Chemical Engineering 374

Fluid Mechanics

Fluid Statics

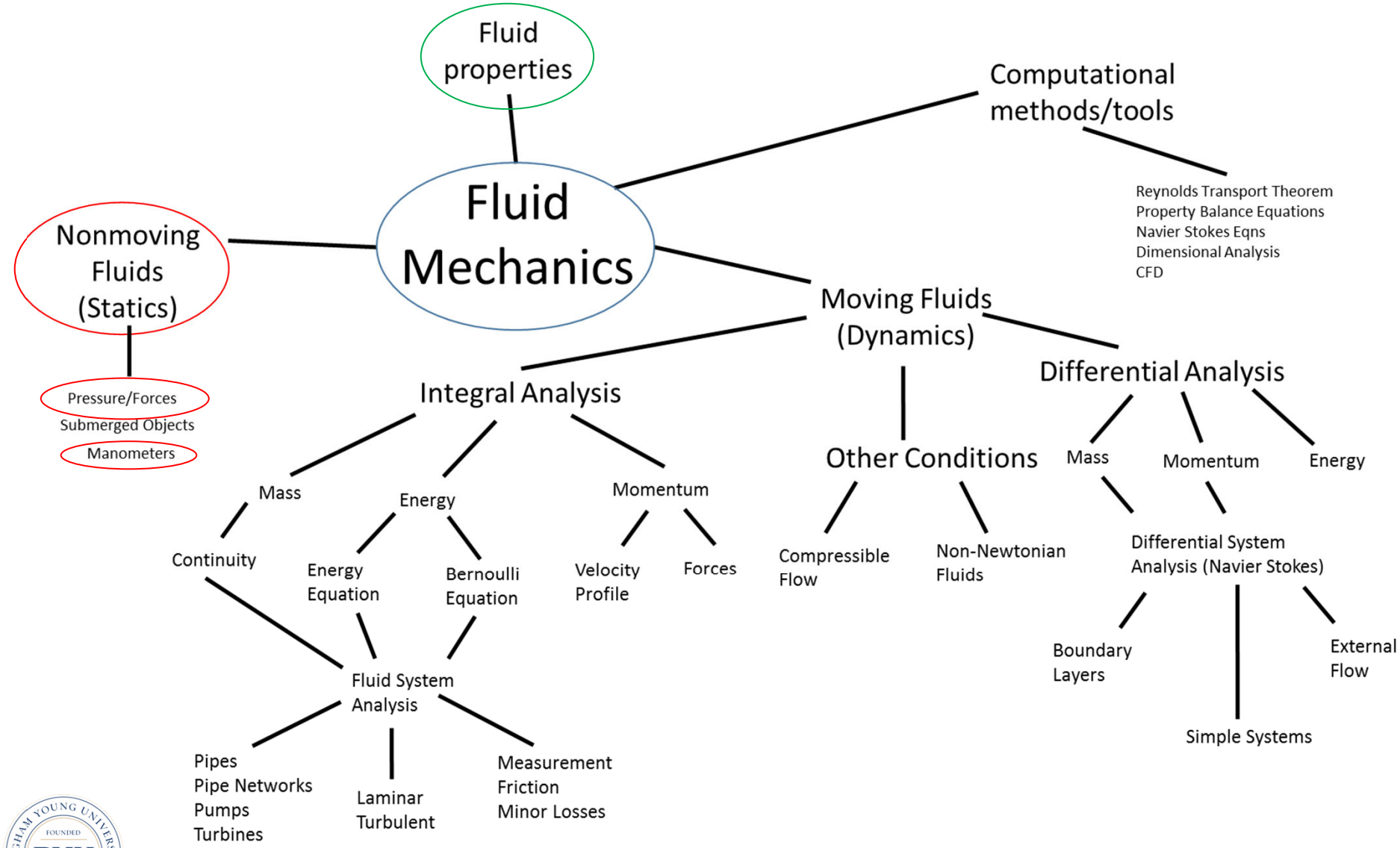
Spiritual Thought

Alma 34:31

31…for behold, now is the time and the day of your salvation; and therefore, if ye will repent and harden not your hearts, immediately shall the great plan of redemption be brought about unto you.

32 For behold, this life is the time for men to prepare to meet God; yea, behold the day of this life is the day for men to perform their labors.

Fluids Roadmap



Key Points

- Statics – No fluid motion
- Pressure characteristics
  - Scalar, all directions, normal to surfaces, units
- Barometric Equation
- Derivation
  - Constant properties
  - Non-constant properties
- Examples

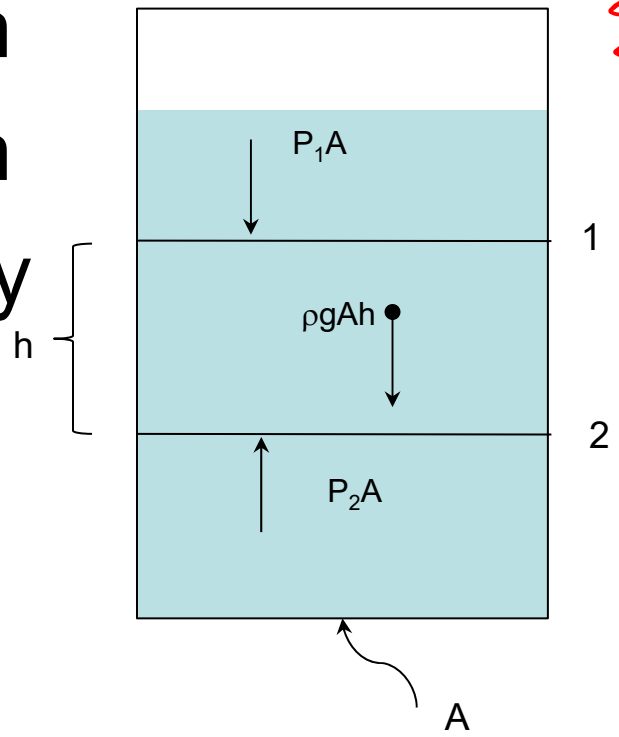
Pressure

- $P = F/A$
- Isotropic – same in all directions
  - Force depends on area – next lecture
- Units
  - $F/A = N/m^2 = Pa = kg/m/s^2$ ***or*** $lbf/in^2 = psi$
- Absolute

- Gauge

Pressure Derivation

- Start with slice of fluid; FORCE BALANCE!
- x-direction
- y-direction
- how many Forces? - 3



$\Sigma F_x = 0 \Rightarrow$ same on both sides

$\Sigma F_y = 0 \Rightarrow$ weight + top P = bottom P

$0 = -P_1 A + P_2 A - \rho g h A$

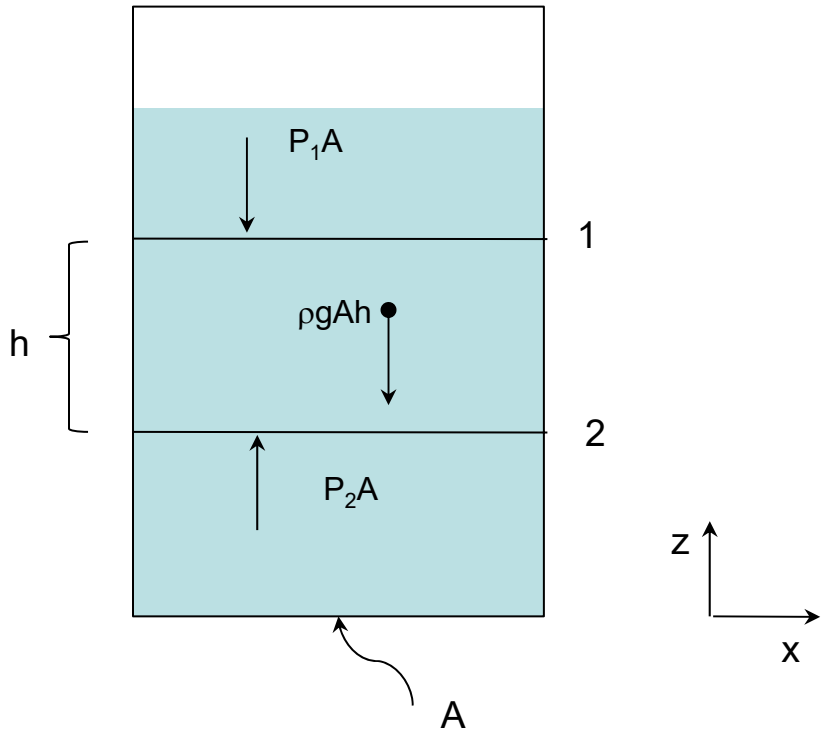
z
x

$P_1 - P_2 = -\rho g h$

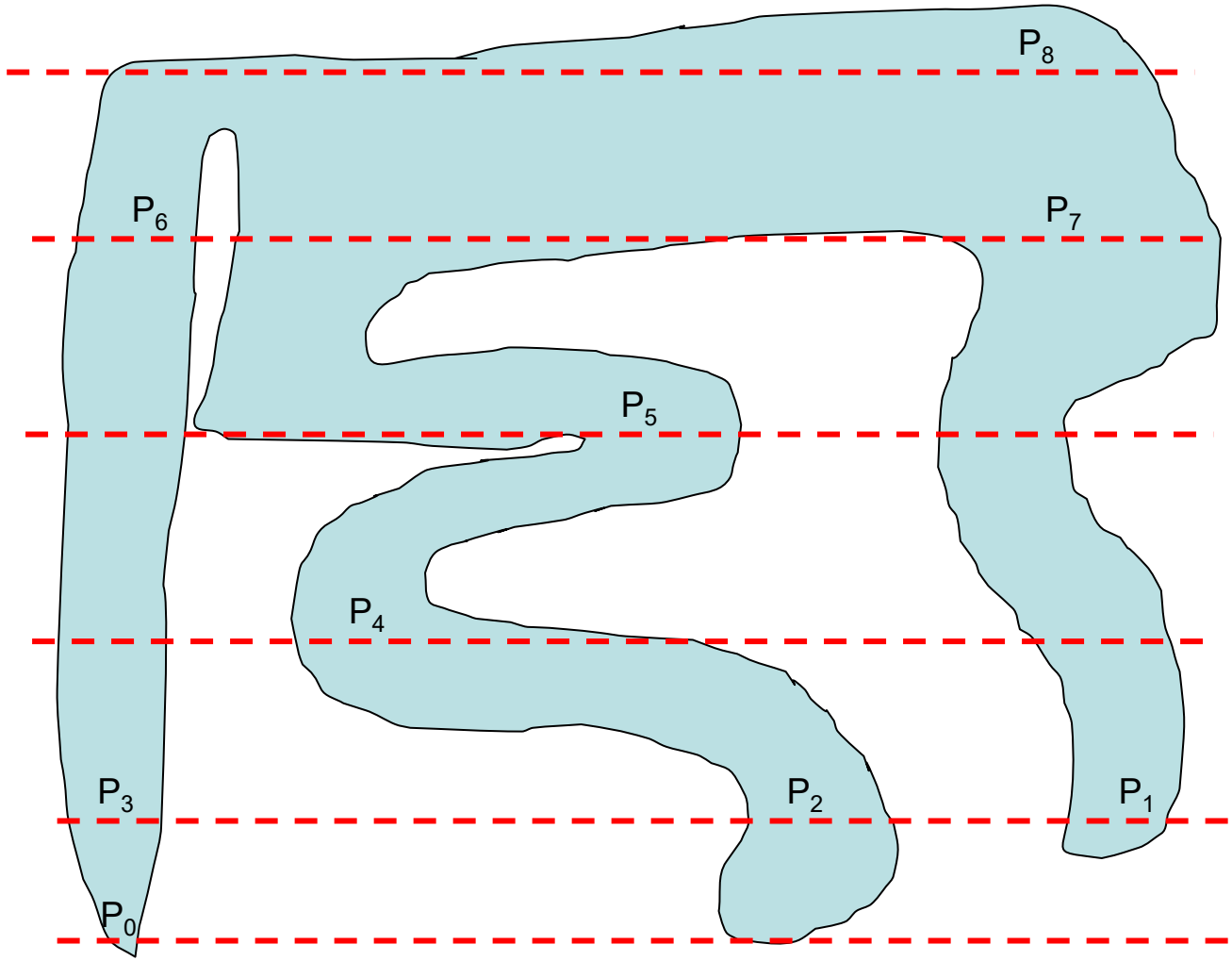
$\Delta P = P_2 - P_1 = \rho g h$

What if ρ isn't constant?

- Same equation, but shrunk to infinitesimal point
- $\Delta P = +\rho g h$
- h = z_1-z_2
  = -Δz
- $\Delta P = -\rho g \Delta z$
- Lim $\Delta z \rightarrow 0$



Pressure Intuition



Examples

Turns out there are aliens at the bottom of the marina trench (10,994 m)! We of course want to go say “hi”, Abyss-style, but need to make sure our sub can get there. What should the design pressure of our sub be?

$$\Delta P = \rho g h = (1000\ kg/m^3)(9.8\ m/s^2)(10{,}994\ m)$$

$$\Delta P = 1063\ atm$$

Example 2

- Develop and expression for the pressure in the atmosphere as a function of elevation:
  - P(z)
  - Where $P(0) = P_{atm}$.
  - Assume constant temperature T.
  - Use $dP/dz = -\rho g$
- Strategy
  - Go with with you know:
  - What do you expect to happen?
  - Draw a picture
  - What is an expression or law that relates the physics?
  - What additional information or assumptions are needed.
  - Talk to your neighbor ☺

Hint: Use $\rho = MP/RT$

$$\frac{dP}{dz} = -\rho g \Rightarrow \frac{dP}{dz} = -\frac{MP}{RT}g$$

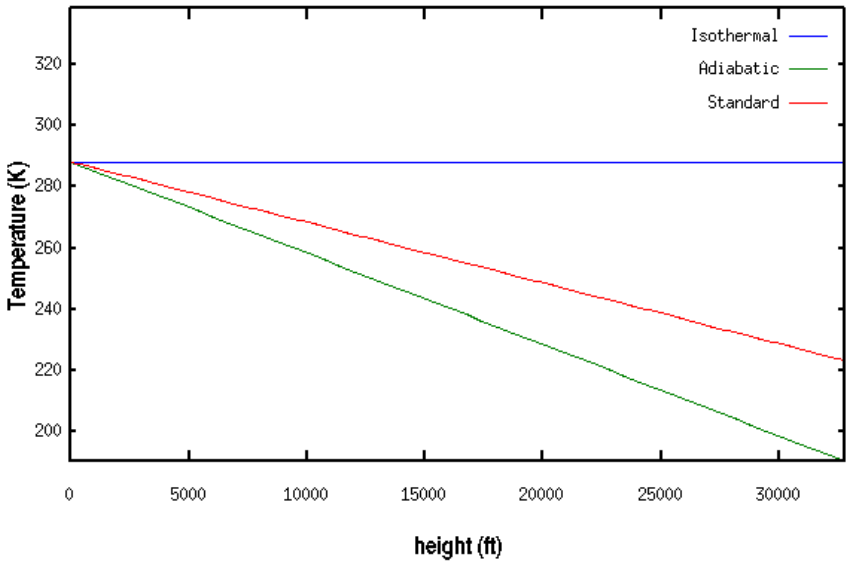
$M, R, T, g \rightarrow$ all constant

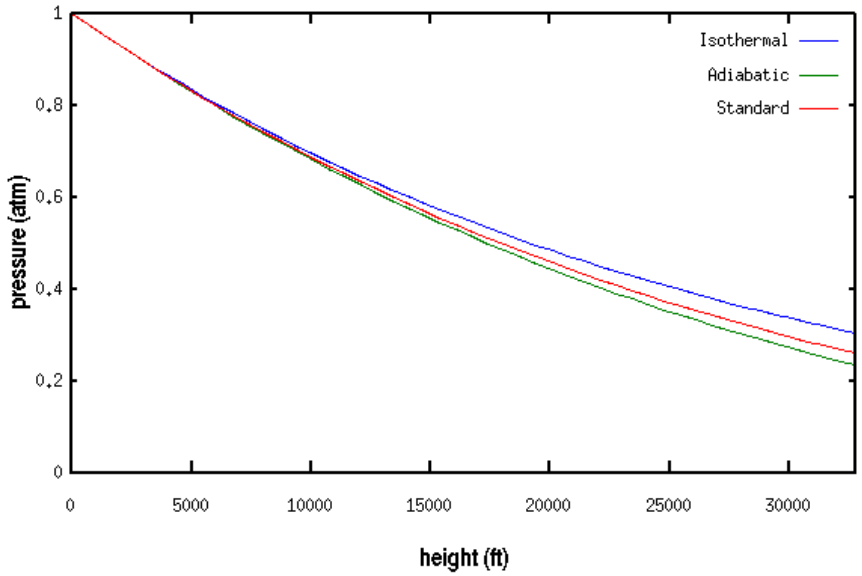
$$\therefore \frac{Mg}{RT} = \alpha \Rightarrow \frac{dP}{dz} = -\alpha P$$

$$\int_{P_{atm}}^{P} \frac{dP}{P} = -\alpha \int_{0}^{z} dz \Rightarrow \ln\frac{P}{P_{atm}} = -\alpha z \Rightarrow P = P_{atm} e^{(-\alpha z)}$$

$$\boxed{P = P_{atm} e^{\left(-\frac{Mg}{RT} z\right)}}$$

Temperature, Pressure in Atmosphere





Ht (ft)	P iso (atm)	P adia	P std
0.00e+00	1.000e+00	1.0000e+00	1.0000e+00
1.7268e+03	9.3941e-01	9.3888e-01	9.3906e-01
3.4537e+03	8.8250e-01	8.8048e-01	8.8117e-01
5.1805e+03	8.2903e-01	8.2472e-01	8.2620e-01
6.9074e+03	7.7880e-01	7.7153e-01	7.7405e-01
8.6342e+03	7.3161e-01	7.2084e-01	7.2459e-01
1.0361e+04	6.8729e-01	6.7257e-01	6.7773e-01
1.2088e+04	6.4565e-01	6.2665e-01	6.3336e-01
1.3815e+04	6.0653e-01	5.8302e-01	5.9137e-01
1.5542e+04	5.6978e-01	5.4160e-01	5.5166e-01
1.7268e+04	5.3526e-01	5.0233e-01	5.1414e-01
1.8995e+04	5.0283e-01	4.6514e-01	4.7872e-01
2.0722e+04	4.7236e-01	4.2995e-01	4.4530e-01
2.2449e+04	4.4374e-01	3.9671e-01	4.1380e-01
2.4176e+04	4.1686e-01	3.6535e-01	3.8413e-01
2.5903e+04	3.9160e-01	3.3580e-01	3.5621e-01
2.7629e+04	3.6788e-01	3.0800e-01	3.2995e-01
2.9356e+04	3.4559e-01	2.8188e-01	3.0529e-01
3.1083e+04	3.2465e-01	2.5738e-01	2.8214e-01
3.2810e+04	3.0498e-01	2.3444e-01	2.6044e-01