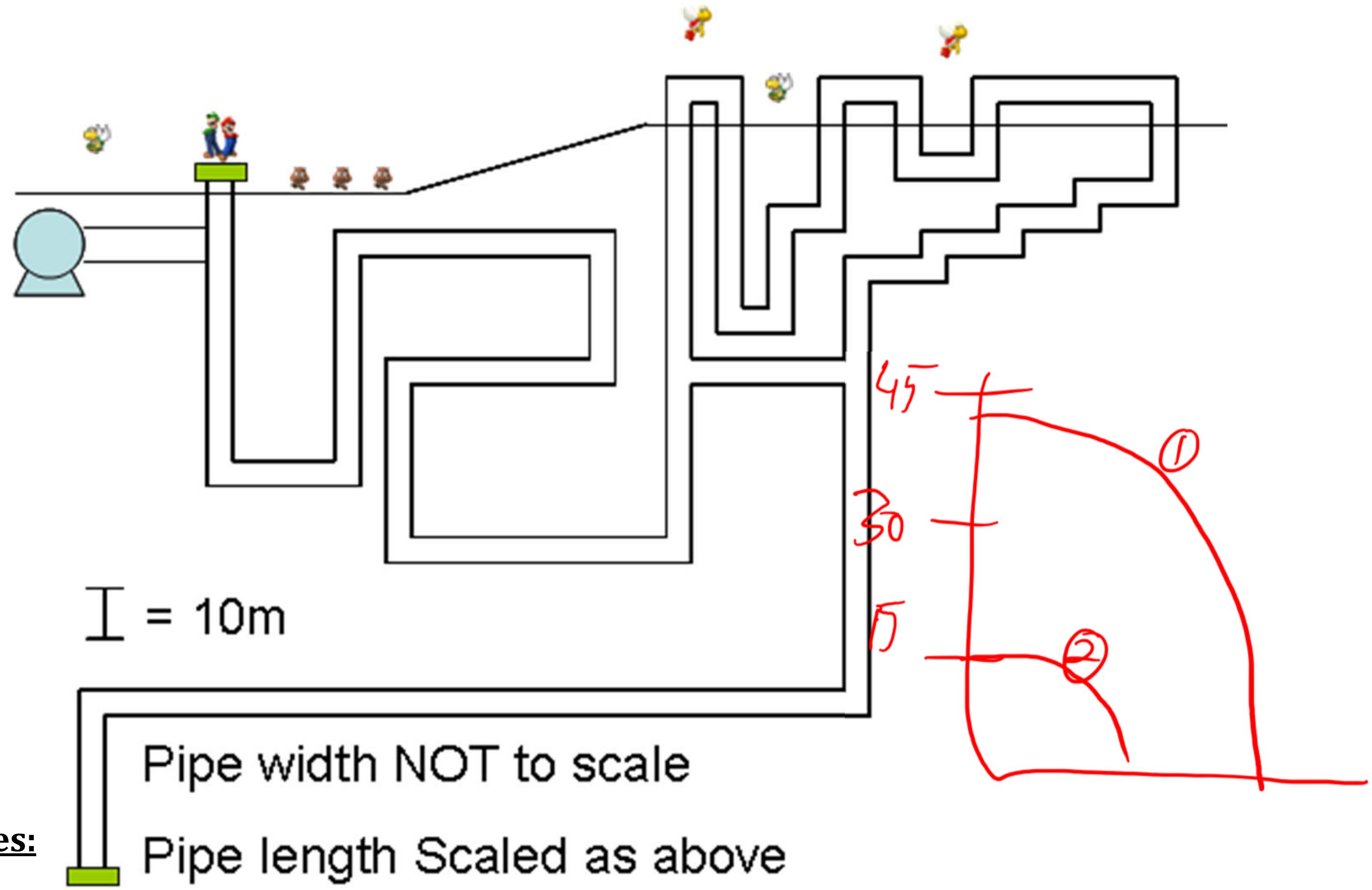
$$\boxed{P_{2,g} = -660\ \text{Pa}}$$

Practice Problem #3

- A Marvelously Massive plumber and his Ludicrously Lanky brother are on an urgent mission to save their beloved princess from a killer turtle that breathes fire. Frequently throughout their journeys, they have traveled from above ground locations to underground caverns via large pipe networks. Many of you have likely seen the amazing adventures of Mario and Luigi (courtesy of Nintendo, of course), but you probably don't know what actually happens when they drop down into the pipes… they drop into a large water network with pumps that push the fluid to the pipe outlet in question. A scale drawing of the actual pipe network that they use is shown in the figure. In order to pump the fluid through these pipes, a sequence of pumps is installed off to the side of the pipe entrance. Given the two different pump options indicated below, which pump should be used to power the pipe network? Assuming that the citizens of the Mushroom Kingdom are energy conscious and conservation minded, how many pumps should be used and in what configuration (parallel or series) should they be used?

- (HINT – Because you can't play Mario right now, you may assume that Mario travels from the entrance to the exit of the pipe network in 3 seconds.)

Practice Problem #3 (cont.)



Pump curves:

Pump 1 - H(m) = 40000 – 0.6*Q^2 (m^3/s)
Pump 2 - H(m) = 15000 – 0.8*Q^2 (m^3/s)

a) What is the problem asking for?

Practice Problem #3 (still more...)

b) What assumptions are you making in order to solve the problem?

c) Provide an answer with justification based on your assumptions.

d) i) How oversized (in percent) is the pump system you selected in part c?

ii) What is the power required to run these pumps?

iii) Is this power reasonable? What practical limitations might prevent such flows and speeds indicated by this problem?

a) The most efficient # & configuration of pumps to maintain a flow rate corresponding to a V of 3 m/s.

b) SS, water is continuosly being supplied to the pumps, ν is constant, ρ = constant, V = 3 m/s = bulk average flow, bends are 90° miter bends (K_L = 1.1), Mario moves @ same flow rate as the water, Δz = 150 m, Steel Pipes (ε = .002 mm)

D = 1 m, L = 715 m,

$P_1 = P_2 = 1$ atm

$V_1 = V_2$ = same

2 Tee Lines straight → K_L = 1.0 each

2 Tee Lines, branch → K_L = 0.2 each

10 90° Bends

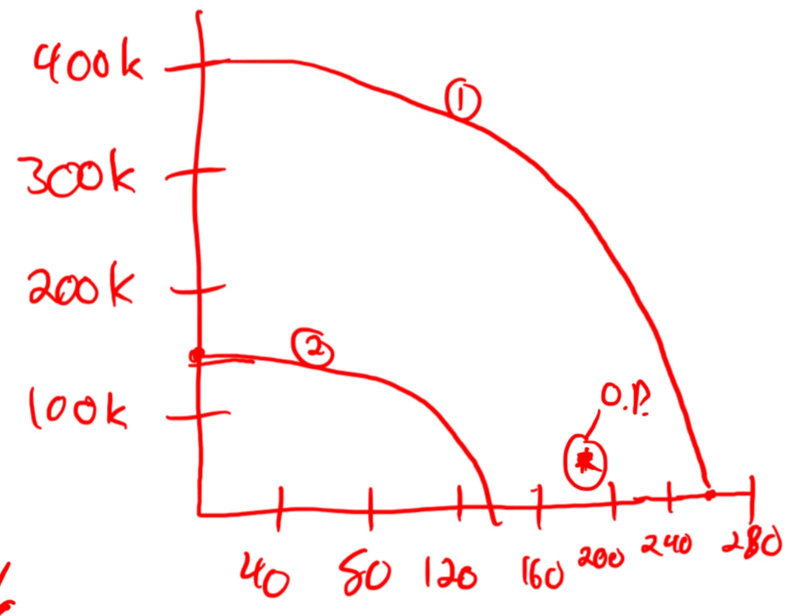
$K_{L,tot} = 13.4$

Head (System Curve)

$$\frac{\cancel{\Delta P}^{\,0}}{\rho g} + \frac{\cancel{\Delta V^2}^{\,0}}{2g} + \Delta z = h_L + h_p$$

$$h_p = z_2 - z_1 + \left(f\frac{L}{D} + k\right)\frac{V^2}{2g}$$

$$h_p = 150\,m + \left(f\frac{715\,m}{1\,m} + 13.4\right)\frac{v^2}{2(9.8\,m/s^2)}$$



Operating point → $v = \frac{L}{t} = \frac{715\,m}{3\,s} = 238.3\ m/s$

$$\dot{V} = v \cdot A = (238\ m/s)\left(\frac{\pi}{4}(1\,m)^2\right) = \underline{187\ m^3/s}$$

System Head @ operating point

$Re = \frac{\rho v D}{\mu}$ $\quad f = \left[1.8 \log_{10}\left(\frac{6.9}{Re} + \left[\frac{(\varepsilon/D)}{3.7}\right]^{1.11}\right)\right]^{-2}$ $\quad \underline{f = 0.0066}$ $\quad \boxed{h_p = 52{,}256\,m}$

Pump 2 requires 2 pumps in parallel, while pump 1 requires only 1 pump

d) i) Pump 1 Head, $400{,}000 - 6(187 \, m^3/s)^2 = 190{,}186 \, m$

$$\frac{190{,}186 \, m}{52.256 \, m} = 3{,}634 \text{ times bigger}$$

363,9% oversized! Too big!

ii) $P = \rho v H \dot{V} = (1000 \, kg/m^3)(238.3 \, m/s)(190186 \, m)(187 \, m^3/s)$

$P = 3.48 \times 10^5 \, MW = \underline{\underline{348 \, GW}}$! Woah!!

iii) Speed is $238.3 \, m/s$ → very high, nearly sonic!

Also, Massive power is required for this pump! → ≈ 330 Nuke Plants!

Way too large!!!