Chemical Engineering 374

Fluid Mechanics

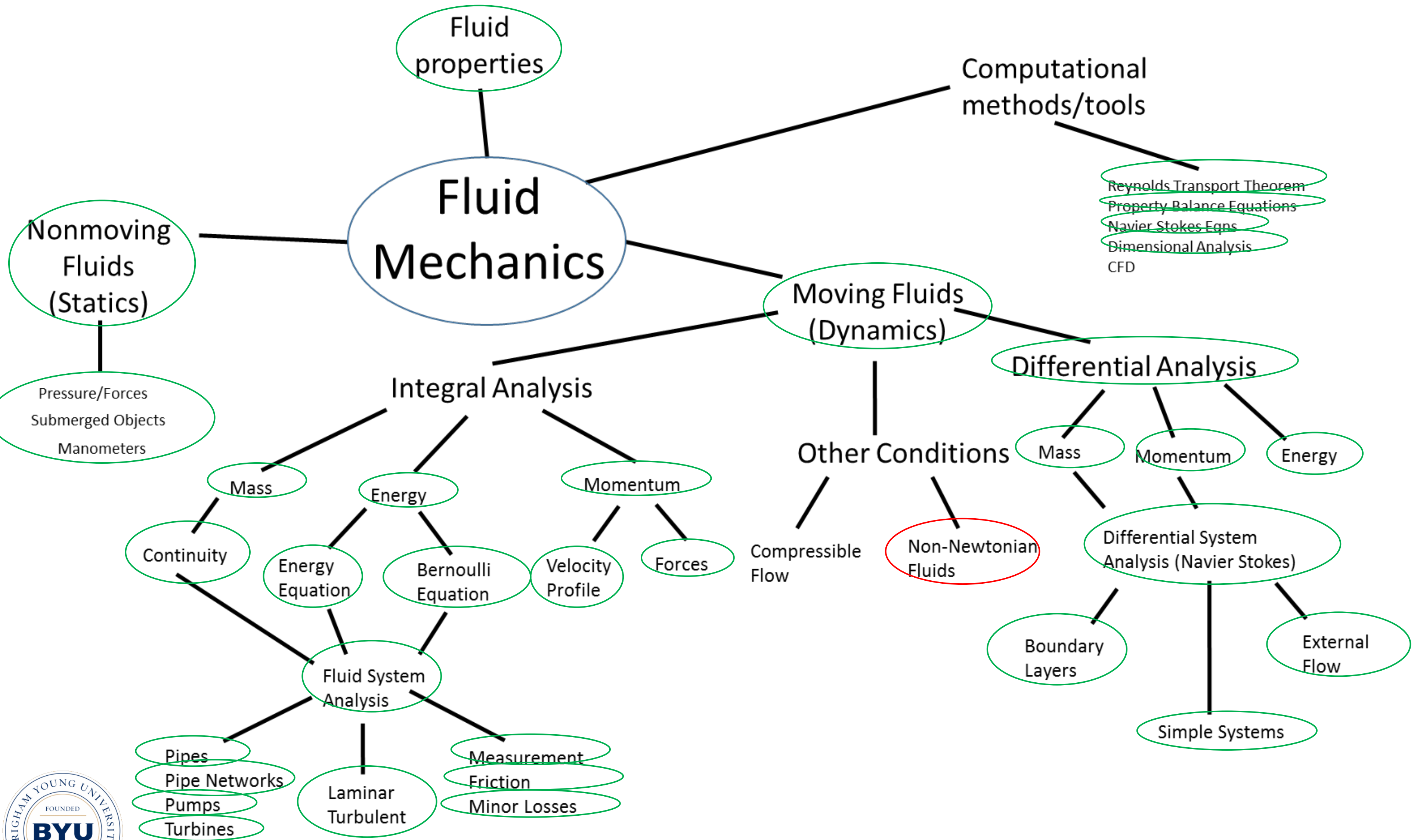
NonNewtonian Fluids

Spiritual Thought

"The recent emphasis of making the Sabbath a delight is a direct result of inspiration from the Lord."

M. Russell Ballard

Fluids Roadmap



OEP 10 Clip

OEP 10

Open Ended Problem #10

The Avengers

INDIVIDUAL WORK ONLY, Due 12/6/24 at beginning of class

(Don't be afraid to "Google" good assumptions!)

The Avengers

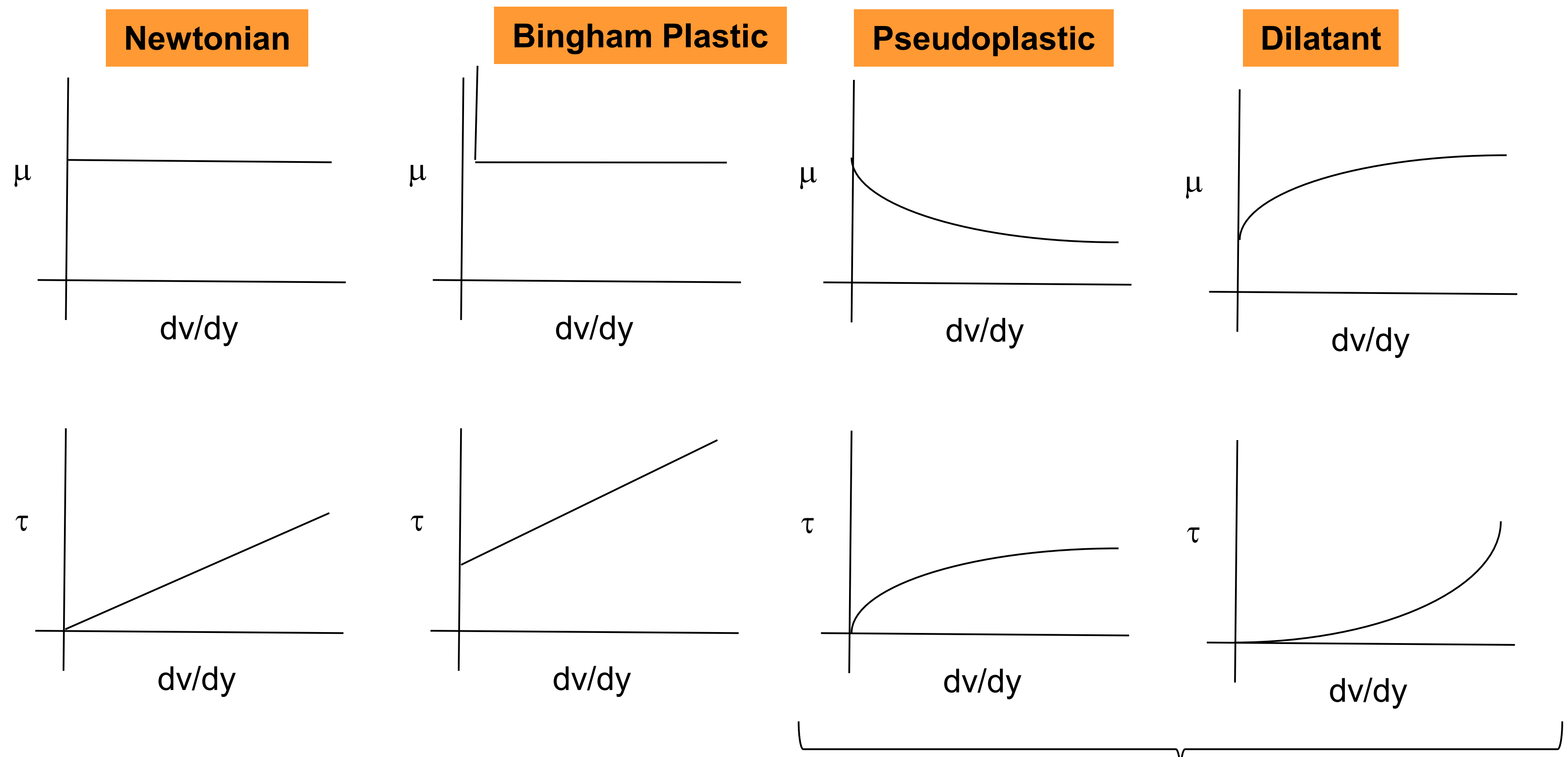
In a thematic scene of pure awesome, the Avengers stop Loki and his army of Chitauri from conquering earth though a wormhole opened by the Tesseract (we won't go into the inaccuracies of the nuke and the blast in the Chitauri mother-ship… in this class) successfully closing the portal and saving the earth from alien invaders (how cliché). However, there is a serious problem with this scene. Although mass is supposed to be moving through the portal, no air is going through the wormhole, which it would be if a wormhole were opened in the atmosphere of earth! Given this, calculate the rate at which air passes through the portal when it is first opened. Does earth survive Loki, only to be suffocated by the open portal into space? Why, or why not?

Key Points

- Types and properties of non-Newtonian Fluids
- Pipe flows for non-Newtonian fluids
- Velocity profile / flow rate
- Pressure drop
  - Friction factor
  - Pump power
- Rheological Parameters

Power Law Fluids

Non-Newtonian Fluids



$\tau = \mu * dv/dy$

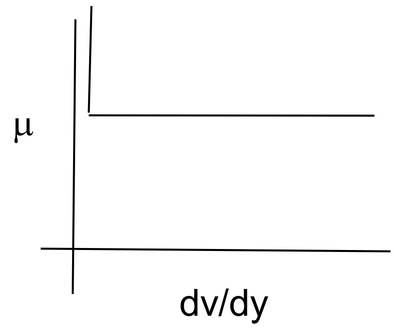
$\tau = \mu * dv/dy + \tau_y$

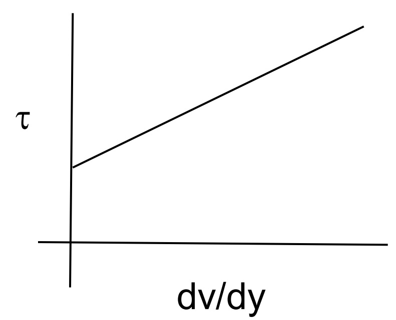
$\tau = \kappa * |dv/dy|^n$

Bingham Plastic

- 3D elastic structures. Weak solid structures that must be broken
- Resists small shear, but structure "breaks apart" with large shear.
- Then τ is ~ linear with du/dx
- Some slurries (coal, grain slurries), sewage sludge.
  - Toothpaste (no drip)
- Larger particles → weak solid structure → breaks

Bingham Plastic



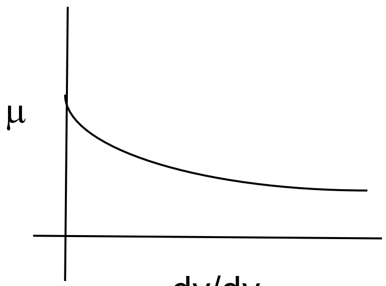


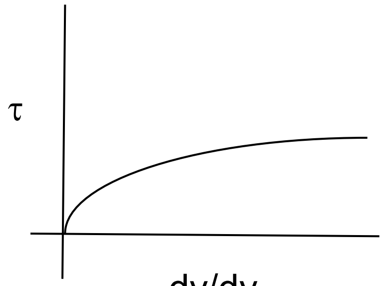
$\tau = \mu * dv/dy + \tau_y$

Pseudoplastic

- Most common
- Dissolved or dispersed particles, like dissolved long chain molecules.
- Have a random orientation in the fluid at rest, but line up when the fluid is sheared.
  - τ decreases with strain rate
  - μ drops as molecules align
- Polymer melts, paper pulp suspensions, pigment suspensions, hair gel, blood, muds, most slurries
- "Shear-Thinning"
  - motor oil

Pseudoplastic



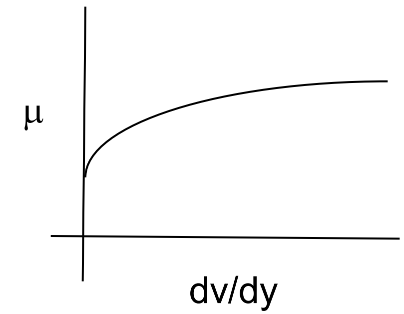


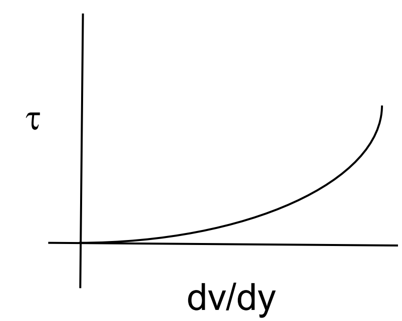
$\tau = K * (-dv/dy)^n$

Dilatant

- Rare
- Slurries of solid particles with barely enough liquid to keep apart. (corn starch, water squeezed out at high shear)
- At low strain rates, the fluid can lubricate solids; at high strain rates, this lubrication breaks down.
- μ increases with strain $\rightarrow$ τ increases.
- "Shear thickening".

Dilitant



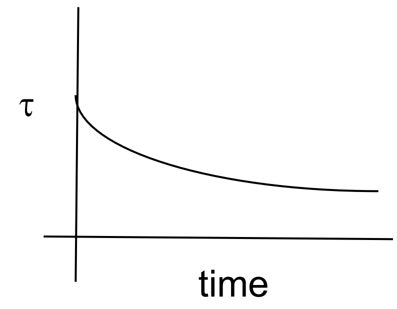


$\tau = K * (-dv/dy)^n$

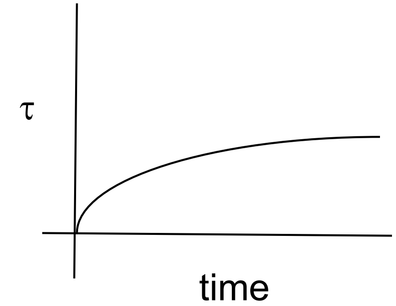
Dilatant Example

Time dependence

- Thixotropic
  - Slurries/solutions of polymers
  - Many known fluids
  - Most are pseudoplastic
  - Alignable particles/molecules with weak bonds (H-bonding)
  - Paint



- Rheopectic
  - Rare
  - Fewer known examples
  - Usually, fluids only show this behavior under mild shearing



- Changes occur within the first 60 sec. for most processes.
- Hard to describe
- Viscoelastic

Power Law Fluids

- Governing equations are "correct" in terms of τ
  - Expression for τ is the model.
  - Called a "constitutive relation"
    - Also have these for mass and heat fluxes in heat and mass transfer.
  - Newtonian flow

$$\tau = -\mu \frac{dv}{dy}$$

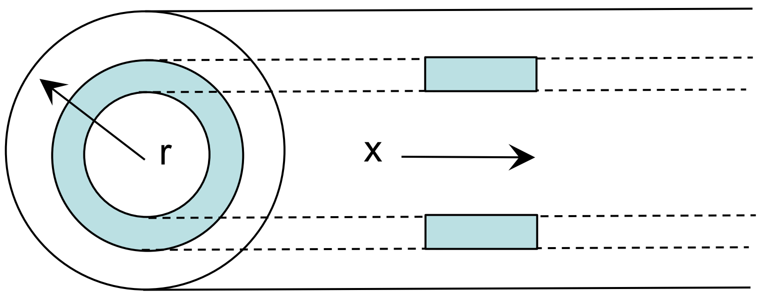
  - For dilitant and pseudoplastic fluids (most common)—***Power Law***

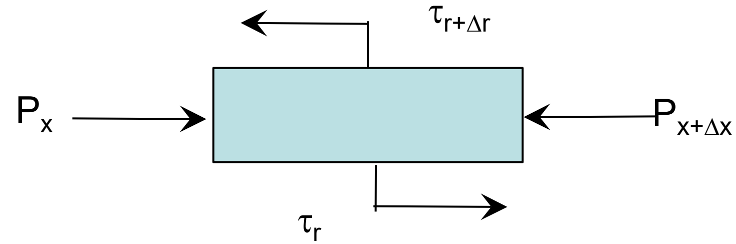
$$\tau = K\left(-\frac{dv}{dy}\right)^n$$

n>1 → Dilitant
n<1 → Psuedoplastic
n=1, K=μ → Newtonian

    - K, n are empirical constants
  - Many other forms
    - Simpler ones have 3 parameters and give a better fit, but are more complex than power law form.
      - See Handout of Book Chapter on Webpage.

Laminar Pipe Flow





Force Balance: Pressure, stress

$$(P_x - P_{x+\Delta x})(2\pi r \Delta r) + (2\pi \Delta x r)\tau_r - (2\pi \Delta x)(r + \Delta r)\tau_{r+\Delta r} = 0$$

Divide $2\pi\Delta r\Delta x$

$$r\frac{P_x - P_{x+\Delta x}}{\Delta x} + \frac{r\tau_r - (r+\Delta r)\tau_{r+\Delta r}}{\Delta r} = 0$$

Limit Δx, $\Delta r \rightarrow 0$

$$-\frac{dP}{dx} = \frac{1}{r}\frac{d(r\tau)}{dr} = C$$

Separate variables and integrate with τ=0 at r=0 $\qquad \tau = -\frac{r}{2}\frac{dP}{dx}$

Non-Newtonian Pipe Flow

- Most non-Newtonian flows are laminar.
- Key results: (remember, Q is just volumetric flow rate-Vdot)

  - Force balance:
  - Power law constitutive relation
  - Integrate with B.C. v=0 at r=R

$$\tau = -\frac{r}{2}\frac{dP}{dx}$$

$$\tau = K\left(-\frac{dv}{dr}\right)^n$$

$$v = \left(-\frac{1}{2K}\frac{dP}{dx}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$$

  - $\dot{V}$= Q = Av_{avg}

$$Q = \frac{\pi n D^3}{8(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n} \qquad V_{avg} = \frac{nD}{2(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$$

  - Kinetic Energy Correction Factor: $\alpha = \frac{3(3n+1)^2)}{(5n+3)(2n+1)}$

  - Momentum Flux Correction Factor: $\beta = \frac{3n+1}{2n+1}$

Pressure Drop—Laminar Flow

Define $$f = \frac{4\tau_w}{\frac{1}{2}\rho v_{avg}^2}$$ **1**

Force Balance $$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$ **2**

$$V_{avg} = -\frac{1}{\pi R^2}\int_A v(r)dA$$

Newtonian

$$v = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$ **3**

$$V_{avg} = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$ **4**

$$\tau_w = \frac{4\mu V_{avg}}{R}$$

$$f = \frac{64}{Re}$$

Solve 4 for dP/dx and insert into 2

Insert into 1 for τ_w

Non-Newtonian

$$v = \left(-\frac{1}{2K}\frac{dP}{dx}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$$

$$V_{avg} = \frac{nD}{2(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$$

τ_w=(some complex expression)

$$f = \frac{8K}{\rho V_{avg}^2}\left(\frac{2(3n+1)V_{avg}}{nD}\right)^n$$

Turbulent Flow

- Define the friction factor as before: $f = \dfrac{8\tau_w}{\rho V_{avg}^2} = \dfrac{\Delta P D/L}{\frac{1}{2}\rho V_{avg}^2}$
  - (Laminar or Turbulent)
- For turbulent flow we had f = f(Re, ε/D) from dimensional analysis.
- Question: Will this work for non-Newtonian Flow?
- Question: What is the Reynolds number?
  - No clear definition of Re since μ is not constant (depends on the strain rate dv/dr, which depends on V_{avg})
- Use the same definition as the laminar friction factor: Re=64/f

$$f = \frac{8K}{\rho V_{avg}^2}\left(\frac{2(3n+1)V_{avg}}{nD}\right)^n \longrightarrow \boxed{Re = \frac{8\rho V_{avg}^2}{K}\left(\frac{nD}{2(3n+1)V_{avg}}\right)^n}$$

  - (Definition based on laminar Newtonian, but used for all regimes)
- Plot friction factor versus Re as for Newtonian flows, using the red definition of Re.

Non-Newtonian Friction Factor (Power Law)

$f_{Fanning} = (1/4)\, f_{Darcy}$

$$Re = \frac{8\rho V_{avg}^2}{K}\left(\frac{nD}{2(3n+1)V_{avg}}\right)^n$$

Rheological Parameters (power law)

$$\tau = K\left(-\frac{dv}{dr}\right)^n$$

- **Problem:** Non-Newtonian fluid has:
  - **How to find K, and n for a given fluid?**
- You need to measure something (what?)
- Try a pipe flow
  - D, Q, dP/dx
- Here's what we know:

$$\tau = K\left(-\frac{dv}{dr}\right)^n \qquad v = \left(-\frac{1}{2K}\frac{dP}{dx}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$$

$$\tau = -\frac{r}{2}\frac{dP}{dx} \qquad Q = \frac{\pi n D^3}{8(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$$

$$\tau_w = K\left(-\frac{dv}{dr}\right)_w^n \qquad V_{avg} = \frac{nD}{2(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$$

$$\tau_w = -\frac{R}{2}\frac{dP}{dx}$$

- D, Q, dP/dx → V_{avg}, τ_w.
- Then relate these to K, n: $\tau_w = K\left(-\frac{dv}{dr}\right)_w^n$
  - Compute $(-dv/dr)_w$ from v(r)

Rheological Parameters (power law)

- From v(r), we get: $\left(-\frac{dv}{dr}\right)_w = \frac{2(3n+1)V_{avg}}{nD}$
- Now $\tau = K\left(-\frac{dv}{dr}\right)^n \longrightarrow \ln(\tau_w) = \ln(K) + n\ln(-dv/dr)_w$
- So a plot of $\ln(\tau_w)$ versus $\ln(-dv/dr)_w$ is linear with slope n, and intercept ln(K).
- But, note that $(-dv/dr)_w$ involves n, which is unknown → what to do?
- Just rearrange:

$$\ln(\tau_w) = \ln(K) + n\ln(2(3n+1)V_{avg}/nD)$$

$$\ln(\tau_w) = n\ln(V_{avg}) + \{\ln(K) + n\ln(2(3n+1)/nD)\}$$

- Now, a plot of $\ln(\tau_w)$ versus $\ln(V_{avg})$ is linear with slope n.
- Once n is known, K can be computed from the intercept (term in {}), or just compute it analytically from $\tau = K\left(-\frac{dv}{dr}\right)^n$ and $\left(-\frac{dv}{dr}\right)_w = \frac{2(3n+1)V_{avg}}{nD}$ which give

$$K = \frac{\tau_w}{(2(3n+1)V_{avg}/nD)^n}$$

Recap

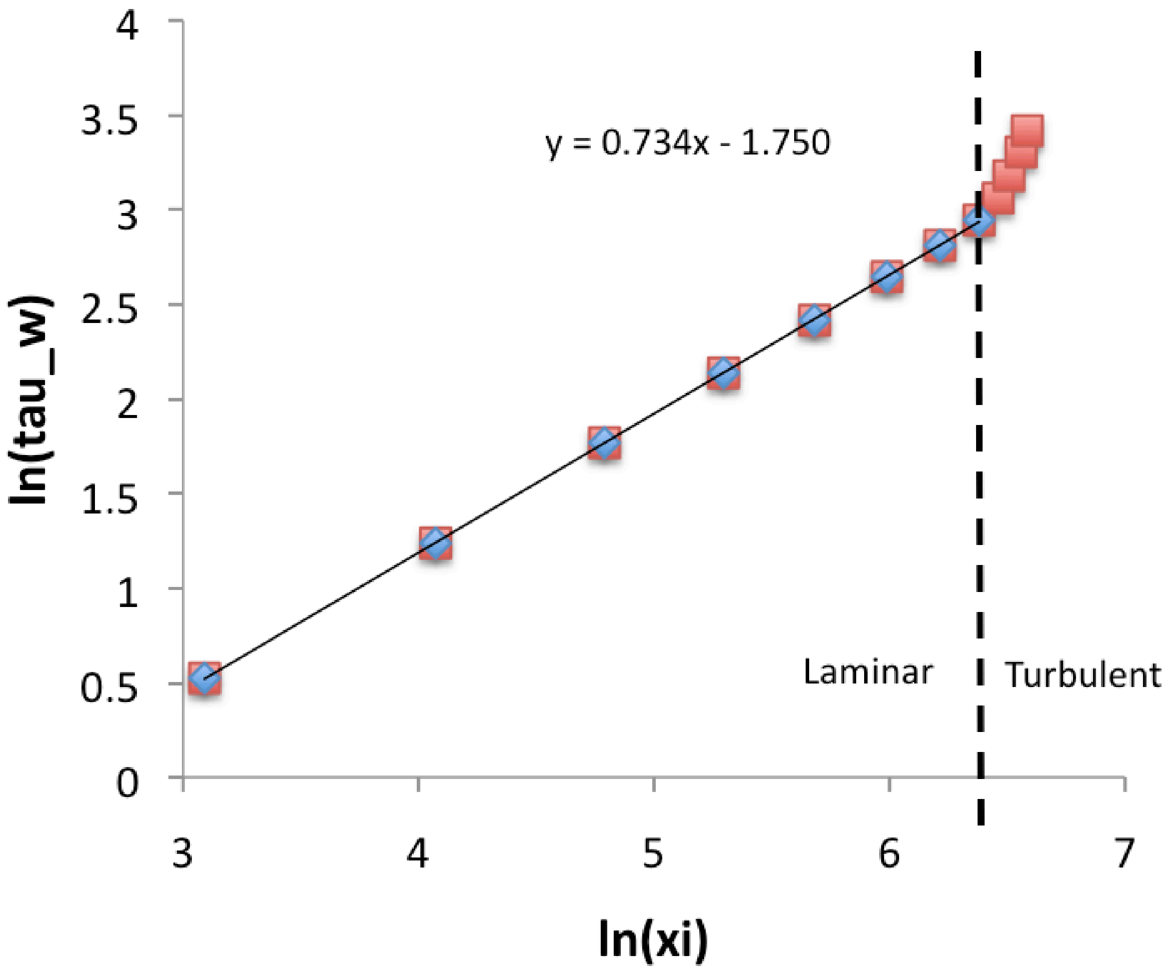
- To compute K, n for a non-Newtonian fluid
- Measure Q, D, dP/dx
- Compute V_{avg} from Q and D (area), that is, $Q=A*V_{avg}$
- Compute τ_w from $\tau_w = -\frac{R}{2}\frac{dP}{dx}$
- Plot $\ln(\tau_w)$ versus $\ln(V_{avg})$
- Fit a line to the data (the linear part of the data)
- The slope is n
- K is computed from the intercept, or from

$$K = \frac{\tau_w}{(2(3n+1)V_{avg}/nD)^n}$$

- Note, the units on K are ($kg*s^{n-2}/m$)

Example

- Given:
  - Diameter
  - Pressure Drop
  - Flow Rate
- Compute:
  - K, n
  - Re
  - Power through a given pipe is as usual, Q*ΔP



Note, here xi = 8*V_{avg}/D, and instead of plotting ln(tau_w) versus ln(V_{avg}), I'm plotting ln(tau_w) versus ln(xi). The approach is the same, but the intercept has a different formula for getting K. By the way, xi = 8*V_{avg}/D is $(-dv/dr)_w$ for Newtonian fluids, hence that choice here.